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COMMON FIXED POINTS OF f-WEAK CONTRACTIONS IN CONE METRIC SPACES[†]

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Abstract. Recently, Choudhury and Metiya proved some fixed point theorems for weak contractions in cone metric spaces. Weak contractions are generalizations of Banach's contraction mapping, which have been studied by several authors. Here, we introduce the notion of f-weak contractions and also establish a coincidence and common fixed point result for f-weak contractions in cone metric spaces. Our result is supported by an example which includes and generalizes the results of Choudhury and Metiya's work.

1. **Introduction**

The Banach's fixed point theorem first appeared in explicit form in Banach's thesis in 1922 [6], where it was used to establish the existence of a solution for an integral equation. Banach's fixed point theorem plays an important role in several branches of mathematics. For instance, it has been used to show the existence of solutions of nonlinear Volterra integral equations, nonlinear integro-differential equations in Banach spaces, and to show the convergence of algorithms in computational mathematics. Because of its importance and usefulness for mathematical theory, its has become a very popular tool in solving existence

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problems in many branches of mathematical analysis and extended in many directions. Several authors have obtained various extensions and generalizations of Banach's theorem by considering contractive mappings on many different metric spaces.

The classical contraction mapping principle of Banach states that if (X, d) is a complete metric space and $T : X \to X$ is a contraction mapping (i.e., $d(Tx, Ty) \leq \alpha d(x, y)$, for all $x, y \in X$, where $0 \leq \alpha < 1$), then T has a unique fixed point. This principle has been generalized in different directions in different spaces by mathematicians over the years. Also, in contemporary research, it is still seriously investigated. The works noted in [4, 8, 10, 18, 20, 21, 23] are some examples.

In [3], Alber and Guerre-Delabriere introduced the concept of weak contraction in Hilbert spaces. Rhoades [19] showed that the result which Alber et al. had proved in [3] was also valid in complete metric spaces. We state the result of Rhoades as follows:

A mapping $T : X \to X$ is said to be weakly contractive if

$$d(Tx, Ty) \leq d(x, y) - \phi(d(x, y)), \tag{1.1}$$

for $x, y \in X$, where $\phi : [0, \infty) \to [0, \infty)$ is a continuous and nondecreasing function with $\phi(t) = 0$ if and only if $t = 0$.

If we take $\phi(t) = kt$, where $0 < k < 1$, then weak contraction reduces to contraction mapping.

Weak inequalities of the above type have been used to establish fixed point results in a number of subsequent works, some of which are noted in [7, 9, 12, 19, 22, 25].

Cone metric spaces are generalizations of metric spaces, where each pair of points is assigned to a member of a real Banach space with a cone. This cone naturally induces a partial order in the Banach space. The concept of cone metric space was introduced in the work of Huang and Zhang [13] where they also established the Banach's contraction mapping principle in such spaces. Afterwards, several authors have studied fixed point problems in cone metric spaces. Some of these works are noted in [1, 2, 5, 14, 15, 24]. Recently, Choudhury and Metiya [11] established a fixed point result for weak contractions in cone metric spaces. Here, we give the notion of f-weak contractions and establish a coincidence and common fixed point result for f-weak contractions in cone metric spaces. We also illustrate our result by an example.

2. **Preliminaries**

In this section, we shall give the notion of cone metric spaces and some related properties.

Definition 2.1 ([13]). *Let E always be a real Banach space and P a subset of E. P is called a* cone *if and only if*

(i) *P is closed, nonempty and $P \neq \{0\}$;*
(ii) *$a, b \in \mathbb{R}$, $a, b \geq 0$, $x, y \in P \Longrightarrow ax + by \in P$;*
(iii) *$x \in P$ and $-x \in P \Longrightarrow x = 0$.*

Given a cone $P \subset E$, define a partial ordering $\leq$ with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall write $x < y$ to indicate that $x \leq y$, but $x \neq y$, while $x \ll y$ will stand for $y - x \in \text{int } P$, with int P denoting the interior of P.

The cone P is called *normal* if there is a number $K > 0$ such that for all $x, y \in E$,

$$0 \leq x \leq y \Longrightarrow \|x\| \leq K\|y\|.$$

The least positive number satisfying the above inequality is called the normal constant of P.

The cone P is called *regular* if every increasing sequence bounded from above is convergent. That is, if $\{x_n\}$ is a sequence such that

$$x_1 \leq x_2 \leq ... \leq x_n \leq ... \leq y,$$

for some $y \in E$, then there is $x \in E$ such that $\|x_n - x\| \to 0$, as $n \to \infty$. Equivalently, the cone P is regular if and only if every decreasing sequence bounded from below is convergent. It is well known that a regular cone is a normal cone.

In the following, we always suppose E is a Banach space, P is a cone in E with int $P \neq \emptyset$ and $\leq$ is a partial ordering with respect to P.

Definition 2.2 ([13]). *Let X be a nonempty set. Suppose that the mapping $d : X \times X \to E$ satisfies*

(i) *$0 \leq d(x, y)$, for all $x, y \in X$, and $d(x, y) = 0$ if and only if $x = y$;*
(ii) *$d(x, y) = d(y, x)$, for all $x, y \in X$;*
(iii) *$d(x, y) \leq d(x, z) + d(z, y)$, for all $x, y, z \in X$.*

Then, d is called a cone metric *on X, and (X, d) is called a* cone metric space*.*

Definition 2.3 ([13]). *Let (X, d) be a cone metric space. Let $\{x_n\}$ be a sequence in X and $x \in X$. If for every $c \in E$ with $0 \ll c$ there exists*

$N \in \mathbb{N}$ such that for all $n > N$, $d(x_n, x) \ll c$, then $\{x_n\}$ is said to be convergent and $\{x_n\}$ converges to x, and x is the limit of $\{x_n\}$. We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x, 1$ as $n \to \infty$.

Lemma 2.4 ([13]). *Let (X, d) be a cone metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ be a sequence in X. Then, $\{x_n\}$ converges to x if and only if $d(x_n, x) \to 0$, as $n \to \infty$.*

Lemma 2.5 ([13]). *Let (X, d) be a cone metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ be a sequence in X. If $\{x_n\}$ converges to x and $\{x_n\}$ converges to y, then $x = y$. That is, the limit of $\{x_n\}$ is unique.*

Definition 2.6 ([13]). *Let (X, d) be a cone metric space and $\{x_n\}$ be a sequence in X. If for any $c \in E$ with $0 \ll c$, there exists $N \in \mathbb{N}$ such that for all $n, m > N$, $d(x_n, x_m) \ll c$, then $\{x_n\}$ is called a Cauchy sequence in X.*

Definition 2.7 ([13]). *Let (X, d) be a cone metric space and $\{x_n\}$ be a sequence in X. If every Cauchy sequence is convergent in X, then X is called a complete cone metric space.*

Lemma 2.8 ([13]). *Let (X, d) be a cone metric space and $\{x_n\}$ be a sequence in X. If $\{x_n\}$ converges to $x \in X$, then $\{x_n\}$ is a Cauchy sequence.*

Lemma 2.9 ([13]). *Let (X, d) be a cone metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ be a sequence in X. Then, $\{x_n\}$ is a Cauchy sequence if and only if $d(x_n, x_m) \to 0$, as $n, m \to \infty$.*

Lemma 2.10 ([13]). *Let (X, d) be a cone metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X and $x_n \to x$, $y_n \to y$, as $n \to \infty$. Then, $d(x_n, y_n) \to d(x, y)$, as $n \to \infty$.*

Lemma 2.11 ([15]). *If P is a normal cone in E, then*

(i) *if $0 \le x \le y$ and $a \ge 0$, where a is real number, then $0 \le ax \le ay$;*
(ii) *if $0 \le x_n \le y_n$, for $n \in \mathbb{N}$, and $x_n \to x$, $y_n \to y$, then $0 \le x \le y$.*

Lemma 2.12 ([17]). *If E is a real Banach space with cone P in E, then for $a, b, c \in E$,*

(i) *if $a \le b$ and $b \ll c$, then $a \ll c$;*
(ii) *if $a \ll b$ and $b \ll c$, then $a \ll c$.*

Definition 2.13 ([16]). *Let $(Y, \leq)$ be a partially ordered set. Then, a function $F : Y \to Y$ is said to be* monotone increasing *if it preserves ordering, i.e., given $x, y \in Y$, $x \leq y$ implies that $Fx \leq Fy$.*

Definition 2.14. *Let f and T be self mappings of a nonempty set X. If $w = fx = Tx$ for some $x \in X$, then x is called a coincidence point of f and T, and w is called a point of coincidence of f and T. If $w = x$, then x is called a common fixed point of f and T.*

3. Main results

We first introduce the notion of f-weak contraction mappings.

Definition 3.1. *Let (X, d) is a metric space and $f : X \to X$. A mapping $T : X \to X$ is an said to be f-weak contraction if*

$$d(Tx, Ty) \leq d(fx, fy) - \phi(d(fx, fy)), \tag{3.1}$$

for $x, y \in X$, where $\phi : [0, \infty) \to [0, \infty)$ is a continuous and nondecreasing function with $\phi(t) = 0$ if and only if $t = 0$.

Remark 3.2. *If f is an identity mapping, then (3.1) reduce to (1.1).*

Theorem 3.3. *Let (X,d) be a cone metric space with regular cone P such that $d(x, y) \in \text{int } P$, for $x, y \in X$ with $x \neq y$. Let $f : X \to X$ and $T : X \to X$ be a mapping satisfying the inequality*

$$d(Tx, Ty) \leq d(fx, fy) - \phi(d(fx, fy)), \tag{3.2}$$

for $x, y \in X$, where $\phi : \text{int } P \cup \{0\} \to \text{int } P \cup \{0\}$ is a continuous and monotone increasing function with

(i) *$\phi(t) = 0$ if and only if $t = 0$;*
(ii) *$\phi(t) \ll t$, for $t \in \text{int } P$;*
(iii) *either $\phi(t) \leq d(fx, fy)$ or $d(fx, fy) \leq \phi(t)$, for $t \in \text{int } P \cup \{0\}$ and $x, y \in X$.*

If $TX \subseteq fX$ and fX is a complete subspace of X, then f and T have a unique point of coincidence in X. Moreover, f and T have a unique common fixed point in X if $ffz = fz$ for the coincidence point z.

Proof. Let $x_0 \in X$. Since $TX \subseteq fX$, we construct the sequence $\{fx_n\}$, where $fx_n = Tx_{n-1}$, $n \geq 1$.

If $fx_{n+1} = fx_n$, for some n, then trivially f and T have a coincidence point in X.

If $fx_{n+1} \neq fx_n$, for $n \in \mathbb{N}$, by the given condition, we have

$$\begin{aligned} d(fx_n, fx_{n+1}) &= d(Tx_{n-1}, Tx_n) \\ &\leq d(fx_{n-1}, fx_n) - \phi(d(fx_{n-1}, fx_n)). \end{aligned}$$

By the property of ϕ, that is, $0 \leq \phi(t)$, for all $t \in int\, P \cup \{0\}$, we have

$$d(fx_n, f_x n + 1) \leq d(fx_{n-1}, fx_n).$$

It follows that the sequence $\{d(fx_n, fx_{n+1})\}$ is monotonically decreasing. Since cone P is regular and $0 \leq d(fx_n, fx_{n+1})$, for all $n \in \mathbb{N}$, there exists $r \geq 0$ such that

$$d(fx_n, fx_{n+1}) \to r \text{ as } n \to \infty.$$

Since ϕ is continuous and

$$d(fx_n, fx_{n+1}) \leq d(fx_{n-1}, fx_n) - \phi(d(fx_{n-1}, fx_n)),$$

by making $n \to \infty$, we get

$$r \leq r - \phi(r),$$

which is a contradiction, unless $r = 0$. Therefore, $d(fx_n, fx_{n+1}) \to 0$, as $n \to \infty$.

Let $c \in E$ with $0 \ll c$ be arbitrary. Since $d(fx_n, fx_{n+1}) \to 0$, as $n \to \infty$, there exists $m \in \mathbb{N}$ such that

$$d(fx_m, fx_{m+1}) \ll \phi(\phi(c/2)).$$

Let $B(fx_m, c) = \{fx \in X : d(fx_m, fx) \ll c\}$. Clearly, $fx_m \in B(fx_m, c)$. Therefore, $B(fx_m, c)$ is nonempty. Now, we will show that $Tx \in B(fx_m, c)$, for $fx \in B(fx_m, c)$.

Let $x \in B(fx_m, c)$. By property (iii) of ϕ, we have the following two possible cases.

Case (i): $d(fx, fx_m) \leq \phi(c/2)$, and

Case (ii): $\phi(c/2) < d(fx, fx_m) \ll c$.

We have:

Case (i):

$$\begin{aligned} d(Tx, fx_m) &\leq d(Tx, Tx_m) + d(Tx_m, fx_m) \\ &\leq d(fx, fx_m) - \phi(d(fx, fx_m)) + d(Tx_m, fx_m) \\ &\leq d(fx, fx_m) - \phi(d(fx, fx_m)) + d(fx_{m+1}, fx_m) \\ &\leq d(fx, fx_m) + d(fx_{m+1}, fx_m) \\ &\leq \phi(c/2) + d(fx_{m+1}, fx_m) \\ &\leq \phi(c/2) + \phi(\phi(c/2)) \\ &\ll \phi(c/2) + \phi(c/2) \\ &\ll c/2 + c/2 \\ &= c. \end{aligned}$$

Case (ii):

$$\begin{aligned} d(Tx, fx_m) &\leq d(Tx, Tx_m) + d(Tx_m, fx_m) \\ &\leq d(fx, fx_m) - \phi(d(fx, fx_m)) + d(Tx_m, fx_m) \\ &\leq d(fx, fx_m) - \phi(d(fx, fx_m)) + d(fx_{m+1}, fx_m) \\ &\leq d(fx, fx_m) - \phi(\phi(c/2)) + \phi(\phi(c/2)) \\ &\leq d(fx, fx_m) \\ &\ll c. \end{aligned}$$

Therefore, T is a self mapping of $B(fx_m, c)$. Since $fx_m \in B(fx_m, c)$ and $fx_n = Tx_{n-1}$, $n \geq 1$, it follows that $fx_n \in B(fx_m, c)$, for all $n \geq m$. Again, c is arbitrary. This establishes that $\{fx_n\}$ is a Cauchy sequence in fX. It follows from completeness of fX that $fx_n \to fx$, for some $x \in X$. Now, we observe that

$$\begin{aligned} d(fx_n, Tx) &= d(Tx_{n-1}, Tx) \\ &\leq d(fx_{n-1}, fx) - \phi(d(fx_{n-1}, fx)). \end{aligned}$$

By making $n \to \infty$, we have $d(fx, Tx) \leq 0$. Therefore, $d(fx, Tx) = 0$, that is, $fx = Tx$. Hence, x is a coincidence point of f and T.

For uniqueness of the coincidence point of f and T, let, if possible, $y \in X$ $(y \neq x)$ be another coincidence point of f and T.

We note that

$$\begin{aligned} d(fx, fy) &= d(Tx, Ty) \\ &\leq d(fx, fy) - \phi(d(fx, fy)). \end{aligned}$$

Hence, $\phi(d(fx, fy)) \leq 0$, which is a contradiction, by the property of ϕ. Therefore, f and T have a unique point of coincidence in X.

Let z be a coincidence point of f and T. It follows from $ffz = fz$ and z being a coincidence point of f and T that $ffz = fz = Tz$.

From inequality (3.2), we get

$$\begin{aligned} d(Tfz, Tz) &\leq d(ffz, fz) - \phi(d(ffz, fz)) \\ &= 0 - \phi(0) \\ &= 0. \end{aligned}$$

Therefore, $Tfz = Tz$, that is, $fz = ffz = Tfz$. Hence, fz is a common fixed point of f and T. The uniqueness of the common fixed point is easy to establish from (3.2). This completes the proof. □

If we take $f : X \to X$ in Theorem 3.3 as an identity mapping, then Theorem 3.3 reduces to Theorem 2.1 in [11].

Corollary 3.4. (Theorem 2.1 in [11]) *Let (X,d) be a complete cone metric space with regular cone P such that $d(x, y) \in \text{int } P$, for $x, y \in X$ with $x \neq y$. Let $T : X \to X$ be a mapping satisfying the inequality*

$$d(Tx, Ty) \leq d(x, y) - \phi(d(x, y)), \tag{3.3}$$

for $x, y \in X$, where $\phi : \text{int } P \cup \{0\} \to \text{int } P \cup \{0\}$ is a continuous and monotonically increasing function with

(i) *$\phi(t) = 0$ if and only if $t = 0$;*
(ii) *$\phi(t) \ll t$, for $t \in \text{int } P$;*
(iii) *either $\phi(t) \leq d(x, y)$ or $d(x, y) \leq \phi(t)$, for $t \in \text{int } P \cup \{0\}$ and $x, y \in X$.*

Then, T has a unique fixed point in X.

Remark 3.5. *Theorem 3.3 is a generalization of the common fixed point theorems of f-weakly contraction in metric space to cone metric space. Moreover, Theorem 3.3 is a generalization of Theorem 2.1 of Choudhury and Metiya* [11].

Next, we show that Theorem 3.3 generalizes Theorem 2.1 of Choudhury and Metiya [11] by the following example.

Example 3.6. *Let $X = [0, 1]$, $E = \mathbb{R} \times \mathbb{R}$, with usual norm, be a real Banach space, $P = \{(x, y) \in E : x, y \geq 0\}$ be a regular cone and the partial ordering $\leq$ with respect to the cone P be the usual partial ordering in E. Define $d : X \times X \to E$ as:*

$$d(x, y) = (|x - y|, |x - y|), \text{ for } x, y \in X.$$

Then, (X,d) is a complete cone metric space with $d(x,y) \in int\ P$, for $x, y \in X$ with $x \neq y$.

Let us define $\phi : int\ P \cup \{0\} \to int\ P \cup \{0\}$ as:

$$\phi(t) = \begin{cases} (t_1^2, t_1^2), & \text{for } t = (t_1, t_2) \in int\ P \cup \{0\} \text{ with } t_1 \leq t_2; \\ (t_2^2, t_2^2), & \text{for } t = (t_1, t_2) \in int\ P \cup \{0\} \text{ with } t_1 > t_2. \end{cases}$$

Clearly, ϕ has all the required properties. Let us define $f : X \to X$ and $T : X \to X$ as

$$fx = \frac{x}{2} \text{ and } Tx = \frac{x}{2} - \frac{x^2}{4}, \text{ for } x \in X.$$

Without loss of generality, take $x, y \in X$ with $x > y$.

Now, we observe that

$$\begin{aligned}
d(Tx,Ty) &= d\Big(\frac{x}{2} - \frac{x^2}{4}, \frac{y}{2} - \frac{y^2}{4}\Big) \\
&= \Big(\Big(\frac{x}{2} - \frac{x^2}{4}\Big) - \Big(\frac{y}{2} - \frac{y^2}{4}\Big), \Big(\frac{x}{2} - \frac{x^2}{4}\Big) - \Big(\frac{y}{2} - \frac{y^2}{4}\Big)\Big) \\
&= \Big(\frac{x-y}{2} - \frac{x^2-y^2}{4}, \frac{x-y}{2} - \frac{x^2-y^2}{4}\Big) \\
&= \Big(\frac{x-y}{2} - \frac{(x-y)(x+y)}{4}, \frac{x-y}{2} - \frac{(x-y)(x+y)}{4}\Big) \\
&\leq \Big(\frac{x-y}{2} - \frac{(x-y)(x-y)}{4}, \frac{x-y}{2} - \frac{(x-y)(x-y)}{4}\Big) \\
&= \Big(\frac{x-y}{2}, \frac{x-y}{2}\Big) - \Big(\frac{(x-y)(x-y)}{4}, \frac{(x-y)(x-y)}{4}\Big) \\
&= d(fx,fy) - \phi(d(fx,fy)).
\end{aligned}$$

Here, $0 \in X$ is the unique common fixed point of f and T.

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References

[1] M. Abbas and G. Jungck, Common fixed point results for noncommuting mappings without continuity in cone metric space, *J. Math. Anal. Appl.* **341** (2008), no. 1, 416–420.

[2] M. Abbas and B.E. Rhoades, Fixed and periodic point results in cone metric spaces, *Appl. Math. Lett.* **22** (2009), no. 4, 511–515.

[3] Ya. I. Alber and S. Guerre-Delabriere, Principles of weakly contractive maps in Hilbert spaces, in: I. Gohberg and Yu Lyubich *New Results in Operator Theory and it's Applications, in: Oper. Theory Adv. Appl.* Birkhäuser, Basel, 1997.

[4] A. D. Arvanitakis, A proof of the generalized Banach contraction conjecture, *Proc. Amer. Math. Soc.* **131** (2003), no. 12, 3647-3656.

[5] A. Azam and M. Arshad, Common fixed points of generalized contractive maps in cone metric space, *Bull. Iran. Math. Soc.* **35** (2009), no. 2, 225-264.

[6] S. Banach, Sur les opérations dans les ensembles abstraits et leurs applications aux équations intégrales, *Fund. Math.* **3** (1922) 133-181.

[7] I. Beg and M. Abbas, Coincidence point and invariant approximation for mappings satisfying generalized weak contractive condition, *Fixed Point Theory Appl.* (2006). Article ID 74503, 7 pages.

[8] D. W. Boyd and J. S. W. Wong, On nonlinear contractions, *Proc. Amer. Math. Soc.* **20** (1969) 458-464.

[9] C.E. Chidume, H. Zegeye and S.J. Aneke, Approximation of fixed points of weakly contractive non self maps in Banach spaces, *J. Math. Anal. Appl.* **270** (2002), no. 1, 189-199.

[10] B. S. Choudhury and K.P. Das, A new contraction principle in Menger spaces, *Acta Math. Sin.* **24** (2008), no. 8, 1379-1386.

[11] B. S. Choudhury and N. Metiya, Fixed points of weak contractions in cone metric spaces, *Nonlinear Analysis* **72** (2010), no. 3-4, 1589-1593.

[12] P. N. Dutta and B. S. Choudhury, A generalisation of contraction principle in metric spaces, *Fixed Point Theory Appl.* 2008 (2008). Art. ID 406368, 8 pp.

[13] L.-G. Huang and X. Zhang, Cone metric spaces and fixed point theorems of contractive mappings, *J. Math. Anal. Appl.* **332** (2007) 1468-1476.

[14] D. Ilić, V. Rakočević, Common fixed point for maps on cone metric space, *J. Math. Anal. Appl.* **341** (2008), no. 2, 876-882.

[15] D. Ilić, V. Rakočević, Quasi-contraction on a cone metric space, *Appl. Math. Lett.* **22** (2009) 728-731.

[16] J. Jachymski, Order-Theoretic Aspects of Metric Fixed Point Theory, *Handbook of Metric Fixed Point Theory*, Kluwer Academic Publishers, Dordrecht, 2001.

[17] G. Jungck, S. Radenović, S. Radojević, V. Rakočević, Common fixed point theorems for weakly compatible pairs on cone metric spaces, *Fixed Point Theory Appl.* (2009). Art. ID 643840, 13 pp.

[18] J. Merryfield, B. Rothschild and J. D. Stein Jr, An application of Ramsey's theoremto the Banach contraction principle, *Proc. Amer. Math. Soc.* **130** (2002), no. 4, 927-933.

[19] B. E. Rhoades, Some theorems on weakly contractive maps, *Nonlinear Anal. TMA* **47** (2001), no. 4, 2683-2693.

[20] W. Sintunavart, P. Kumam, Coincidence and common fixed points for hybrid strict contractions without the weakly commuting condition, *Appl. Math. Lett.* **22** (2009), no. 12, 1877-1881.

[21] W. Sintunavarat, P. Kumam, P. Patthanangkoor, Common random fixed points for multivalued random operators without S- and T-weakly commuting random operators, *Random Operators Stochastic Eqs.* **17** (2009) 381-388.

[22] Y. Song, Coincidence points for noncommuting f -weakly contractive mappings, *Int. J. Comput. Appl. Math.* **2** (2007), no. 1, 51-57.

[23] T. Suzuki, A generalized Banach contraction principle that characterizes metric completeness, *Proc. Amer. Math. Soc.* **136** (2008), no. 5, 1861-1869.

[24] D. Wardowski, Endpoints and fixed points of set-valued contractions in cone metric spaces, *Nonlinear Anal.* **71** (2009), no. 1-2, 512-516.

[25] Q. Zhang and Y. Song, Fixed point theory for generalized ϕ-weak contractions, *Appl. Math. Lett.* **22** (2009), no. 1, 75-78.

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