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THE nc-SUPPLEMENTED SUBGROUPS OF FINITE GROUPS†

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Abstract. A subgroup H is said to be nc-supplemented in a group G if there exists a subgroup $K \leq G$ such that $HK \lhd G$ and $H \cap K$ is contained in H_G, the core of H in G. We characterize the supersolubility of finite groups G with that every maximal subgroup of the Sylow subgroups is nc-supplemented in G.

1. **Introduction**

In this paper the word group always means finite group.

A subgroup H is said to be complemented in G if there exists a subgroup K such that $G = HK$ and $H \cap K = 1$. Hall proved that a group is soluble if and only if every Sylow subgroup is complemented [7]. Ramadan in [13] proved that if G/H is supersoluble and all maximal subgroups of the Sylow subgroups of H are normal in G, then G is supersoluble. A subgroup H is $c-$normal in G if there exists a normal subgroup N of G such that $HN = G$ and $H \cap N$ is contained in H_G, the core of H in G (see [17]). Obviously $c-$normality is weaker than normality. A subgroup H is said to be $c-$supplemented in a group G if

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there exists a subgroup K such that $HK = G$ and $H \cap K$ is contained in H_G, the core of H in G (see [3]). The notion of c-supplementation is a generalization of the notions of complement and c-normality. Li et al. in [12] defined the following concept: A subgroup H is said to be nc-supplemented in a group G if there exists a subgroup $K \leq G$ such that $HK \lhd G$ and $H \cap K$ is contained in H_G, the core of H in G.

In this note, we give some generalization of supersolubility based on the concept of nc-supplementation.

We will prove the following theorem:

Theorem 1.1. *Suppose that G is a group with a normal subgroup H such that G/H is supersoluble. If every maximal subgroup of every Sylow subgroup of H is nc-supplemented in G, then G is supersoluble.*

A class of finite group $\mathfrak{F}$ is said to be a formation if every epimorphic image of an $\mathfrak{F}$-group is an $\mathfrak{F}$-group and if $G/N_1 \cap N_2$ belongs to $\mathfrak{F}$ whenever G/N_1 and G/N_2 belong to $\mathfrak{F}$. A formation $\mathfrak{F}$ is said to be saturated if a finite group $G \in \mathfrak{F}$ whenever $G/\Phi(G) \in \mathfrak{F}$ (see [14, p. 277]). The class of supersoluble group is a saturated formation (see [14, 9.4.5]). Let $\mathfrak{U}$ denote the class of all supersoluble groups.

Also we prove:

Theorem 1.2. *Let $\mathfrak{F}$ be a saturated formation containing $\mathfrak{U}$. Suppose that G is a group with a normal subgroup H such that $G/H \in \mathfrak{F}$. If every maximal subgroup of all Sylow subgroups of H is nc-supplemented in G, then $G \in \mathfrak{F}$.*

Further definitions and notations are standard, please refer to [11] and [9].

2. Preliminaries

In this section, we give some concepts and some lemmas.

Definition 2.1. ([3]) *A subgroup H is said to be $c-$supplemented in group G if there exists a subgroup K such that $HK = G$ and $H \cap K$ is contained in $Core_G(H)$. Then we say that K is a $c-$supplement of H in G.*

Definition 2.2. ([12]) *Let G be a group and H a subgroup of G. Then H is said to be $nc-$supplemented in G if there is a subgroup K of G such that $HK \trianglelefteq G$ and $H \cap K \leq H_G$. We say that K is a $nc-$supplement of H in G.*

Remark 2.3. *If H is a maximal subgroup of G, then an nc-supplement of H in G is a c-supplement of H in G.*

Proof. If H is nc-supplemented in G, then there exists a subgroup K such that $HK \lhd G$ and $H \cap K \leq H_G$. The maximality of H implies that $HK = G$ or $HK = H$. In the former case, H is c-supplemented in G. In the latter case, $H \lhd G$ and so H is also c-supplemented in G. □

Remark 2.4. *Being nc-supplement is weaker than c-supplementation and normality.*

nc-supplemented is a generalized c-supplemented. In general, nc-supplementation does not imply c-supplementation. For example (see [12, Example 3]), let $G = A_4$ and $B = \{(1), (12)(34), (13)(24), (14)(23)\}$. Let $C = \{(1), (12)(34)\}$ and $H = \{(1), (13)(24)\}$. Then $B = CH \trianglelefteq G$ and C is $nc-$supplemented in G but not c-supplemented in G since $C_G = 1$ and G has no subgroup of order 6.

Lemma 2.5. *(* [12, Lemma 4]*) If H is nc$-$supplemented in G, then there exists a subgroup C of G such that $H \cap C = H_G$ and $HC \trianglelefteq G$.*

Lemma 2.6. *(* [12, Lemma 5]*) Let G be a group. Then*

(1) If $H \leq M \leq G$ and H is nc$-$supplemented in G, then H is nc$-$supplemented in M.

(2) If $N \trianglelefteq G$ and $N \leq H$, then H is nc$-$supplemented in G if and only if H/N is nc$-$supplemented in G/N.

(3) If $N \trianglelefteq G$ and $(|N|, |H|) = 1$. If H is nc$-$supplemented in G , then HN/N is nc$-$supplemented in G/N.

Lemma 2.7. *(*[16, 2.16]*) Let $\mathfrak{F}$ be a formation containing $\mathfrak{U}$ and let G be a group with a normal subgroup H such that $G/H \in \mathfrak{F}$. If H is cyclic, then $G \in \mathfrak{F}$.*

3. Main results and their applications

In this section, we give the proofs of the main theorems.

The proof of Theorem 1.1

Proof. Suppose that G is a counter-example of minimal order. We have:

Step 1. Every proper subgroup M of G containing H is supersoluble and G is soluble.

Since $H \leq M$, it follows that M/H is a proper subgroup of G/H. Since G/H is supersoluble, it follows that M/H is supersoluble. Thus

M satisfies the hypotheses of the theorem, and by the minimality of G, M is supersoluble. In particular, H is supersoluble and so G is soluble by [4].

Step 2. $\Phi(G) < H$ and $\Phi(G) = 1$.

Since the class of supersoluble group is a saturated formation by [14, 9.4.5], it is easy to get the result.

In the following, let L be a minimal normal subgroup of G contained in H. Then, by Step 1 and [10, Lemma 8. 6, p. 102] L is an elementary abelian p-group for some prime divisor p of $|G|$.

Step 3. G/L is supersoluble and L is the unique minimal normal subgroup of G which is contained in H.

First, we check that $(G/L, H/L)$ satisfies the hypothesis as (G, H). Let $\overline{Q} = QL/L$ be a Sylow q-subgroup of $H/L = \overline{H}$. Then $\overline{G} = G/L$. Hence we assume that Q is a Sylow q-subgroup of H.

Case a. If $p = q$, we assume that $L < P$, then $P = Q > L$. Let P_1 be a maximal subgroup of P. By hypothesis P_1 is nc-supplemented in G, and by Lemma 2.6, $\overline{P_1}$ is nc-supplemented in $\overline{G}$. The minimality of G implies that $\overline{G}$ is supersoluble.

Case b. Assume that $p \neq q$. Let $\overline{Q_1}$ be a maximal subgroup of a Sylow q-subgroup $\overline{Q}$ of $\overline{H}$. Without loss of generality, we assume that $\overline{Q_1} = Q_1L/L$. Since Q_1 is nc-supplemented in G, it follows, by Lemma 2.6, that $\overline{Q_1}$ is nc-supplemented in $\overline{G}$. The minimality of G implies that $\overline{G}$ is supersoluble.

Now, let R be another minimal normal subgroup of G contained in H. Then G/R is supersoluble by Step 3. Since $G/R \cap L \lesssim G/R \times G/L$, it follows, from [1, Theorem 3] that, $G/R \cap L$ is supersoluble. On the other hand, $R \cap L \leq L$ and so $R \cap L = 1$ or $R \cap L = L$ by the minimality of L. In the former case, $G/1 \cong G$ is supersoluble, a contradiction. In the latter, L is unique.

Step 4. $L = F(H) = C_H(L)$.

Since L is an elementary abelian normal subgroup of G, $L \leq H$. So by [11, 6.5.4], $F(H)$, the Fitting subgroup of H contains every minimal normal subgroup of H. By [6, Theorem 1.9.17] and Step 2, $F(H)$ is the direct product of minimal normal subgroups of G contained in H. Then $L = F(H)$ by Step 3. Since G is soluble by Step 1, $F(H) \leq C_H(L) = C_H(F(H)) \leq F(H)$ by [19, Lemma 2.3].

Step 5. L is a Sylow subgroup of H.

Let q be the largest prime divisor of $|H|$ and let Q be a Sylow q-subgroup of H. Since H/L is supersoluble, it follows, by [9, VI-9.1(c)], that LQ/L is characteristic in G/L and so $LQ \trianglelefteq G$. Thus we have:

Case a. If $p = q$, then $L \le P = Q \lhd G$. Therefore, by Step 1 and [4, Hilfssatz C], $L = Q$ is a Sylow subgroup of H.

Case b. If $p < q$, then $L \le P$ and $PQ = PLQ$ is a subgroup of G. Since every maximal subgroup of all Sylow subgroups of PQ is nc-supplemented in PQ by Lemma 2.2(1), PQ satisfies the hypothesis of the theorem. Then we have:

Subcase a. If $PQ < G$, then, by Step 1, PQ is supersoluble and so $Q \lhd PQ$ by [9, VI-9.1]. Hence $LQ = L \times Q$ and so $Q \le C_G(L) \le L$ by [19, Lemma 2.3], a contradiction.

Subcase b. Assume that $PQ = H = G$ and $L < P$ in the case $Q \ntrianglelefteq G$. Since $L \cap N_G(Q) = 1$ and LQ is characteristic in $H = PQ = G$, it follows that $G = [L]N_G(Q)$. Let P_2 be a Sylow p-subgroup of $N_G(Q)$. Then LP_2 is a Sylow p-subgroup of G. Choose a maximal subgroup P_1 of LP_2 with $P_2 \le P_1$. Obviously, $L \nleq P_1$ and $P_{1G} = 1$. Otherwise, $L = P_{1G}$, which contradicts that $L \cap N_G(Q) = 1$. By hypotheses, P_1 is nc-supplemented in G, then there exists a subgroup K such that $P_1K \lhd G$ and so $P_1 \cap K \le P_{1G} = 1$. Hence if K is a q-subgroup of a Sylow q-subgroup Q of G, then P_1K is supersoluble by Step 1 and K is characteristic in P_1K which is normal in G. Then $LK = L \times K$ and so, by [19, Lemma 2.3], $K \le C_G(L) \le L$, a contradiction. Thus we assume that K is not a q-group. Since $|K|_p = |G : P_1|_p = p$, it follows that K has a normal p-complement Q^*. Obviously, P_1Q^* is a subgroup of G. By Step 1, P_1Q^* is supersoluble. And so, by [9, VI-9.1], $Q^* \lhd P_1Q^*$. Thus $LQ^* = L \times Q^*$ and $Q^* \le C_{P_1Q^*}(L) \le L$ by [19, Lemma 2.3], a contradiction. So we have $P_1K = G$. Now $|K|_p = |G : P_1|_p = p$ implies that K has a normal p-complement Q_1 which is also a Sylow q-subgroup of G. By [8, Theorem 4.2.2], there exists a $g \in LP_2 = P$ such that $Q_1^g = Q$. Since $P_1 \lhd P$, we have $G = P_1K = (P_1K)^g = P_1K^g$ and $P_1 \cap K^g = 1$. Since $K^g \cong K$ has a normal p-complement and $Q_1^g = Q \le K^g$, it follows that $K^g \le N_G(Q)$. Since $P = LP_2 = P_1LP_2 = P_1LP_2 \cap G = P_1(LP_2 \cap K^g)$, if $P_1(LP_2 \cap K^g) \le P_2$, then $LP_2 \le P_1P_2 \le P_2$, a contradiction. So $P_1(LP_2 \cap K^g) \nleq P_2$ and P_2 must be a proper subgroup of $P_3 = \langle P_2, LP_2 \cap K^g \rangle$, where P_3 is a subgroup of a Sylow p-subgroup P. Thus P_2 and K^g are contained in $N_G(Q)$ and so P_3 is a p-subgroup of G containing a proper Sylow p-subgroup P_2 of $N_G(Q)$, a contradiction.

Thus L is a Sylow subgroup of H.

Step 6. $|L| = p$.

Let L_1 be a maximal subgroup of L. Then, by hypothesis, L_1 is *nc*-supplemented in G and so, by Lemma 2.5, there exists a subgroup K of G such that $L_1K \trianglelefteq G$ and $L_1 \cap K \le L_{1G}$. By Step 3, $L_1K \ge L$, and so $L = L \cap (L_1K) = L_1(L \cap K)$. It follows that $L \cap K = L$ or $L \cap K < L$. In the first case, it is easy to get $L \cap K \lhd G$. In the second case, $L_1 \cap K < L_1 < L$, and so $L_1 \cap K = L_1 \cap K \cap K < L \cap K < L$. Since $L_1 \cap K \lhd G$ and $L \lhd G$, it follows that $L(L_1 \cap K) \lhd G$. As $L(L_1 \cap K) = (LL_1) \cap K = L \cap K$, we have $L \cap K \lhd G$ and so $L \cap K \ge L$ by the minimality and uniqueness of L. Then $L \cap K = L$ and so $L \le K$. Hence $L_1 \cap K \le L \cap K = L$ and so $L_1 \cap K = 1$. Thus $L_1 = 1$ and $|L| = p$.

Step 7. The final contradiction.

By Step 3, G/L is supersoluble. By Step 6, L is a cyclic subgroup of prime order. Then by Lemma 2.7, G is supersoluble, a contradiction.

The final contradiction completes the proof. □

Remark 3.1. *The condition of Theorem 1.1 " G/H is supersoluble " cannot be replaced by " G/H is soluble ". Let $G = A_4 \times C_5$, where A_4 is the alternating group of degree 4 and C_5 is a cyclic group of order 5. Then $G/C_5 \cong A_4$ is soluble. Obviously, C_5 satisfies the hypotheses, but G is not supersoluble.*

Corollary 3.2. ([3, Theorem 3.3]) *Let G be a finite group and let N be a normal subgroup of G such that G/N is supersoluble. If every maximal subgroup of every Sylow subgroup of N is c-supplemented in G, then G is supersoluble.*

Corollary 3.3. ([17, Theorem 1.1]) *Let G be a finite group. Suppose P_1 is c-normal in G for every Sylow subgroup P of G and every maximal subgroup P_1 of P. Then G is supersoluble.*

Corollary 3.4. ([2, Theorem 3.2]) *Let G be a finite solvable group. Then G is supersoluble if and only if G/H is supersoluble and all maximal subgroups of every Sylow subgroup of $F(H)$ are normal in G.*

Corollary 3.5. ([15, Theorem 1]) *Let G be a finite group such that all maximal subgroups of Sylow subgroups are normal in G. Then G is supersoluble.*

Corollary 3.6. ([13, Theorem 3.5]) *Assume that G/H is supersolvable and all maximal subgroups of the Sylow subgroups of H are normal in G. Then G is supersolvable.*

The proof of the theorem 1.2

Proof. Assume that the theorem is false. And suppose that G is a counter-example of minimal order. By Lemma 2.6, we have that every maximal subgroup of the Sylow subgroups of H is nc-supplemented in H and so G is soluble. Then by [12, Theorem 11], H is soluble. We consider the following two cases:

Case 1. H is a p-group for some prime number p.

Step 1. Let N be the $\mathfrak{F}$-residual subgroup of G. Then $N = C_H(N) = F(H)$.

Let M be a nontrivial normal subgroup of G and let B be a maximal subgroup of MH with $M \leq B$. Then $B = M(H \cap B)$. Since $p = |MH : B| = |MH : M(H \cap B)| = |H : H \cap B|$, it follows that $H \cap B$ is a maximal subgroup of H. By hypothesis, $H \cap B$ is nc-supplemented in G and so is B. Thus B/M is nc-supplemented in G/M by Lemma 2.6(2). The minimal choice of G implies that $G/M \in \mathfrak{F}$. Since N is the $\mathfrak{F}$-residual subgroup of G, it follows that $\Phi(G) = 1$ and N is an elementary abelian subgroup of G since $\mathfrak{F}$ is a saturated formation. Obviously $N \leq H$. Let $F(H)$ be the Fitting subgroup of H. Then $N = F(H)$ since $\mathfrak{F}$ is a saturated formation. Then $F(H) \leq C_H(N) \leq N$ since H is solvable. Thus $N = C_H(N) = F(H)$ is a minimal normal nontrivial p-subgroup of G.

Step 2. H is a Sylow p-subgroup of G.

Suppose that H is not a Sylow p-subgroup of G and G is soluble. It follows, from [5, Theorem 3.5, p. 229], that there exists a Hall $\{p, q\}$-subgroup of G, where q is a prime which is not equal to p, and that HQ is a subgroup of G since H is normal in the Sylow p-subgroup of G and $H \lhd G$. Since G/H is supersoluble, HQ/H is supersoluble . If $HQ < G$, then HQ is supersoluble and so is NQ. Then $N \cap Q = 1$, and $NQ = N \times Q$ since $N \lhd NQ$ and NQ is supersoluble. By [5, Theorem 1.3, p. 218], $Q \leq C_G(N) \leq N$, a contradiction. So H is a Sylow p-subgroup of G.

Step 3. $|N| = p$.

Let H_1 be a maximal subgroup of H. Then $N < H_1$. Otherwise, $N = H_1 \lhd G$, it follows, from [17, Theorem 1.1], that $G \in \mathfrak{F}$. H_1 is nc-supplemented in G by hypothesis and so there exists a subgroup K of G such that $H_1K \lhd G$ and $H_1 \cap K \leq H_{1G}$. Thus we have that $H_1 \cap K = 1$ or $H_1 \cap K = N$. If the former, $H_1K \geq H$ or $H_1K = H_1$ and so $K \geq H$ or $H_1 \geq K$, which contradicts $H_1 \cap K = 1$.

Hence $N \leq K$ and N is a Sylow p-subgroup of K. If N is not a Sylow p-subgroup of K, then there is a Sylow p-subgroup P_K of G with $N < P_K$, and so $H_1P_K = H$ or $H_1P_K = H_1$. In the former case, $P_K = H$ and so $H_1 \cap K = H_1 \cap H = H_1 \lhd G$. It follows, from [13, Theorem 3.5], that G is supersoluble, a contradiction. In the latter, $N < P_K \leq H_1$ and so $N = H_1 \cap K = H_1 \cap P_K = P_K > N$, another contradiction. Thus N is a normal Sylow p-subgroup of K. By Step 2, $K < G$ and so $HK < G$. Since HK/H is supersoluble and every maximal subgroup of H is nc-supplemented in HK, it follows, from the minimal choice of G that, HK is supersoluble and so K is supersoluble. Let Q be a Sylow q-subgroup of K, where q is the largest prime of $|K|$. Thus Q is normal in K, and $NQ = N \times Q$. This means $Q \leq C_K(N) \leq N$, a contradiction. Hence there does not exist non-trivial maximal subgroup of H, that is, H is a Sylow p-subgroup of G of order p. Namely, $|H| = |N| = p$.

Step 4. The final contradiction.

By Step 3, H is a cyclic subgroup. By Lemma 2.7, $G \in \mathfrak{F}$, a contradiction.

Case 2. H is not of prime power order.

Let P be a Sylow p-subgroup of H. Then by hypothesis and Lemma 2.6(1), the maximal subgroups of every Sylow subgroup of H are nc-supplemented in H. Then by Theorem 1.1, H is supersoluble, and so by [4, Hillssatz C] H has a normal Sylow subgroup P

Since P is characteristic in H and $H \lhd G$, it follows that $P \lhd G$. Clearly, $(G/P)/(H/P) \cong G/H \in \mathfrak{F}$. By the minimality of G, $G/P \in \mathfrak{F}$. But now $G \in \mathfrak{F}$ by Case 1, a contradiction.

So the minimal counter-example does not exist.

This completes the proof. □

Remark 3.7. *The condition of Theorem 1.2, "* $\mathfrak{U}$ *" cannot be replaced by "* $\mathfrak{N}$*", where* $\mathfrak{N}$ *is the class of all nilpotent groups. Let* $G = S_3$ *the symmetric group of degree 3. Then* G *is supersoluble, but* G *not nilpotent.*

Corollary 3.8. *(*[18, Theorem 1]*) Let* $\mathfrak{F}$ *be a saturated formation containing* $\mathfrak{U}$*. Suppose that* G *is a group with a soluble normal subgroup* H *such that* $G/H \in \mathfrak{F}$*. If all maximal subgroups of all Sylow subgroups of* $F(H)$ *are c-normal in* G*, then* $G \in \mathfrak{F}$*.*

Corollary 3.9. *(*[19, Theorem 3.1]*) Let* $\mathfrak{F}$ *be a saturated formation containing* $\mathfrak{U}$*. Suppose that* G *is a group with a normal subgroup* H *such* *that $G/H \in \mathfrak{F}$. If all maximal subgroups of all Sylow subgroups of $F^*(H)$ are c-normal in G, then $G \in \mathfrak{F}$.*

Corollary 3.10. ([20, Theorem 1.2]) *Let $\mathfrak{F}$ be a saturated formation containing $\mathfrak{U}$. Suppose that G is a group G with a normal subgroup H such that $G/H \in \mathfrak{F}$. If all maximal subgroups of all Sylow subgroups of $F^*(H)$ are c-supplemented in G, then $G \in \mathfrak{F}$.*

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