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ON PAIRWISE WEAKLY LINDELÖF BITOPOLOGICAL SPACES

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Abstract. In the present paper we introduce and study the notion of pairwise weakly Lindelöf bitopological spaces and obtain some results. Further, we also study the pairwise weakly Lindelöf subspaces and subsets, and investigate some of their properties. It is proved that a pairwise weakly Lindelöf property is not a hereditary property.

1. **Introduction and Preliminaries**

In the literature there are several generalizations of the notion of Lindelöf spaces and these are studied separately for different reasons and purposes. In 1982, Balasubramaniam [1] introduced and studied the notion of nearly Lindelöf spaces. Then in 1996, Cammaroto and Santoro [3] studied and gave further new results about these spaces which are considered as one of the main generalizations of Lindelöf spaces. Recently the authors introduced and studied the notion of pairwise Lindelöf spaces [10] and pairwise nearly Lindelöf spaces [16] and pairwise weakly regular-Lindelöf spaces [13] as well as pairwise almost Lindelöf spaces in bitopological setting, see [11] and extended some results due to Balasubramaniam [1] and Cammaroto and Santoro [3].

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Our purpose in this paper is to study the decompositions of pairwise continuity, openness and closedness functions and its generalizations and mappings on pairwise nearly Lindelöf spaces in a suitable way of bitopological spaces after the method of Fawakhreh and Kılıçman [5]. We extend most of their results in topological spaces to bitopological spaces.

The concepts of continuous functions and its generalizations have been introduced and studied in topological spaces. In [12, 14], the authors studied the pairwise Lindelöfness and pairwise continuity. They also introduced and studied the pairwise almost regular-Lindelöf bitopological spaces, their subspaces and subsets, and investigated some of their characterizations (see [15]).

The Lindelöf spaces are related to the compact spaces, therefore, in the literature there are several generalizations of the notion of Lindelöf spaces. One of these generalizations is known as weakly Lindelof which was introduced and studied by Frolik [6].

The purpose of this paper is to define the notion of weakly Lindelöf spaces in bitopological spaces, which we call pairwise weakly Lindelöf spaces and investigate some of their properties. Moreover, we study the pairwise weakly Lindelöf subspaces and subsets and also investigate some of their properties.

Throughout this paper, all spaces (X, τ) and (X, τ_1, τ_2) (or simply X) are always assumed to be topological spaces and bitopological spaces, respectively unless explicitly stated. By i-open set, we mean the open set with respect to a topology τ_i in X. We always use ij- to indicate certain properties with respect to topology τ_i and τ_j respectively, where $i, j \in \{1, 2\}$ and $i \neq j$. In this paper, every result in terms of ij- will have pairwise counter-part as a corollary.

By $i\text{-}int\,(A)$ and $i\text{-}cl\,(A)$, we mean the interior and the closure of a subset A of X with respect to the topology τ_i, respectively. We denote by $int\,(A)$ and $cl\,(A)$ the interior and the closure of a subset A of X with respect to the topology τ_i for each $i = 1, 2$, respectively. By i-open cover of X, we mean a cover of X by i-open sets in X; similarly, we treat with the ij-regular open cover of X and etc.

If $S \subseteq A \subseteq X$, then $i\text{-}int_A\,(S)$ and $i\text{-}cl_A\,(S)$ will be used to denote the interior and closure of S in the subspace A with respect to the topology τ_i, respectively. This means that $i\text{-}int\,(S) = i\text{-}int\,(S) \cap A$ and $i\text{-}cl_A\,(S) = i\text{-}cl\,(S) \cap A$.

2. **Pairwise Weakly Lindelöf Spaces**

Definition 2.1. *A bitopological space X is said to be ij-weakly Lindelöf if for every i-open cover $\{U_\alpha : \alpha \in \Delta\}$ of X, there exists a countable subset $\{\alpha_n : n \in \mathbb{N}\}$ of Δ such that $X = j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$. The space X is said pairwise weakly Lindelöf if it is both ij-weakly Lindelöf and ji-weakly Lindelöf.*

Proposition 2.2. *A bitopological space X is ij-weakly Lindelöf if and only if every family $\{C_\alpha : \alpha \in \Delta\}$ of i-closed subsets of X such that $\bigcap_{\alpha\in\Delta} C_\alpha = \emptyset$ admits a countable subfamily $\{C_{\alpha_n} : n \in \mathbb{N}\}$ such that j-$int\left(\bigcap_{n\in\mathbb{N}} C_{\alpha_n}\right) = \emptyset$.*

Proof. Let $\{C_\alpha : \alpha \in \Delta\}$ be a family of i-closed subsets of X such that $\bigcap_{\alpha\in\Delta} C_\alpha = \emptyset$. Then $X = X\setminus \bigcap_{\alpha\in\Delta} C_\alpha = \bigcup_{\alpha\in\Delta} (X \setminus C_\alpha)$, i.e., the family $\{X \setminus C_\alpha : \alpha \in \Delta\}$ is an i-open cover of X. Since X is ij-weakly Lindelöf, there exists a countable subfamily $\{X \setminus C_{\alpha_n} : n \in \mathbb{N}\}$ such that $X = j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} (X \setminus C_{\alpha_n})\right)$. So $X \setminus j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} (X \setminus C_{\alpha_n})\right) = \emptyset$, i.e., j-$int\left(X \setminus \bigcup_{n\in\mathbb{N}} (X \setminus C_{\alpha_n})\right) = \emptyset$. Thus $j\text{-}int\left(\bigcap_{n\in\mathbb{N}} C_{\alpha_n}\right) = \emptyset$.

Conversely, let $\{U_\alpha : \alpha \in \Delta\}$ be an i-open cover of X. Then $X = \bigcup_{\alpha\in\Delta} U_\alpha$ and $\{X \setminus U_\alpha : \alpha \in \Delta\}$ is a family of i-closed subsets of X. Hence $X \setminus \bigcup_{\alpha\in\Delta} U_\alpha = \emptyset$, i.e., $\bigcap_{\alpha\in\Delta} (X \setminus U_\alpha) = \emptyset$. By hypothesis, there exists a countable subfamily $\{X \setminus U_{\alpha_n} : n \in \mathbb{N}\}$ such that $j\text{-}int\left(\bigcap_{n\in\mathbb{N}} (X \setminus U_{\alpha_n})\right) = \emptyset$. So

$$\begin{aligned} X &= X \setminus j\text{-}int\left(\bigcap_{n\in\mathbb{N}} (X \setminus U_{\alpha_n})\right) \\ &= j\text{-}cl\left(X \setminus \bigcap_{n\in\mathbb{N}} (X \setminus U_{\alpha_n})\right) \\ &= j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right). \end{aligned}$$

Therefore X is ij-weakly Lindelöf. $\square$

In particular, we can deduce the following corollary.

Corollary 2.3. *A bitopological space X is pairwise weakly Lindelöf if and only if every family $\{C_\alpha : \alpha \in \Delta\}$ of closed subsets of X such that $\bigcap_{\alpha\in\Delta} C_\alpha = \emptyset$ admits a countable subfamily $\{C_{\alpha_n} : n \in \mathbb{N}\}$ such that $int\left(\bigcap_{n\in\mathbb{N}} C_{\alpha_n}\right) = \emptyset$.*

Proposition 2.4. *A bitoplogical space X is ij-weakly Lindelöf if and only if for every family $\{C_\alpha : \alpha \in \Delta\}$ of i-closed subsets of X, there exists a countable subfamily $\{C_{\alpha_n} : n \in \mathbb{N}\}$ such that when j-int $\left(\bigcap_{n\in\mathbb{N}} C_{\alpha_n}\right) \neq \emptyset$, then intersection $\bigcap_{\alpha\in\Delta} C_\alpha \neq \emptyset$.*

Proof. Let $\{C_\alpha : \alpha \in \Delta\}$ be a family of i-closed subsets of X and exists a countable subfamily $\{C_{\alpha_n} : n \in \mathbb{N}\}$ such that j-$int\left(\bigcap_{n\in\mathbb{N}} C_{\alpha_n}\right) \neq \emptyset$. Suppose that $\bigcap_{\alpha\in\Delta} C_\alpha = \emptyset$. Hence $X = X \setminus \bigcap_{\alpha\in\Delta} C_\alpha = \bigcup_{\alpha\in\Delta} (X \setminus C_\alpha)$. Thus $\{X \setminus C_\alpha : \alpha \in \Delta\}$ forms an i-open cover for X. Since X is ij-weakly Lindelöf, there exists a countable subset $\{\alpha_n : n \in \mathbb{N}\}$ of Δ such that $X = j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} (X \setminus C_{\alpha_n})\right)$. Hence $X \setminus j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} (X \setminus C_{\alpha_n})\right) = \emptyset$, i.e., $j\text{-}int\left(X \setminus \bigcup_{n\in\mathbb{N}} (X \setminus C_{\alpha_n})\right) = \emptyset$. Thus $j\text{-}int\left(\bigcap_{n\in\mathbb{N}} C_{\alpha_n}\right) = \emptyset$ which is a contradiction.

We give two methods to prove the converse of the Proposition.

The First Method: Assume that the condition is holds. By contrapositive, this condition is equivalent to the following statement: if for every family $\{C_\alpha : \alpha \in \Delta\}$ of i-closed subsets of X such that the intersection $\bigcap_{\alpha\in\Delta} C_\alpha = \emptyset$, then there exists a countable subfamily $\{C_{\alpha_n} : n \in \mathbb{N}\}$ such that $j\text{-}int\left(\bigcap_{n\in\mathbb{N}} C_{\alpha_n}\right) = \emptyset$. Let $\{U_\alpha : \alpha \in \Delta\}$ be an i-open cover of X. Then $X = \bigcup_{\alpha\in\Delta} U_\alpha$ and $\{X \setminus U_\alpha : \alpha \in \Delta\}$ be a family of i-closed subsets of X. Hence $X \setminus \bigcup_{\alpha\in\Delta} U_\alpha = \emptyset$, i.e., $\bigcap_{\alpha\in\Delta} (X \setminus U_\alpha) = \emptyset$. By hypothesis, there exists a countable subfamily $\{X \setminus U_{\alpha_n} : n \in \mathbb{N}\}$ such that $j\text{-}int\left(\bigcap_{n\in\mathbb{N}} (X \setminus U_{\alpha_n})\right) = \emptyset$. So $X = X \setminus j\text{-}int\left(\bigcap_{n\in\mathbb{N}} (X \setminus U_{\alpha_n})\right) = j\text{-}cl\left(X \setminus \bigcap_{n\in\mathbb{N}} (X \setminus U_{\alpha_n})\right) = j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$. Therefore X is ij-weakly Lindelöf.

The Second Method: Suppose that X is not ij-weakly Lindelöf. Then there exists an i-open cover $\{U_\alpha : \alpha \in \Delta\}$ of X with no countable subfamily $\{U_{\alpha_n} : n \in \mathbb{N}\}$ such that $X = j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$. Hence $X \neq j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$ for any countable subfamily $\{U_{\alpha_n} : n \in \mathbb{N}\}$. It follows that $X \setminus j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right) \neq \emptyset$, i.e., $j\text{-}int\left(X \setminus \bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right) \neq \emptyset$, or, $j\text{-}int\left(\bigcap_{n\in\mathbb{N}} (X \setminus U_{\alpha_n})\right) \neq \emptyset$. Thus $\{X \setminus U_\alpha : \alpha \in \Delta\}$ is a family of i-closed subsets of X and satisfies $j\text{-}int\left(\bigcap_{n\in\mathbb{N}} (X \setminus U_{\alpha_n})\right) \neq \emptyset$ for a countable subfamily $\{X \setminus U_{\alpha_n} : n \in \mathbb{N}\}$. By hypothesis, $\bigcap_{\alpha\in\Delta} (X \setminus U_\alpha) \neq \emptyset$, and so $X \setminus \bigcup_{\alpha\in\Delta} U_\alpha \neq \emptyset$, i.e., $X \neq \bigcup_{\alpha\in\Delta} U_\alpha$. This is a contradiction with the fact that $\{U_\alpha : \alpha \in \Delta\}$ is an i-open cover of X. Therefore X is ij-weakly Lindelöf. □

Corollary 2.5. *A bitoplogical space X is pairwise weakly Lindelöf if and only if for every family $\{C_\alpha : \alpha \in \Delta\}$ by closed subsets of X, there exists a countable subfamily $\{C_{\alpha_n} : n \in \mathbb{N}\}$ such that when $int\left(\bigcap_{n\in\mathbb{N}} C_{\alpha_n}\right) \neq \emptyset$, the n intersection $\bigcap_{\alpha\in\Delta} C_\alpha \neq \emptyset$.*

Definition 2.6. *A subset E of a bitopological space X is said to be i-dense in $F \subseteq X$ if $F \subseteq i\text{-}cl\,(E)$. In particular, E is i-dense in X or is an i-dense subset of X if $i\text{-}cl\,(E) = X$. The subset E is said to be dense in F if it is i-dense in F for each $i = 1, 2$. In particular, E is dense in X or is a dense subset of X if it is i-dense in X or is an i-dense subset of X for each $i = 1, 2$.*

Proposition 2.7. *Let (X, τ_1, τ_2) be a bitopological space. Then if*

(i) *X is ij-weakly Lindelöf;*

(ii) *every ij-regular open cover $\{U_\alpha : \alpha \in \Delta\}$ of X admits a countable subfamily $\{U_{\alpha_n} : n \in \mathbb{N}\}$ with j-dense union in X;*

(iii) *every family $\{C_\alpha : \alpha \in \Delta\}$ of ij-regular closed subsets of X such that $\bigcap\limits_{\alpha\in\Delta} C_\alpha = \emptyset$ admits a countable subfamily $\{C_{\alpha_n} : n \in \mathbb{N}\}$ such that $j\text{-}int\left(\bigcap\limits_{n\in\mathbb{N}} C_{\alpha_n}\right) = \emptyset$;*

then we have the relation: $(i) \Rightarrow (ii) \Leftrightarrow (iii)$. Moreover, if X is ij-semiregular, then $(ii) \Rightarrow (i)$.

Proof. $(i) \Rightarrow (ii)$: It is obvious by the definition since an ij-regular open set is also i-open set.

$(ii) \Leftrightarrow (iii)$: If $\{C_\alpha : \alpha \in \Delta\}$ is a family of ij-regular closed subsets of X such that $\bigcap\limits_{\alpha\in\Delta} C_\alpha = \emptyset$, then $X = X \setminus \bigcap\limits_{\alpha\in\Delta} C_\alpha = \bigcup\limits_{\alpha\in\Delta} (X \setminus C_\alpha)$, i.e., the family $\{X \setminus C_\alpha : \alpha \in \Delta\}$ is an ij-regular open cover of X. By (ii), there exists a countable subfamily $\{X \setminus C_{\alpha_n} : n \in \mathbb{N}\}$ with j-dense union in X, i.e., $X = j\text{-}cl\left(\bigcup\limits_{n\in\mathbb{N}} (X \setminus C_{\alpha_n})\right)$. So $X \setminus j\text{-}cl\left(\bigcup\limits_{n\in\mathbb{N}} (X \setminus C_{\alpha_n})\right) = \emptyset$, i.e.,

$$j\text{-}int\left(X \setminus \bigcup_{n\in\mathbb{N}} (X \setminus C_{\alpha_n})\right) = \emptyset.$$

Thus $j\text{-}int\left(\bigcap\limits_{n\in\mathbb{N}} C_{\alpha_n}\right) = \emptyset$. Conversely, let $\{U_\alpha : \alpha \in \Delta\}$ be an ij-regular open cover of X. Then $X = \bigcup\limits_{\alpha\in\Delta} U_\alpha$ and $\{X \setminus U_\alpha : \alpha \in \Delta\}$ is a family of ij-regular closed subsets of X. Hence $X \setminus \bigcup\limits_{\alpha\in\Delta} U_\alpha = \emptyset$, i.e., $\bigcap\limits_{\alpha\in\Delta} (X \setminus U_\alpha) = \emptyset$. By (iii), there exists a countable subfamily $\{X \setminus U_{\alpha_n} : n \in \mathbb{N}\}$ such that $j\text{-}int\left(\bigcap\limits_{n\in\mathbb{N}} (X \setminus U_{\alpha_n})\right) = \emptyset$. So $X = X \setminus j\text{-}int\left(\bigcap\limits_{n\in\mathbb{N}} (X \setminus U_{\alpha_n})\right) = j\text{-}cl\left(X \setminus \bigcap\limits_{n\in\mathbb{N}} (X \setminus U_{\alpha_n})\right) = j\text{-}cl\left(\bigcup\limits_{n\in\mathbb{N}} U_{\alpha_n}\right)$ and (ii) proved.

$(ii) \Rightarrow (i)$: Let $\{U_\alpha : \alpha \in \Delta\}$ be an i-open cover of X. Since X is ij-semiregular, we can assume that U_α is ij-regular open set for each α. By (ii), there exists a countable subfamily $\{U_{\alpha_n} : n \in \mathbb{N}\}$ with j-dense union in X, i.e., $X = j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$. Thus X is ij-weakly Lindelöf and this completes the proof. □

Corollary 2.8. *Let (X, τ_1, τ_2) be a bitopological space. For the following conditions*

(i) X is pairwise almost Lindelöf;

(ii) every pairwise regular open cover $\{U_\alpha : \alpha \in \Delta\}$ of X admits a countable subfamily $\{U_{\alpha_n} : n \in \mathbb{N}\}$ with a dense union in X;

(iii) every family $\{C_\alpha : \alpha \in \Delta\}$ of pairwise regular closed subsets of X such that $\bigcap_{\alpha\in\Delta} C_\alpha = \emptyset$ admits a countable subfamily $\{C_{\alpha_n} : n \in \mathbb{N}\}$ such that $int\left(\bigcap_{n\in\mathbb{N}} C_{\alpha_n}\right) = \emptyset$;

we have that $(i) \Rightarrow (ii) \Leftrightarrow (iii)$ and if X is pairwise semiregular, then $(ii) \Rightarrow (i)$.

Proof. The proof is obvious by observing that every ij-almost Lindelöf space is ij-weakly Lindelöf since

$$\bigcup_{n\in\mathbb{N}} j\text{-}cl\,(U_{\alpha_n}) \subseteq j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right).$$

□

Now we can list a couple of open questions that are worth to study in detail.

Question 1. Does ij-weakly Lindelöf property imply ij-almost Lindelöf property? The authors expect that the answer to this question is negative.

Definition 2.9. *A bitopological space X is said to be i-separable if there exists a countable i-dense subset of X. The space X is said separable if it is i-separable for each $i = 1, 2$.*

Proposition 2.10. *If the bitopological space X is j-separable, then it is ij-weakly Lindelöf.*

Proof. Let $\mathcal{U} = \{U_\alpha : \alpha \in \Delta\}$ be an i-open cover of the j-separable space X. Then X has a countable j-dense subset $D = \{x_1, x_2, \ldots, x_n, \ldots\}$.

Now, for every $x_k \in D$, there exists $\alpha_k \in \Delta$ with $x_k \in U_{\alpha_k}$. So $X = j\text{-}cl\,(D) = j\text{-}cl\left(\bigcup_{k\in\mathbb{N}} \{x_k\}\right) = j\text{-}cl\left(\bigcup_{k\in\mathbb{N}} U_{\alpha_k}\right)$. This shows that X is ij-weakly Lindelöf. □

Corollary 2.11. *If the bitopological space X is separable, then it is pairwise weakly Lindelöf.*

Definition 2.12. *A bitopological space X is called ij-weak P-space if for each countable family $\{U_n : n \in \mathbb{N}\}$ of i-open sets in X, we have $j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right) = \bigcup_{n\in\mathbb{N}} j\text{-}cl\,(U_{\alpha_n})$. The space X is called pairwise weak P-space if it is both ij-weak P-space and ji-weak P-space.*

Proposition 2.13. *In ij-weak P-spaces, ij-almost Lindelöf property is equivalent to ij-weakly Lindelöf property.*

Proof. The proof follows immediately from the fact that in ij-weak P-spaces, $\bigcup_{n\in\mathbb{N}} j\text{-}cl\,(U_{\alpha_n}) = j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$ for any countable family $\{U_n : n \in \mathbb{N}\}$ of i-open sets in X. □

Corollary 2.14. *In pairwise weak P-spaces, pairwise almost Lindelöf property is equivalent to pairwise weakly Lindelöf property.*

Lemma 2.15 (see [11])**.** *An ij-almost regular space is ij-almost Lindelöf if and only if it is ij-nearly Lindelöf.*

Proposition 2.16. *An ij-weakly Lindelöf, ij-almost regular and ij-weak P-space is ij-nearly Lindelöf.*

Proof. This is a direct consequence of Proposition 5 and Lemma 1. □

Corollary 2.17. *A pairwise weakly Lindelöf, pairwise almost regular and pairwise weak P-space is pairwise nearly Lindelöf.*

Lemma 2.18. *[11]] An ij-regular space is ij-almost Lindelöf if and only if it is i-Lindelöf.*

Proposition 2.19. *An ij-weakly Lindelöf, ij-regular and ij-weak P-space is i-Lindelöf.*

Proof. This is a direct consequence of Proposition 2.13 and Lemma 2.18. □

Corollary 2.20. *A pairwise weakly Lindelöf, pairwise regular and pairwise weak P-space is Lindelöf.*

Definition 2.21 (see [3], [4]). *Let X be a space. A cover $\mathcal{V} = \{V_j : j \in J\}$ of X is a refinement of another cover $\mathcal{U} = \{U_\alpha : \alpha \in \Delta\}$ if for each $j \in J$, there exists an $\alpha(j) \in \Delta$ such that $V_j \subseteq U_{\alpha(j)}$, i.e., each $V \in \mathcal{V}$ is contained in some $U \in \mathcal{U}$.*

Definition 2.22 (see [3], [4]). *A family $\mathcal{U} = \{U_\alpha : \alpha \in \Delta\}$ of subsets of a space X is locally finite if for every point $x \in X$, there exists a neighbourhood U_x of x such that the set $\{\alpha \in \Delta : U_x \cap U_\alpha \neq \emptyset\}$ is finite, i.e., each $x \in X$ has a neighbourhood U_x meeting only finitely many $U \in \mathcal{U}$.*

Definition 2.23. *A bitopological space X is said to be ij-nearly paracompact if every cover of X by ij-regular open sets admits a locally finite refinement.*

Proposition 2.24. *An ij-weakly Lindelöf, ij-semiregular and ij-nearly paracompact bitopological space X is ij-almost Lindelöf.*

Proof. Let $\{U_\alpha : \alpha \in \Delta\}$ be an ij-regular open cover of X. Since X is ij-nearly paracompact, this cover admits a locally finite refinement $\{V_\lambda : \lambda \in \Lambda\}$. Since X is ij-weakly Lindelöf, there exists a countable subfamily $\{V_{\lambda_n} : n \in \mathbb{N}\}$ such that $X = j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} V_{\lambda_n}\right)$. Since the family $\{V_{\lambda_n} : n \in \mathbb{N}\}$ is locally finite, then $j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} V_{\lambda_n}\right) = \bigcup_{n\in\mathbb{N}} j\text{-}cl\,(V_{\lambda_n})$ (see [4]). Choosing, for each $n \in \mathbb{N}$, $\alpha_n \in \Delta$ such that $V_{\lambda_n} \subseteq U_{\alpha_n}$, we obtain $X = \bigcup_{n\in\mathbb{N}} j\text{-}cl\,(V_{\lambda_n}) = \bigcup_{n\in\mathbb{N}} j\text{-}cl\,(U_{\alpha_n})$. By Proposition 2.7, X is ij-almost Lindelöf. □

Corollary 2.25. *A pairwise weakly Lindelöf, pairwise semiregular and pairwise nearly paracompact bitopological space X is pairwise almost Lindelöf.*

Proposition 2.26. *An ij-weakly Lindelöf, ij-regular and ij-nearly paracompact bitopological space X is i-Lindelöf.*

Proof. Let $\{U_\alpha : \alpha \in \Delta\}$ be an i-open cover of X. Since X is ij-regular, then it is ij-semiregular. So X is ij-almost Lindelöf by Proposition 2.24. Since X is ij-regular, then X is i-Lindelöf by Lemma 2.18. □

Corollary 2.27. *A pairwise weakly Lindelöf, pairwise regular and pairwise nearly paracompact bitopological space X is Lindelöf.*

3. Pairwise Weakly Lindelöf Subspaces and Subsets

A subset S of a bitopological space X is said to be ij-weakly Lindelöf (resp. pairwise weakly Lindelöf) if S is ij-weakly Lindelöf (resp. pairwise weakly Lindelöf) as a subspace of X, i.e., S is ij-weakly Lindelöf (resp. pairwise weakly Lindelöf) with respect to the inducted bitopology from the bitopology of X.

Definition 3.1. *A subset S of a bitopological space X is said to be ij-weakly Lindelöf relative to X if for every cover $\{U_\alpha : \alpha \in \Delta\}$ of S by i-open subsets of X such that $S \subseteq \bigcup_{\alpha\in\Delta} U_\alpha$, there exists a countable subset $\{\alpha_n : n \in \mathbb{N}\}$ of Δ such that $S \subseteq j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$. The subset S is said to be pairwise weakly Lindelöf relative to X if it is both ij-weakly Lindelöf relative to X and ji-weakly Lindelöf relative to X.*

Proposition 3.2. *Let S be a subset of a bitopological space X. Then S is ij-weakly Lindelöf relative to X if and only if for every family $\{C_\alpha : \alpha \in \Delta\}$ of i-closed subsets of X such that $\left(\bigcap_{\alpha\in\Delta} C_\alpha\right) \cap S = \emptyset$, there exists a countable subfamily $\{C_{\alpha_n} : n \in \mathbb{N}\}$ such that $j\text{-}int\left(\bigcap_{n\in\mathbb{N}} C_{\alpha_n}\right) \cap S = \emptyset$.*

Proof. Let $\{C_\alpha : \alpha \in \Delta\}$ be a family of i-closed subsets of X such that $\left(\bigcap_{\alpha\in\Delta} C_\alpha\right) \cap S = \emptyset$. Then $S \subseteq X \setminus \left(\bigcap_{\alpha\in\Delta} C_\alpha\right) = \bigcup_{\alpha\in\Delta} (X \setminus C_\alpha)$, so $\{X \setminus C_\alpha : \alpha \in \Delta\}$ forms a family of i-open subsets of X covering S. By hypothesis, there exists a countable subfamily $\{X \setminus C_{\alpha_n} : n \in \mathbb{N}\}$ such that $S \subseteq j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} (X \setminus C_{\alpha_n})\right)$. Hence $\left(X \setminus j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} (X \setminus C_{\alpha_n})\right)\right) \cap S = \emptyset$, i.e., $j\text{-}int\left(X \setminus \bigcup_{n\in\mathbb{N}} (X \setminus C_{\alpha_n})\right) \cap S = \emptyset$. Thus $j\text{-}int\left(\bigcap_{n\in\mathbb{N}} C_{\alpha_n}\right) \cap S = \emptyset$. Conversely, let $\{U_\alpha : \alpha \in \Delta\}$ be a family of i-open subsets in X such that $S \subseteq \bigcup_{\alpha\in\Delta} U_\alpha$. Then $\left(X \setminus \bigcup_{\alpha\in\Delta} U_\alpha\right) \cap S = \emptyset$, i.e., $\left(\bigcap_{\alpha\in\Delta} X \setminus U_\alpha\right) \cap S = \emptyset$. Since $\{X \setminus U_\alpha : \alpha \in \Delta\}$ is a family of i-closed subsets of X, by hypothesis there exists a countable subfamily $\{X \setminus U_{\alpha_n} : n \in \mathbb{N}\}$ such that $j\text{-}int\left(\bigcap_{n\in\mathbb{N}}(X\setminus U_{\alpha_n})\right)\cap S=\emptyset$. Therefore,

$S\subseteq X\setminus j\text{-}int\left(\bigcap_{n\in\mathcal{N}}(X\setminus U_{\alpha_n})\right)=j\text{-}cl\left(X\setminus\left(\bigcap_{n\in\mathbb{N}}(X\setminus U_{\alpha_n})\right)\right)=j\text{-}cl\left(\bigcup_{n\in\mathbb{N}}U_{\alpha_n}\right)$.

This completes the proof. □

Corollary 3.3. *Let S be a subset of a bitopological space X. Then S is pairwise weakly Lindelöf relative to X if and only if for every family $\{C_\alpha:\alpha\in\Delta\}$ of closed subsets of X such that $\left(\bigcap_{\alpha\in\Delta}C_\alpha\right)\cap S=\emptyset$, there exists a countable subfamily $\{C_{\alpha_n}:n\in\mathbb{N}\}$ such that $int\left(\bigcap_{n\in\mathbb{N}}C_{\alpha_n}\right)\cap S=\emptyset$.*

Proposition 3.4. *Let X be a bitopological space and $S\subseteq X$. For the following conditions*

(i) S is ij-weaklyLindelöf relative to X;

(ii) every family by ij-regular open subsets $\{U_\alpha:\alpha\in\Delta\}$ of X that cover S admits a countable subfamily $\{U_{\alpha_n}:n\in\mathbb{N}\}$ with j-dense union in S;

(iii) every family $\{C_\alpha:\alpha\in\Delta\}$ of ij-regular closed subsets of X such that

$$\left(\bigcap_{\alpha\in\Delta}C_\alpha\right)\cap S=\emptyset$$

admits a countable subfamily $\{C_{\alpha_n}:n\in\mathbb{N}\}$ such that $j\text{-}int\left(\bigcap_{n\in\mathbb{N}}C_{\alpha_n}\right)\cap S=\emptyset$; we have that $(i)\Rightarrow(ii)\Leftrightarrow(iii)$. Furthermore, if X is ij-semiregular, then $(ii)\Rightarrow(i)$.

Proof. $(i)\Rightarrow(ii)$: It is obvious by the definition since an ij-regular open set is also i-open.

$(ii)\Leftrightarrow(iii)$: If $\{C_\alpha:\alpha\in\Delta\}$ is a family of ij-regular closed subsets of X such that $\left(\bigcap_{\alpha\in\Delta}C_\alpha\right)\cap S=\emptyset$, then $S\subseteq X\setminus\bigcap_{\alpha\in\Delta}C_\alpha=\bigcup_{\alpha\in\Delta}(X\setminus C_\alpha)$, i.e., the family $\{X\setminus C_\alpha:\alpha\in\Delta\}$ is an ij-regular open subsets of X that cover S. By (ii), there exists a countable subfamily $\{X\setminus C_{\alpha_n}:n\in\mathbb{N}\}$ such that $S\subseteq j\text{-}cl\left(\bigcup_{n\in\mathbb{N}}(X\setminus C_{\alpha_n})\right)=X\setminus j\text{-}int\left(X\setminus\bigcup_{n\in\mathbb{N}}(X\setminus C_{\alpha_n})\right)=X\setminus j\text{-}int\left(\bigcap_{n\in\mathbb{N}}C_{\alpha_n}\right)$. So, $j\text{-}int\left(\bigcap_{n\in\mathbb{N}}C_{\alpha_n}\right)\cap S=\emptyset$. Conversely, let

$\{U_\alpha : \alpha \in \Delta\}$ be a family of ij-regular open subsets of X that cover S. Then $S \subseteq \bigcup_{\alpha\in\Delta} U_\alpha$ and $\{X \setminus U_\alpha : \alpha \in \Delta\}$ is a family of ij-regular closed subsets of X. Hence $\left(X \setminus \bigcup_{\alpha\in\Delta} U_\alpha\right) \cap S = \emptyset$, i.e., $\left(\bigcap_{\alpha\in\Delta} (X \setminus U_\alpha)\right) \cap S = \emptyset$. By (iii), there exists a countable subfamily $\{X \setminus U_{\alpha_n} : n \in \mathbb{N}\}$ such that $j\text{-}int\left(\bigcap_{n\in\mathbb{N}} (X \setminus U_{\alpha_n})\right) \cap S = \emptyset$. So $S \subseteq X \setminus j\text{-}int\left(\bigcap_{n\in\mathbb{N}} (X \setminus U_{\alpha_n})\right) = j\text{-}cl\left(X \setminus \bigcap_{n\in\mathbb{N}} (X \setminus U_{\alpha_n})\right) = j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$.

$(ii) \Rightarrow (i)$: Let $\{U_\alpha : \alpha \in \Delta\}$ be a family of i-open subsets of X that cover S. Since X is an ij-semiregular, we can assume that U_α is an ij-regular open set for each α. By (ii), there exists a countable subfamily $\{U_{\alpha_n} : n \in \mathbb{N}\}$ such that $S \subseteq j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$. This completes the proof. $\square$

Corollary 3.5. *Let X be a bitopological space and $S \subseteq X$. For the following conditions*

(i) *S is pairwise weakly Lindelöf relative to X;*

(ii) *every family by pairwise regular open subsets $\{U_\alpha : \alpha \in \Delta\}$ of X that cover S admits a countable subfamily $\{U_{\alpha_n} : n \in \mathbb{N}\}$ with dense union in S;*

(iii) *every family $\{C_\alpha : \alpha \in \Delta\}$ of pairwise regular closed subsets of X such that $\left(\bigcap_{\alpha\in\Delta} C_\alpha\right) \cap S = \emptyset$ admits a countable subfamily $\{C_{\alpha_n} : n \in \mathbb{N}\}$ such that $int\left(\bigcap_{n\in\mathbb{N}} C_{\alpha_n}\right) \cap S = \emptyset$;*

we have that $(i) \Rightarrow (ii) \Leftrightarrow (iii)$. Furthermore, if X is pairwise semiregular, then $(ii) \Rightarrow (i)$.

Proposition 3.6. *Let X be a bitopological space and let A be any subset of X. If A is ij-weakly Lindelöf, then it is ij-weakly Lindelöf relative to X.*

Proof. Let $\{U_\alpha : \alpha \in \Delta\}$ be a family of i-open subsets of X that cover A. Then for each α, we can find an i-open set V_α of A with $U_\alpha \cap A = V_\alpha$. Thus $\{V_\alpha : \alpha \in \Delta\}$ is a cover of A by i-open subsets of A. Since A is ij-weakly Lindelöf, then there exists a countable subset $\{\alpha_n : n \in \mathbb{N}\}$ of

Δ such that $A = j\text{-}cl_A\left(\bigcup_{n\in\mathbb{N}} V_{\alpha_n}\right) \subseteq j\text{-}cl_X\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$. Therefore A is ij-weakly Lindelöf relative to X. □

Corollary 3.7. *Let X be a bitopological space and let A be any subset of X. If A is pairwise weakly Lindelöf, then it is pairwise weakly Lindelöf relative to X.*

Question 2. Is the converse of Proposition 3.6 above true?

The authors expect that the answer is negative.

The converse of Proposition 3.6 holds if $A \subseteq X$ is i-open. We prove this in the following proposition.

Proposition 3.8. *Let X be a bitopological space and A an i-open subset of X. Then A is ij-weakly Lindelöf if and only if it is ij-weakly Lindelöf relative to X.*

Proof. The proof of necessity was shown in Proposition 3.6. For the sufficiency, let $\{U_\alpha : \alpha \in \Delta\}$ be an i-open cover of A. Since i-open subsets of an i-open subspace of X is i-open in X, then $\{U_\alpha : \alpha \in \Delta\}$ is a cover of A by i-open subsets of X. Since A is ij-weakly Lindelöf relative to X, there exists a countable subset $\{\alpha_n : n \in \mathbb{N}\}$ of Δ such that $A \subseteq j\text{-}cl_X\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$. Then $A = j\text{-}cl_X\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right) \cap A = j\text{-}cl_X\left(\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right) \cap A\right) = j\text{-}cl_X\left(\bigcup_{n\in\mathbb{N}} (U_{\alpha_n} \cap A)\right) = j\text{-}cl_A\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$. Therefore A is ij-weakly Lindelöf. □

Corollary 3.9. *Let X be a bitopological space and A an open subset of X. Then A is pairwise weakly Lindelöf if and only if it is pairwise weakly Lindelöf relative to X.*

Remark 3.10. *The above result shows that in an i-open subset of a bitopological space X, ij-weakly Lindelöf property and ij-weakly Lindelöf relative to X property are equivalent.*

Remark 3.11. *The space X in Proposition 3.6, Proposition 3.8, Corollary 3.7 and Corollary 3.8 is any bitopological space.*

If we consider X itself is an ij-weakly Lindelöf space, we have the following results.

Proposition 3.12. *Every ij-regular closed and j-open subset of an ij-weakly Lindelöf and ij-semiregular space X is ij-weakly Lindelöf relative to X.*

Proof. Let A be an ij-regular closed and j-open subset of X. If $\{U_\alpha : \alpha \in \Delta\}$ is a cover of A by ij-regular open subsets of X, then $X = \left(\bigcup_{\alpha\in\Delta} U_\alpha\right) \cup (X \setminus A)$. Hence the family $\{U_\alpha : \alpha \in \Delta\} \cup \{X \setminus A\}$ forms an ij-regular open cover of X. Since X is ij-weakly Lindelöf by proposition 2.7, there will be a countable subfamily $\{X \setminus A, U_{\alpha_1}, U_{\alpha_2}, \ldots\}$ such that

$$\begin{aligned} X &= j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} (U_{\alpha_n} \cup X \setminus A)\right) \\ &= j\text{-}cl\left(\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right) \cup X \setminus A\right) \\ &= j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right) \cup j\text{-}cl\,(X \setminus A) \\ &= j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right) \cup (X \setminus A)\,. \end{aligned}$$

But A and $X \setminus A$ are disjoint; hence $A \subseteq j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$. This shows that A is ij-weakly Lindelöf relative to X and completes the proof. □

Corollary 3.13. *Every pairwise regular closed and open subset of a pairwise weakly Lindelöf and pairwise semiregular space X is pairwise weakly Lindelöf relative to X.*

Proposition 3.14. *An i-clopen and j-open subset of an ij-weakly Lindelöf space X is ij-weakly Lindelöf.*

Proof. Let F be an i-clopen and j-open subset of X. By Proposition 3.8 above, it is sufficient to prove that F is ij-almost Lindelöf relative to X. Let $\{U_\alpha : \alpha \in \Delta\}$ be a family of i-open subsets of X that cover F. Then $\{U_\alpha : \alpha \in \Delta\} \cup \{X \setminus F\}$ forms an i-open cover of X. Since X is ij-weakly Lindelöf, there exists a countable subset $\{\alpha_n : n \in \mathbb{N}\}$ of Δ

such that

$$\begin{aligned} X &= j\text{-}cl\left(\bigcup_{n\in\mathbb{N}}(U_{\alpha_n}\cup X\setminus F)\right) \\ &= j\text{-}cl\left(\left(\bigcup_{n\in\mathbb{N}}U_{\alpha_n}\right)\cup(X\setminus F)\right) \\ &= j\text{-}cl\left(\bigcup_{n\in\mathbb{N}}U_{\alpha_n}\right)\cup j\text{-}cl\,(X\setminus F) \\ &= j\text{-}cl\left(\bigcup_{n\in\mathbb{N}}U_{\alpha_n}\right)\cup(X\setminus F)\,. \end{aligned}$$

But F and $X \setminus F$ are disjoint; hence $F \subseteq j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$. This shows that F is ij-weakly Lindelöf relative to X and completes the proof. □

Corollary 3.15. *A clopen subset of a pairwise weakly Lindelöf space X is pairwise weakly Lindelöf.*

Question 3. Is i-closed subset of an ij-weakly Lindelöf space X ij-weakly Lindelöf?

Question 4. Is ij-regular open subset of an ij-weakly Lindelöf space X ij-weakly Lindelöf? The authors expect that the answer of both questions are negative.

Observe that, the condition in Proposition 3.12 that a subset should be ij-regular closed and j-open and in Proposition 3.14 that a subset should be i-clopen and j-open are necessary and it is not sufficient to be only i-open as the following example shows. In general, arbitrary subsets of ij-wekly Lindelöf spaces need not be ij-weakly Lindelöf relative to the spaces and so not ij-weakly Lindelöf by Proposition 3.6.

Example 3.16. *Let Ω denote the set of ordinals which are less than or equal to the first uncountable ordinal ω_1. This Ω is an uncountable well-ordered set with a largest element ω_1, having the property that if $\alpha \in \Omega$ with $\alpha < \omega_1$, then $\{\beta \in \Omega : \beta \leq \alpha\}$ is countable. Since Ω is a totally ordered space, it can be considered by its order topology. Let us denote this order topology by τ_1. Let $\mathcal{B}$ be a collection of all sets in Ω of the form $(a,b) = \{\beta \in \Omega : a < \beta < b\}$, $(a_0, \omega_1] = \{\beta \in \Omega : a_0 < \beta \leq \omega_1\}$ and $[1, b_0) = \{\beta \in \Omega : 1 \leq \beta < b_0\}$. Then the collection $\mathcal{B}$ is a base for the* *order topology τ_1 for Ω. Choose the discrete topology as another topology for Ω denoted by τ_2. So (Ω, τ_1, τ_2) form a bitopological space. Now Ω is a 1-Lindelöf space (see [19]), so it is 12-weakly Lindelöf. The subset $\Omega_0 = \Omega \setminus \{\omega_1\}$, however, is not 1-Lindelöf (see [19]). We notice that Ω_0 is 1-open subset of Ω since $\Omega_0 = [1, \omega_1) = \{\beta \in \Omega : 1 \leq \beta < \omega_1\}$. So Ω_0 is not 12-wekly Lindelöf by Proposition 2.19 since it is 12-regular and 12-weak P-space. Moreover Ω_0 is not 12-weakly Lindelöf relative to Ω by Proposition 3.8.*

So we can say that in general, an ij-weakly Lindelöf property is not a hereditary property and therefore pairwise weakly Lindelöf property is not either.

Definition 3.17. *A bitopological space X is said to be hereditary ij-weakly Lindelöf if every subspace of X is ij-weakly Lindelöf. The space X is said to be hereditary pairwise weakly Lindelöf if it is both hereditary ij-weakly Lindelöf and hereditary ji-weakly Lindelöf.*

Proposition 3.18. *Let X be an ij-semiregular bitopological space. Then X is i-open hereditary ij-weakly Lindelöf if and only if any $A \in \tau_{ij}^s$ is ij-weakly Lindelöf.*

Proof. Let X be an ij-semiregular and i-open hereditary ij-weakly Lindelöf space. Since $\tau_{ij}^s \subseteq \tau_i$, it is obvious that any $A \in \tau_{ij}^s$ implies $A \in \tau_i$ and hence A is ij-weakly Lindelöf. Conversely, let $B \subseteq X$ be an i-open subset of ij-weakly Lindelöf space X. By Proposition 3.8, it is sufficient to prove that B is ij-weakly Lindelöf relative to X. Let $\mathcal{U} = \{U_\alpha : \alpha \in \Delta\}$ be a family of ij-regular open subsets of X such that $B \subseteq \bigcup_{\alpha\in\Delta} U_\alpha$. The set $A = \bigcup_{\alpha\in\Delta} U_\alpha \in \tau_{ij}^s$, so by the hypothesis A is ij-weakly Lindelöf. Hence there exists a countable subfamily $\{U_{\alpha_n} : n \in \mathbb{N}\}$ of $\mathcal{U}$ such that $A = j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$ and therefore $B \subseteq j\text{-}cl\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$. This completes the proof. □

Corollary 3.19. *A bitopological space X is open hereditary pairwise weakly Lindelöf if and only if any $A \in \tau^s$ is pairwise weakly Lindelöf.*

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