Bulletin of the Iranian Mathematical Society Vol. 38 *No. 1* (*2012*), *pp 159-168.*

ON HEYTING ALGEBRAS AND DUAL BCK-ALGEBRAS

Y. H. YON AND K. H. KIM*

Communicated by Ali Enayat

Abstract. A Heyting algebra is a distributive lattice with implication and a dual BCK-algebra is an algebraic system having as models logical systems equipped with implication. The aim of this paper is to investigate the relation of Heyting algebras between dual BCK-algebras. We define notions of i-invariant and m-invariant on dual BCK-semilattices and prove that a Heyting semilattice is equivalent to an i-invariant and m-invariant dual BCK-semilattices, and show that a commutative Heyting algebra is equivalent to a bounded implicative dual BCK-algebra.

1. **Introduction**

A Heyting semilattice is an algebraic system equipped with implication and conjunction. The prepositions of Heyting semilattices in algebraic logic were clearly displayed by H. B. Curry([3]) and systematically studied in [7] and [8]. A dual BCK-algebra($DBCK$-algebra) is an algebraic system having as models logical systems equipped with implication, which is the dual concept of BCK-algebra [4, 5], and it is a generalization of Heyting algebra. Heyting algebras(or Brouwerian lattices) were investigated by H. B. Curry[3] and G. Birkhoff [1], and all the important rules of computation with implication are contained in [3]. The notion of $DBCK$-algebra was studied and generalized in

MSC(2010): Primary: 06D20; Secondary: 06A99, 06F35
Keywords: Heyting semilattice, Heyting algebra, dual BCK-algebra.
Received: 30 April 2009, Accepted: 21 June 2010
*Corresponding author

[2, 6, 11], and more relationships among Heyting semilattice, Hilbert algebra, L-algebra and $DBCK$-algebra can be found in [9, 10].

In this paper, we define notions of i-invariant and m-invariant on $DBCK$-semilattices and investigate the relation between Heyting algebras and $DBCK$-algebras. We prove that a Heyting semilattice is equivalent to an i-invariant and m-invariant $DBCK$-semilattices, and show that a commutative Heyting algebra is equivalent to a bounded implicative $DBCK$-algebra.

2. Preliminaries

A *$DBCK$-algebra* is an algebraic system $(X, \circ, 1)$ satisfying the following axioms.

DBCK1. $(x \circ y) \circ ((y \circ z) \circ (x \circ z)) = 1$,

DBCK2 $x \circ ((x \circ y) \circ y) = 1$,

DBCK3. $x \circ x = 1$,

DBCK4. $x \circ y = 1$ and $y \circ x = 1$ imply $x = y$,

DBCK5. $x \circ 1 = 1$.

A $DBCK$-algebra is a poset with the binary relation "$\leq$" defined by $x \leq y$ if and only if $x \circ y = 1$, and 1 is the greatest element.

A *Heyting semilattice* (or *implicative semilattice*) is a (meet-)semilattice with a binary operation "$\circ$" satisfying the axiom :

H. $z \wedge x \leq y$ if and only if $z \leq x \circ y$.

Proposition 2.1. [5, 6, 7, 8] *A Heyting semilattice and $DBCK$-algebra have the following common properties.*

(CP1) $x \circ (y \circ z) = y \circ (x \circ z)$,

(CP2) $y \leq x \circ y$

(CP3) $x \leq y$ *implies* $z \circ x \leq z \circ y$ *and* $y \circ z \leq x \circ z$,

(CP4) $x \leq y \circ z$ *implies* $y \leq x \circ z$,

(CP5) $1 \circ x = x$.

Proposition 2.2. [5, 6] *A $DBCK$-algebra has the following properties.*

(DP1) $x \circ y \leq (y \circ z) \circ (x \circ z)$,

(DP2) $x \leq (x \circ y) \circ y$,

(DP3) $x \circ y \leq (z \circ x) \circ (z \circ y)$,

(DP4) $(x \circ y) \circ y) \circ y = x \circ y$.

In a $DBCK$-algebra, $(x \circ y) \circ y$ is an upper bound of x and y by (DP2) and (CP2).

Proposition 2.3. [7, 8] *A Heyting semilattice has a greatest element* 1 *and has the following properties.*

(HP1) $a \leq b$ *if and only if* $a \circ b = 1$,
(HP2) $x \circ x = 1$,
(HP3) $x \wedge (x \circ y) = x \wedge y$,
(HP4) $x \circ (y \wedge z) = (x \circ y) \wedge (x \circ z)$,
(HP5) $x \circ (y \circ z) = (x \wedge y) \circ z$.

A $DBCK$-algebra $(X, \circ, 1)$ is said to be *bounded* if there exists an element 0 in X such that $0 \circ x = 1$ for all $x \in X$. For any element x in a bounded $DBCK$-algebra X, the element $x \circ 0$ will be denoted by x^* and $x^{**} = (x^*)^*$.

Proposition 2.4. [6] *A bounded* $DBCK$*-algebra has the following properties.*

(1) $1^* = 0$ *and* $0^* = 1$,
(2) $x \leq x^{**}$ *and* $x^{***} = x^*$,
(3) $x \circ y \ \leq \ y^* \circ x^*$,
(4) $x \leq y$ *implies* $y^* \leq x^*$,
(5) $x \circ y^* = y \circ x^*$.

A $DBCK$-algebra is said to be *commutative* if it satisfies $(x \circ y) \circ y = (y \circ x) \circ x$ for every $x, y \in X$.

Proposition 2.5. [6] *A bounded commutative* $DBCK$*-algebra* X *has the following properties.*

(1) X *is a lattice with* $x \vee y = (x \circ y) \circ y$ *and* $x \wedge y = (x^* \vee y^*)^*$,
(2) $x = x^{**}$,
(3) $x \circ y = y^* \circ x^*$.

3. Heyting semilattices and $DBCK$-semilattices

Definition 3.1. *A* $DBCK$*-algebra* X *is called a* $DBCK$*-semilattice if every finite subset of* X *has the greatest lower bound. A* $DBCK$*-semilattice* X *is said to be implication-invariant, shortly i-invariant, if* $x \wedge y = x \wedge (x \circ y)$ *for all* $x, y \in X$, *and meet-invariant, shortly m-invariant, if* $x \circ y = x \circ (x \wedge y)$ *for all* $x, y \in X$.

Those axioms of the i-invariant and the m-invariant $DBCK$-semilattice are independent, as the following examples show.

Example 3.2. *(1) Let* $N_5 = \{0, a, b, c, 1\}$ *be a* $DBCK$*-semilattice with a binary operation "*$\circ$*" and Hasse diagram given by Figure 1. Then* N_5

$\circ$	0	a	b	c	1
0	1	1	1	1	1
a	b	1	b	c	1
b	a	a	1	1	1
c	0	a	b	1	1
1	0	a	b	c	1

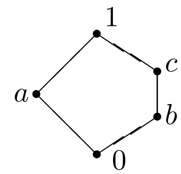

FIGURE 1. Cayley table and Hasse diagram of $DBCK$-semilattice N_5

is i-invariant but not m-invariant, in fact $a \circ (a \wedge c) = b \neq c = a \circ c$.

(2) Let $X = \{0, a, b, 1\}$ *be a* $DBCK$*-semilattice with a binary operation "*$\circ$*" and Hasse diagram given by Figure 2. Then* X *is m-invariant*

$\circ$	0	a	b	1
0	1	1	1	1
a	b	1	1	1
b	a	b	1	1
1	0	a	b	1

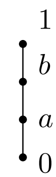

FIGURE 2. Cayley table and Hasse diagram of $DBCK$-semilattice X

but not i-invariant for $a \wedge 0 = 0 \neq a = a \wedge (a \circ 0)$.

Theorem 3.3. *Every Heyting semilattice is an i-invariant* $DBCK$*-semilattice.*

Proof. Suppose that X is a Heyting semilattice. Then DBCK3, DBCK4 and DBCK5 are trivial from (HP2) and (HP1). For any $x, y \in X$, $x \circ ((x \circ y) \circ y) = (x \circ y) \circ (x \circ y) = 1$ by (CP1). It is DBCK2. By (HP1) and (CP3), it implies that $y \leq (y \circ z) \circ z$ and $x \circ y \leq x \circ ((y \circ z) \circ z) = (y \circ z) \circ (x \circ z)$ for every $x, y, z \in X$, hence $(x \circ y) \circ ((y \circ z) \circ (x \circ z)) = 1$. Thus X is a $DBCK$-semilattice. Also, we have $x \wedge y \leq x \wedge (x \circ y)$ by (CP2). By axiom H, $x \wedge (x \circ y) \leq y$ since $x \circ y \leq x \circ y$. It implies $x \wedge (x \circ y) \leq x \wedge y$. Hence $x \wedge y = x \wedge (x \circ y)$ and X is i-invariant. $\square$

We have two types of distributive law on $DBCK$-semilattices with respect to "$\circ$" and "$\wedge$" respectively :

$$x \circ (y \circ z) = (x \circ y) \circ (x \circ z) \quad \text{(self-distributive)},$$
$$x \circ (y \wedge z) = (x \circ y) \wedge (x \circ z) \quad \text{(meet-distributive)}.$$

In $DBCK$-semilattice the following inequalities are true.

$$x \circ (y \circ z) \geq (x \circ y) \circ (x \circ z) \text{ and } x \circ (y \wedge z) \leq (x \circ y) \wedge (x \circ z).$$

Proposition 3.4. *Let X be a $DBCK$-semilattice. Then*

(1) *If X satisfies the self-distributive law, then X is i-invariant,*
(2) *If X satisfies the meet-distributive law, then X is m-invariant.*

Proof. (1) Suppose that X satisfies the self-distributive law and $x, y \in X$. Let $u = x \wedge (x \circ y)$. Then it is clear that $x \wedge y \leq x \wedge (x \circ y) = u$ since $y \leq x \circ y$. Also $u \leq x$ and $u \leq x \circ y$. It imply that $u \circ x = 1$ and

$$1 \circ (u \circ y) = (u \circ x) \circ (u \circ y) = u \circ (x \circ y) = 1$$

by hypothesis. It follows that $u \circ y = 1$ and $u \leq y$. Hence u is a lower bound of x and y, i.e., $u \leq x \wedge y$. Therefore $x \wedge (x \circ y) = u = x \wedge y$.

(2) If X satisfies meet-distributive, then $x \circ (x \wedge y) = (x \circ x) \wedge (x \circ y) = 1 \wedge (x \circ y) = x \circ y$ for any $x, y \in X$, □

Proposition 3.5. *Let X be a $DBCK$-semilattice. Then X is i-invariant if and only if it satisfies $x \wedge (x \circ y) \leq y$ for all $x, y \in X$.*

Proof. Suppose that X is i-invariant. Then $x \wedge (x \circ y) = x \wedge y \leq y$. Conversely, suppose that $x \wedge (x \circ y) \leq y$ for all $x, y \in X$. Then it is clear that $x \wedge (x \circ y)$ is a lower bound of x and y. It implies $x \wedge (x \circ y) \leq x \wedge y$. Also $x \wedge y \leq x \wedge (x \circ y)$ since $y \leq x \circ y$. Hence $x \wedge (x \circ y) = x \wedge y$. □

Theorem 3.6. *Let X be a $DBCK$-semilattice. Then the following are equivalent.*

(1) *X is i-invariant and m-invariant.*
(2) *X satisfies $x \circ (y \circ z) = (x \wedge y) \circ z$ for all $x, y, z \in X$.*
(3) *X is a Heyting semilattice.*

Proof. ((1)⇒(2)) Suppose that X is i-invariant, m-invariant and $x, y, z \in X$. Let $u = x \circ (y \circ z)$. Then by definition of i-invariant, we have

$$\begin{aligned}(x \wedge y) \wedge u &= y \wedge [x \wedge (x \circ (y \circ z))] = y \wedge [x \wedge (y \circ z)] \\ &= x \wedge [y \wedge (y \circ z)] = x \wedge (y \wedge z) = (x \wedge y) \wedge z.\end{aligned}$$

It implies that by (CP2) and definition of m-invariant,

$$u \leq (x \wedge y) \circ u = (x \wedge y) \circ ((x \wedge y) \wedge u) = (x \wedge y) \circ ((x \wedge y) \wedge z) \leq (x \wedge y) \circ z.$$

Hence $x \circ (y \circ z) \leq (x \wedge y) \circ z$. To show that $(x \wedge y) \circ z \leq x \circ (y \circ z)$, let $v = (x \wedge y) \circ z$. Then by definition of i-invariant,

$$(x \wedge y) \wedge v = (x \wedge y) \wedge [(x \wedge y) \circ z] = (x \wedge y) \wedge z$$

and by definition of m-invariant,

$$\begin{aligned} x \wedge v \leq y \circ (x \wedge v) &= y \circ (y \wedge (x \wedge v)) = y \circ ((x \wedge y) \wedge v) \\ &= y \circ ((x \wedge y) \wedge z) \leq y \circ z. \end{aligned}$$

It implies $v \leq x \circ v = x \circ (x \wedge v) \leq x \circ (y \circ z)$. Hence $(x \wedge y) \circ z = x \circ (y \circ z)$.

(2)⇒(3) Suppose that X satisfies $x \circ (y \circ z) = (x \wedge y) \circ z$ for all $x, y, z \in X$. Then we have

$$x \wedge y \leq z \iff (x \wedge y) \circ z = 1 \iff x \circ (y \circ z) = 1 \iff x \leq y \circ z.$$

Hence X is a Heyting semilattice.

(3)⇒(1) Suppose that $(X, \circ, \wedge, 1)$ is a Heyting semilattice. Then it is an i-invariant $DBCK$-semilattice by Theorem 3.3, and it is m-invariant by (HP4) and Proposition 3.4(2). □

Proposition 3.7. *Let X be a $DBCK$-semilattice. Then the following properties are equivalent.*

(1) *X is i-invariant and m-invariant.*
(2) *X satisfies the self-distributive and the meet-distributive law.*

Proof. It is clear that (2) implies (1) by (1) and (2) of Proposition 3.4.

Conversely, suppose that X is i-invariant, m-invariant and $x, y, z \in X$. Then by Theorem 3.6(2) and definition of i-invariant,

$$\begin{aligned} x \circ (y \circ z) &= (x \wedge y) \circ z = [x \wedge (x \circ y)] \circ z \\ &= [(x \circ y) \wedge x] \circ z = (x \circ y) \circ (x \circ z). \end{aligned}$$

Hence X satisfies the self-distributive law. Also, X is a Heyting semilattice by Theorem 3.6. Hence X satisfies (HP4), i.e., X satisfies the meet-distributive law. □

Corollary 3.8. *A semilattice X is a Heyting semilattice if and only if it is a $DBCK$-semilattice satisfying the self-distributive and the meet-distributive law.*

Proof. It is clear from Theorem 3.6 and Proposition 3.7. □

A *filter* of a $DBCK$-algebra X is a non-empty subset F of X satisfying (1) $1 \in F$, and (2) $x \in F$ and $x \circ y \in F$ implies $y \in F$. A *filter* of a semilattice X is a non-empty subset F of X satisfying (1) $x \wedge y \in F$ for all $x, y \in F$, and (2) $x \in F$ and $x \leq y$ implies $y \in F$.

Proposition 3.9. *If X is an i-invariant $DBCK$-semilattice, then every filter of X as a semilattice is a filter of X as a $DBCK$-algebra.*

Proof. Suppose that F is a filter of X as a semilattice. Then it is clear that $1 \in F$ since $F \neq \emptyset$ and $1 \in X$. Let $x \in F$ and $x \circ y \in F$. Then $x \wedge y = x \wedge (x \circ y) \in F$ and $x \wedge y \leq y$. Hence $y \in F$. □

Proposition 3.10. *If X is a m-invariant $DBCK$-semilattice, then every filter of X as a $DBCK$-algebra is a filter of X as a semilattice.*

Proof. Let F be a filter of X as a $DBCK$-algebra and $x, y \in F$. Since $y \leq x \circ y$, $y \circ (x \circ y) = 1 \in F$ and $y \in F$, hence $x \circ y \in F$. Also, since $x \circ (x \wedge y) = x \circ y \in F$ and $x \in F$, $x \wedge y \in F$. If $x \in F$ and $x \leq y$, then $x \in F$ and $x \circ y = 1 \in F$, hence $y \in F$. Hence F is a filter of X as a semilattice. □

Corollary 3.11. *Let X be an i-invariant and m-invariant $DBCK$-semilattice. Then F is a filter of X as $DBCK$-algebra if and only if it is a filter of X as a semilattice.*

4. On Implicative $DBCK$-algebras

A bounded lattice $(X, \vee, \wedge, 0, 1)$ is called a *Heyting algebra* if there is a binary operation "$\circ$" on X satisfying the axiom H. Every Heyting algebra is a Heyting semilattice and satisfies all properties of Proposition 2.1 and 2.3. Conversely, every bounded Heyting semilattice X with $x \vee y$ for all $x, y \in X$ is a Heyting algebra.

Definition 4.1. *A $DBCK$-algebra X is said to be implicative if it satisfies $x = (x \circ y) \circ x$ for all $x, y \in X$.*

Definition 4.2. *A Heyting algebra is said to be commutative if it satisfies $(x \circ y) \circ y = (y \circ x) \circ x$ for every $x, y \in X$.*

Proposition 4.3. *Let X be a Heyting algebra. Then X is commutative if and only if it satisfies $x = (x \circ y) \circ x$ for all $x, y \in X$.*

Proof. Suppose that X is commutative and $x, y \in X$. Then it is clear that $x \leq (x \circ y) \circ x$ by (CP2), and by commutativity and (HP5),

$$\begin{aligned}[(x \circ y) \circ x] \circ x &= [x \circ (x \circ y)] \circ (x \circ y) = [(x \wedge x) \circ y] \circ (x \circ y) \\ &= (x \circ y) \circ (x \circ y) = 1\end{aligned}$$

It follows $(x \circ y) \circ x \leq x$. Hence $x = (x \circ y) \circ x$.

Conversely, Suppose that X satisfies $x = (x \circ y) \circ x$ for all $x, y \in X$. Then $y = (y \circ x) \circ y$. Since $x \circ y \leq x \circ y$, $x \leq (x \circ y) \circ y$ by (CP4). It follows that

$$(y \circ x) \circ x \leq (y \circ x) \circ ((x \circ y) \circ y) = (x \circ y) \circ ((y \circ x) \circ y) = (x \circ y) \circ y$$

by (CP1). Interchanging the role of x and y, we have $(x\circ y)\circ y \le (y\circ x)\circ x$. Hence $(x \circ y) \circ y = (y \circ x) \circ x$, and X is commutative. $\square$

Proposition 4.4. *Let X be a bounded implicative $DBCK$-algebra. Then it has the following properties.*

(1) X is commutative,

(2) $x = x^* \circ x$ for every $x \in X$,

(3) $x \vee y = y \vee x = x^* \circ y$ for every $x, y \in X$.

Proof. (1) We can prove it by the same way with the converse part of Proposition 4.3.

(2) If X is a bounded implicative $DBCK$-algebra, then $x^* \circ x = (x \circ 0) \circ x = x$ for any $x \in X$.

(3) Let X is a bounded implicative $DBCK$-algebra and $x, y \in X$. Then $0 \le y$ and $x \circ 0 \le x \circ y$ by (CP3). It implies $x \le (x \circ y) \circ y \le (x \circ 0) \circ y = x^* \circ y$ by (DP2) and (CP3). Since $y \le x^* \circ y$ by (CP2), $x^* \circ y$ is an upper bound of x and y. Hence $x \vee y \le x^* \circ y$. Also, by (DP1) and (2) of this proposition, we have that

$$x^* \circ y \le (y \circ x) \circ (x^* \circ x) = (y \circ x) \circ x.$$

Since X is commutative by (1) of this proposition, $(y \circ x) \circ x = y \vee x$ by Proposition 2.5(1), and it implies $x^* \circ y \le y \vee x$. Hence $y \vee x = x^* \circ y$. $\square$

If X is a bounded implicative $DBCK$-algebra, then it is a $DBCK$-semilattice, hence we can consider the notions of i-invariant and m-invariant of X.

Theorem 4.5. *If X is a bounded implicative $DBCK$-algebra, then it is i-invariant and m-invariant.*

Proof. Suppose that X is a bounded implicative $DBCK$-algebra and $x, y \in X$. Then X is commutative by Proposition 4.4(1) and we have

$$\begin{aligned} x^* \vee (x \circ y)^* &= x^{**} \circ (x \circ y)^* \quad \text{(by Proposition 4.4(3))} \\ &= x \circ (x \circ y)^* \quad \text{(by Proposition 2.5(2))} \\ &= (x \circ y) \circ x^* \quad \text{(by Proposition 2.4(5))} \\ &= (y^* \circ x^*) \circ x^* \quad \text{(by Proposition 2.5(3))} \\ &= y^* \vee x^* \quad \text{(by Proposition 2.5(1)).} \end{aligned}$$

Hence $x \wedge (x \circ y) = (x^* \vee (x \circ y)^*)^* = (x^* \vee y^*)^* = x \wedge y$ by Proposition 2.5(1) and X is i-invariant. Also we have that

$$\begin{aligned} x \circ (x \wedge y) &= x \circ (x^* \vee y^*)^* \quad \text{(by Proposition 2.5(1))} \\ &= (x^* \vee y^*) \circ x^* \quad \text{(by Proposition 2.4(5))} \\ &= (y \circ x^*) \circ x^* \quad \text{(by Proposition 4.4(3) and 2.5(2))} \\ &= y \vee x^* \quad \text{(by Proposition 2.5(1))} \\ &= y^* \circ x^* \quad \text{(by Proposition 4.4(3))} \\ &= x \circ y \quad \text{(by Proposition 2.5(3))} \end{aligned}$$

Hence X is m-invariant. □

Corollary 4.6. *If X is a bounded implicative $DBCK$-algebra, then it is a Heyting algebra.*

Proof. If X is a bounded implicative $DBCK$-algebra, then X is a bounded lattice, and it is Heyting algebra by Theorem 4.5 and 3.6. □

The converse of Corollary 4.6 is not true in general, as the following example shows.

Example 4.7. *Let X be a bounded chain with $|X| \geq 3$. We define a binary operation "$\circ$" on X by*

$$x \circ y = \begin{cases} 1 & \text{if } x \leq y \\ y, & \text{otherwise.} \end{cases}$$

Then X is a Heyting algebra which is not implicative $DBCK$-algebra. In fact, for any element $x \in X$ with $0 < x < 1$, $(x \circ 0) \circ x = 0 \circ x = 1 \neq x$.

Theorem 4.8. *A semilattice X is a commutative Heyting algebra if and only if it is a bounded implicative $DBCK$-algebra.*

Proof. If X is a commutative Heyting algebra, then X is an implicative $DBCK$-algebra by Proposition 4.3 and Theorem 3.3.

Conversely, if $(X, \circ, 0, 1)$ is a bounded implicative $DBCK$-algebra, then X is commutative Heyting algebra by Proposition 4.4(1) and Corollary 4.6. □

Acknowledgments

This research was supported by a grant from the Academic Research Program of Chungju National University in 2010.

References

[1] G. Birkhoff, Lattice Theory, American Mathematical Society Colloquium Publications, Providence, R.I., 1967.

[2] R. A. Borzooei and S. Khosravi Shoar, Implication algebras are equivalent to the dual implicative BCK-algebras, *Sci. Math. Jpn.* **63** (2006), no. 3, 429–431.

[3] H. B. Curry, Foundations of Mathematics Logics, McGraw-Hill, New York, 1963.

[4] K. Iséki, An algebra related with a propositional calculus, *Proc. Japan Acad. Ser. A Math. Sci.* **42** (1966) 26–29.

[5] K. Iséki and S. Tanaka, An introduction to the theory of BCK-algebras, *Mathematica Japonica* **23** (1978) 1–26.

[6] K. H. Kim and Y. H. Yon, Dual BCK-algebra and MV-algebra, *Sci. Math. Jpn.* **66** (2007), no. 2, 247–253.

[7] W. C. Nemitz, Implicative semi-lattices, *Trans. Amer. Math. Soc.* **117** (1965) 128–142.

[8] J. Picado, A. Pultr and A. Tozzi, Ideals in Heyting semilattices and open homomorphisms, *Quaest. Math.* **30** (2007), no. 4, 391–405.

[9] W. Rump, Semidirect products in algebraic logic and solutions of the quantum Yang-Baxter equation, *J. Algebra Appl.* **7** (2008), no.4, 471–490.

[10] W. Rump, A general Glivenko theorem, *Algebra Universalis* **61** (2009), no. 3-4, 455–473.

[11] A. Walendziak, On commutative BE-algebras, *Sci. Math. Jpn.* **69** (2009), no. 2, 281–284.

Yong Ho Yon
Engineering Education Innovation Center, Mokwon University,
P.O. Box 302-729, Daejeon, Korea
Email: yhyon@mokwon.ac.kr

Kyung Ho Kim
Department of Mathematics, Chungju National University,
P.O. Box 380-702, Chungju, Korea
Email: ghkim@cjnu.ac.kr