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ERRATUM: TOPOLOGICAL CENTRES OF CERTAIN BANACH MODULE ACTIONS

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For a normed space $\mathcal{X}$, let $J_{\mathcal{X}} : \mathcal{X} \to \mathcal{X}^{**}$ denote the canonical embedding of $\mathcal{X}$ into $\mathcal{X}^{**}$, with the second adjoint $(J_{\mathcal{X}})^{**} : \mathcal{X}^{**} \to \mathcal{X}^{****}$. We defined $\mathfrak{M}_{\mathcal{X}}$ (in Section 3 of the paper [1]) by

$$\mathfrak{M}_{\mathcal{X}} = \{x^{**} \in \mathcal{X}^{**} : J_{\mathcal{X}^{**}}(x^{**}) = (J_{\mathcal{X}})^{**}(x^{**})\}.$$

It is routine to verify that $\mathfrak{M}_{\mathcal{X}}$ is a closed subspace of $\mathcal{X}^{**}$ containing $\mathcal{X}$. We have claimed that

"*it would be desirable to characterize those $\mathcal{X}$ for which $\mathfrak{M}_{\mathcal{X}} = \mathcal{X}$".*

We have earlier surprisingly noticed that this is always true! Indeed: Let $x^{**} \in \mathfrak{M}_{\mathcal{X}}$ and let $\{x_\alpha\}$ be a bounded net in $\mathcal{X}$ such that $\{J_{\mathcal{X}}(x_\alpha)\}$ is w^*-convergent to x^{**}. Then, for each $x^{***} \in \mathcal{X}^{***}$,

$$\begin{aligned}
\lim_\alpha \langle x^{***}, J_{\mathcal{X}}(x_\alpha)\rangle &= \lim_\alpha \langle (J_{\mathcal{X}})^*(x^{***}), x_\alpha\rangle = \lim_\alpha \langle J_{\mathcal{X}}(x_\alpha), (J_{\mathcal{X}})^*(x^{***})\rangle \\
&= \langle x^{**}, (J_{\mathcal{X}})^*(x^{***})\rangle = \langle (J_{\mathcal{X}})^{**}(x^{**}), x^{***}\rangle \\
&= \langle J_{\mathcal{X}^{**}}(x^{**}), x^{***}\rangle \\
&= \langle x^{***}, x^{**}\rangle.
\end{aligned}$$

Therefore, $\{J_{\mathcal{X}}(x_\alpha)\}$ converges weakly to x^{**}. As $\mathcal{X}$ is a weakly closed subspace of $\mathcal{X}^{**}$, we get $x^{**} \in \mathcal{X}$, so that $\mathfrak{M}_{\mathcal{X}} = \mathcal{X}$, as claimed.

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By this erratum we acknowledge that Example 3.2 (consequently Corollary 3.3) of the paper, in which we have claimed that $\mathfrak{M}_{c_0} \neq c_0$, is not correct and its proof is flawed. The actual error occurs when we use the decomposition $\ell^{\infty *} = c^* \oplus c^{\perp}$. Indeed, the right decomposition is $\ell^{\infty *} = {c_0}^* \oplus {c_0}^{\perp} = \ell^1 \oplus {c_0}^{\perp}$, which is known and holds even in more general situations. It is now more desirable that the reader replace $\mathfrak{M}_{\mathcal{X}}$ with $\mathcal{X}$ throughout the paper.

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References

[1] S. Barookkoob, S. Mohammadzadeh and H. R. E. Vishki, Topological centres of certain Banach module actions, *Bull. Iranian Math. Soc.* **35** (2) (2009), 15–26.

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