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FIRST ORDER COHOMOLOGY OF ℓ^1-MUNN ALGEBRAS AND CERTAIN SEMIGROUP ALGEBRAS

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ABSTRACT. We characterize cyclic and weakly amenable ℓ^1-Munn algebras. In the special case of Rees matrix semigroups, we obtain a new proof of the following result due to Blackmore: The semigroup algebra of every Rees matrix semigroup is weakly amenable. Characterizations of Connes-amenable ℓ^1-Munn algebras with square sandwich matrix and semigroup algebras of Rees matrix semigroups are also provided.

1. Introduction

Cohomology of Banach algebras has received extensive study since the major exposition of Hochschild cohomology theory for Banach algebras by B. E. Johnson[15], with emphasis on the first order cohomology groups. Let A be a Banach algebra. Vanishing of the first cohomology groups $H^1(A, X)$ for certain classes of Banach A-modules X has been given different names, depending on the class of Banach A-modules under investigation among which are cyclic, weak and Connes-amenability. The reader may see [1,2,4,14,15,16,18] for more information. Here, we consider these notions for ℓ^1-Munn algebras and semigroup algebras of Rees matrix semigroups.

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Blackmore showed that the semigroup algebra of every Rees matrix semigroup is weakly amenable [3]. Bowling and Duncan [4] considered weak and cyclic amenability of the convolution algebras of Rees matrix semigroups and gave another proof of Blackmore's result. As shown in [9], these convolution algebras are certain types of ℓ^1- Munn algebras, whose introduction in [9], was motivated by two open problems in [8]. Some of the properties and the structure of these algebras have been studied in [9-11]. In Sections 2 and 3 we study weak and cyclic amenable ℓ^1-Munn algebras and semigroup algebras of Rees matrix semigroups. Our results involve Bowling and Duncan's result. Connes-amenability has been proved an appropriate version of amenability for dual Banach algebras. For more information on this subject, see [18]. In the last section we consider Connes amenability of ℓ^1-Munn algebras and semigroup algebras of Rees matrix semigroups.

Before proceeding further we set up our notations.

Let G be a group , I and J be arbitrary nonempty sets, and $G^0 = G\bigcup\{0\}$. An $I \times J$ matrix A over G^0 that has at most one nonzero entry $a = A(i,j)$ is called a Rees $I \times J$ matrix over G^0 and is denoted by $(a)_{ij}$. Let P be a $J \times I$ matrix over G. The set $S = G \times I \times J$ with the composition $(a,i,j) \circ (b,l,k) = (aP_{jl}b,i,k)$, $\quad (a,i,j),(b,l,k) \in S$, is a semigroup that we denote by $M(G,p)$. Similarly, if P is a $J \times I$ matrix over G^0, then $S = G \times I \times J \cup \{0\}$ is a semigroup under the following composition operation which is denoted by $M^0(G,P)$:

$$(a,i,j) \circ (b,l,k) = \begin{cases} (aP_{jl}b,i,k), & \text{if } p_{jl} \neq 0 \\ 0 & \text{if } p_{jl} = 0, \end{cases}$$

$$(a,i,j) \circ 0 = 0 \circ (a,i,j) = 0 \circ 0 = 0.$$

$M^0(G,P)$ is isomorphic to the semigroup of all Rees $I \times J$ matrices over G^0 with binary operation $A \circ B = APB$. $M^0(G,P)$ [resp. $M(G,P)$] is called the Rees $I \times J$ matrix semigroup over G^0 [resp. G] with sandwich matrix P.

Throughout, A is a Banach algebra and A module means Banach A-bimodule. An A-bimodule X is called dual if there is a closed submodule X_* of X^* such that $X = (X_*)^*$. A is called dual if it is dual as a Banach A-bimodule. Let A be a dual Banach algebra and X be a dual A-bimodule. If for each $x \in X$ the maps $a \longmapsto a.x$ and $a \longmapsto x.a$ from A into X are $\omega^* - \omega^*$ continuous, then X is called a normal A-bimodule.

Let X be an A module. We will denote the set of all bounded [resp. inner] derivations from A into X by $Z^1(A,X)$ [resp. $B^1(A,X)$]. Also, set $H^1(A,X) = Z^1(A,X)/B^1(A,X)$. A bounded derivation $D : A \longrightarrow A^*$ is called cyclic if $\langle Da, b\rangle + \langle Db, a\rangle = 0$, for all $a, b \in A$. The set of all cyclic derivations is denoted by $Z^1_\lambda(A, A^*)$ and $Z^1_\lambda(A, A^*)/B^1(A, A^*)$ by $H^1_\lambda(A, A^*)$. A is called weakly amenable [resp. cyclic amenable] if $H^1(A, A^*) = 0$ [resp. $H^1_\lambda(A, A^*)$]. A dual Banach algebra A is called Connes-amenable if for every normal dual Banach A-bimodule X, every bounded $\omega^* - \omega^*$ continuous derivation from A into X is inner.

Suppose A is unital, I and J are arbitrary index sets and P is a $J \times I$ matrix over A such that all of its nonzero entries are invertible and $\| P \|_\infty \leq 1$. The space $\ell^1(I \times J, A)$ with product $X \circ Y = XPY$ is a Banach algebra which we call the ℓ^1-Munn algebra over A with sandwich matrix P and we denote it with $LM(A,P)$. As shown in [9], Proposition 5.6, semigroup algebras of Rees matrix semigroups are concrete examples of ℓ^1-Munn algebras.

2. **Weak and cyclic amenability of ℓ^1-Munn algebras**

Throughout this section, we assume A is unital.

Theorem 2.1. *If A is cyclic amenable , then so is $LM(A,P)$.*

Proof. Suppose $\alpha \in J$, $\beta \in I$ are such that $P_{\alpha\beta} \neq 0$ and $q = P^{-1}_{\alpha\beta}$. Let $D : LM(A,P) \longrightarrow LM(A,P)^*$ be a bounded cyclic derivation and define $\widehat{D}$ by:

$$\widehat{D} : A \longrightarrow A^*, \quad \langle \widehat{D}a, b\rangle = \langle D(qa\varepsilon_{\beta\alpha}), qb\varepsilon_{\beta\alpha}\rangle, \quad a, b \in A.$$

Clearly $\widehat{D}$ is a bounded linear map. Let $a, b, c \in A$. Then,

$$\begin{aligned}\langle \widehat{D}(ab), c\rangle &= \langle D(qa\varepsilon_{\beta\alpha} \circ qb\varepsilon_{\beta\alpha}), qc\varepsilon_{\beta\alpha}\rangle \\ &= \langle D(qa\varepsilon_{\beta\alpha}), qb\varepsilon_{\beta\alpha} \circ qc\varepsilon_{\beta\alpha}\rangle + \langle D(qb\varepsilon_{\beta\alpha}), qc\varepsilon_{\beta\alpha} \circ qa\varepsilon_{\beta\alpha}\rangle \\ &= \langle D(qa\varepsilon_{\beta\alpha}), qbc\varepsilon_{\beta\alpha}\rangle + \langle D(qb\varepsilon_{\beta\alpha}), qca\varepsilon_{\beta\alpha}\rangle \\ &= \langle \widehat{D}a, bc\rangle + \langle \widehat{D}b, ca\rangle \\ &= \langle \widehat{D}a.b + a.\widehat{D}b, c\rangle.\end{aligned}$$

On the other hand, by assumption we have,

$$\langle \widehat{D}a, b\rangle + \langle \widehat{D}b, a\rangle = \langle D(qa\varepsilon_{\beta\alpha}), qb\varepsilon_{\beta\alpha}\rangle + \langle D(qb_{\beta\alpha}), qa\varepsilon_{\beta\alpha}\rangle = 0.$$

So, $\widehat{D}$ is a bounded cyclic derivation. Let $\widehat{\psi} \in A^*$ be such that $\widehat{D}(a) = a.\widehat{\psi} - \widehat{\psi}.a, (a \in A)$. Define,

$$\psi(a\varepsilon_{ij}) = \widehat{\psi}(p_{ji}a) + \langle D(q\varepsilon_{\beta j}), a\varepsilon_{i\alpha}\rangle, \quad i \in I, j \in J, a \in A.$$

It is easy to see that $\psi \in LM(A,P)^*$. Let $S = a\varepsilon_{ij}$ $T = b\varepsilon_{kl}$ be nonzero elements in $LM(A,P)$, $U = q\varepsilon_{\beta j}$, $V = q\varepsilon_{\beta l}$, $X = a\varepsilon_{i\alpha}$ and $Y = qp_{jk}b\varepsilon_{\beta\alpha}$. Then, $S = X \circ U$ and $U \circ T = Y \circ V$. So,

$$\langle D(X), U \circ T\rangle = -\langle D(Y \circ V), X\rangle$$

$$= -\langle D(Y), V \circ X\rangle - \langle D(V), X \circ Y\rangle$$

$$= \langle D(V \circ X), Y\rangle - \langle D(V), X \circ Y\rangle. \tag{2.1}$$

But,

$$\langle D(V \circ X), Y\rangle = \langle D(qp_{li}a\varepsilon_{\beta\alpha}), qp_{jk}b\varepsilon_{\beta\alpha}\rangle = \langle \widehat{D}(p_{li}a), p_{jk}b\rangle$$

$$= \langle (p_{li}a).\widehat{\psi} - \widehat{\psi}.(p_{li}a), p_{jk}b\rangle$$

$$= \langle \widehat{\psi}, p_{jk}bp_{li}a\rangle - \langle \widehat{\psi}, p_{li}ap_{jk}b\rangle. \tag{2.2}$$

So by (2.1) and (2.2), we have,

$$\begin{aligned}\langle DS, T\rangle &= \langle DU, T \circ X\rangle + \langle DX, U \circ T\rangle \\ &= \langle DU, T \circ X\rangle + \langle D(V \circ X), Y\rangle - \langle DV, X \circ Y\rangle \\ &= \langle D(q\varepsilon_{\beta j}), bp_{li}a\varepsilon_{k\alpha}\rangle + \langle \widehat{\psi}, p_{jk}bp_{li}a\rangle \\ &\quad -\langle \widehat{\psi}, p_{li}ap_{jk}b\rangle - \langle D(q\varepsilon_{\beta l}), ap_{jk}b\varepsilon_{i\alpha}\rangle \\ &= \langle \psi, bp_{li}a\varepsilon_{kj}\rangle - \langle \psi, ap_{jk}b\varepsilon_{il}\rangle = \langle \delta_{-\psi}(S), T\rangle.\end{aligned}$$

Therefore, D is inner. □

Lemma 2.2. *If A is weakly amenable, then every bounded derivation $D : LM(A,P) \longrightarrow LM(A,P)^*$ is cyclic.*

Proof. It is enough to show the cyclic identity for Rees matrices. Let $S = a\varepsilon_{ij}$ and $T = b\varepsilon_{kl}$, where a,b$\neq 0$.

Step I: $p_{jk} = 0$. Then, $0 = D0 = D(S \circ T) = DS.T + S.DT$. So, for every $X \in LM(M,P)$,

$$0 = \langle DS.T, X\rangle + \langle S.DT, X\rangle$$

$$= \langle DS, T \circ X \rangle + \langle DT, X \circ S \rangle. \tag{2.3}$$

If $p_{li} \neq 0$, then take $X = E = p_{li}^{-1}\varepsilon_{il}$. Clearly, E is an idempotent, $T \circ E = T$, $E \circ S = S$ and hence if we substitute X with E in (2.3), then we get,

$$\langle DS, T \rangle + \langle DT, S \rangle = 0.$$

So, the cyclic condition holds. If $p_{li} = 0$, then choose $\alpha \in J$ and $\beta \in I$ such that $p_{\alpha\beta} \neq 0$. Let $Y = a\varepsilon_{i\alpha}$ and $Z = p_{\alpha\beta}^{-1}\varepsilon_{\beta j}$. Then, $Y \circ Z = S$ and $Z \circ T = T \circ Y = 0$. So,

$$\begin{aligned}\langle DS, T \rangle &= \langle D(Y \circ Z), T \rangle = \langle DY.Z, T \rangle + \langle Y.DZ, T \rangle \\ &= \langle DY, Z \circ T \rangle + \langle DZ, T \circ Y \rangle = 0.\end{aligned}$$

Similarly, $\langle DT, S \rangle = 0$ and hence the cyclic condition holds.

Step II: $p_{jk} \neq 0$. By the symmetry of the cyclic condition, we may assume $p_{li} \neq 0$ as well. As above, let $E = p_{li}^{-1}\varepsilon_{il}$. Then, we have,

$$\begin{aligned}\langle D(S \circ T), E \rangle &= \langle DS.T, E \rangle + \langle S.DT, E \rangle \\ &= \langle DS, T \circ E \rangle + \langle DT, E \circ S \rangle \\ &= \langle DS, T \rangle + \langle DT, S \rangle. \end{aligned} \tag{2.4}$$

Now, define the map $\phi : A \longrightarrow LM(A, P)$ by $\phi(c) = p_{li}^{-1}c\varepsilon_{il}$. Clearly, ϕ is an injective bounded linear map. Let $c, d \in A$. Then,

$$\phi(cd) = p_{li}^{-1}cd\varepsilon_{il} = (p_{li}^{-1}c\varepsilon_{il}) \circ (p_{li}^{-1}d\varepsilon_{il}) = \phi(c) \circ \phi(d).$$

Therefore, ϕ is a bounded algebra monomorphism. Now, define the map,

$$\widehat{D} : A \longrightarrow A^*, \qquad \langle \widehat{D}a, b \rangle = \langle (D\phi)(a), p_{li}^{-1}b\varepsilon_{il} \rangle \qquad a, b \in A.$$

Then, $\widehat{D}$ is a bounded linear map. If $a, b, c \in A$, then,

$$\begin{aligned}\langle \widehat{D}(ab), c \rangle &= \langle D(\phi(ab)), p_{li}^{-1}c\varepsilon_{il} \rangle \\ &= \langle (D\phi(a)).\phi(b), p_{li}^{-1}c\varepsilon_{il} \rangle + \langle \phi(a).(D\phi(b)), p_{li}^{-1}c\varepsilon_{il} \rangle \\ &= \langle D(\phi(a)), \phi(b) \circ p_{li}^{-1}c\varepsilon_{il} \rangle + \langle D(\phi(b)), p_{li}^{-1}c\varepsilon_{il} \circ \phi(a) \rangle \\ &= \langle D(\phi(a)), p_{li}^{-1}b\varepsilon_{il} \circ p_{li}^{-1}c\varepsilon_{il} \rangle + \langle D(\phi(b)), p_{li}^{-1}c\varepsilon_{il} \circ p_{li}^{-1}a\varepsilon_{il} \rangle \\ &= \langle D(\phi(a)), p_{li}^{-1}bc\varepsilon_{il} \rangle + \langle D(\phi(b)), p_{li}^{-1}ca\varepsilon_{il} \rangle \\ &= \langle \widehat{D}a, bc \rangle + \langle \widehat{D}b, ca \rangle \\ &= \langle \widehat{D}a.b + a.\widehat{D}b, c \rangle.\end{aligned}$$

Therefore, $\widehat{D}$ is a bonded derivation and, by assumption, it is inner. So, for every $a \in A$, $\langle \widehat{D}a, 1 \rangle = 0$, and hence,

$$\langle D(S \circ T), E \rangle = \langle D(ap_{jk}b\varepsilon_{il}), p_{li}^{-1}\varepsilon_{il} \rangle$$

$$= \langle D\phi(p_{li}ap_{jk}b), p_{li}^{-1}\varepsilon_{il}\rangle$$

$$= \langle \widehat{D}(p_{li}ap_{jk}b), 1\rangle = 0. \tag{2.5}$$

Now, by (2.4) and (2.5),

$$\langle DS, T\rangle + \langle DT, S\rangle = \langle D(S \circ T), E\rangle = 0.$$

Therefore, in either case, the cyclic condition holds. □

The following theorem is an immediate consequence of Theorem 2.1 and Lemma 2.2.

Theorem 2.3. *If A is weakly amenable, then so is $LM(A, P)$.*

Remark 2.4. In the proof of the following theorem we use Lemma 3.7 in [9] which is true only for the case that the sandwich matrix P is square; i.e., the index sets I and J are equal. If P is a regular square matrix and $LM(A, P)$ has a bounded approximate identity, then the converse of theorems 2.1 and 2.3 are also true.

Theorem 2.5. *Suppose P is a regular square matrix and $LM(A, P)$ has a bounded approximate identity. Then, $LM(A, P)$ is cyclic [resp. weakly] amenable if and only if A is cyclic [resp. weakly] amenable.*

Proof. we need only to prove the converse. By Lemma 3.7 in [9], the index set I is finite and $LM(A, P)$ is topologically isomorphic to $A\widehat{\otimes}M_n$, for some $n \in N$. If $D : A \longrightarrow A^*$ is a bounded derivation, then $D \otimes 1$ is a bounded derivation from $LM(A, P)$ to $LM(A, P)^*$. Moreover, if D is cyclic, then so is $D \otimes 1$, since the action of M_n on itself as its dual, is componentwise. Now, suppose $LM(A, P)$ is weakly amenable and $D : A \longrightarrow A^*$ is a bounded derivation. There is a $\phi = \sum_{i=1}^{n} f^i \otimes B^i \in A^* \otimes M_n = LM(A, P)^*$ such that $D \otimes 1 = \delta_\phi$. It is easy to see that $D = \delta_f$, where $f = \sum_{i=1}^{m} B^i_{11} f^i$. The argument for cyclic derivations is the same. □

Now, we apply the preceding results to the semigroup algebras of Rees matrix semigroups. The following theorem is (Corollary 5.3 in [3]) comes with a different proof. Another proof can also be found (see Theorem 2.4 in [4]).

Theorem 2.6. *If S is a Rees matrix semigroup, then $\ell^1(S)$ is weakly amenable.*

Proof. Suppose $S = M^\circ(G,P)$. Then, by Proposition 5.6 in [9], $\ell^1(S)/\ell^1(0)$ is isomorphic to $LM(\ell^1(G),P)$. By Johnson' Theorem, $\ell^1(G)$ is weakly amenable. So, by Theorem 2.3, $LM(\ell^1(G),P)$ is weakly amenable, and hence so is $\ell^1(S)$. □

3. Connes-amenability of ℓ^1-Munn algebras and Rees matrix semigroup algebras

Throughout this section, we assume A ia a dual Banach algebra, $I = J$, P and $LM(A,P)$ are as in Section 1. It is well known that $c_\circ(I \times J, A_*)^* = \ell^1(I \times J, A) = LM(A,P)$. Moreover, $c_\circ(I \times J, A_*)$ is an $LM(A,P)$ submodule of $LM(A,P)^* = \ell^\infty(I \times J, A^*)$. Therefore, $LM(A,P)$ is a dual Banach algebra. In the proof of the following theorem, we use Lemma 3.7 in [9], which is true only for the case that the index sets I and J are equal.

Theorem 3.1. *Suppose A is a unital dual Banach algebra and the index sets I and J are equal. Then, $LM(A,P)$ is Connes-amenable if and only if it has a bounded approximate identity and A is Connes-amenable.*

Proof. Suppose $LM(A,P)$ has a bounded approximate identity and A is Connes-amenable. By Lemma 3.7 in [9], $LM(A,P) \simeq A\widehat{\otimes}M_n$. Since $A\widehat{\otimes}M_n = (A_*\check{\otimes}M_n)^*$ and both of A and M_n are Connes-amenable, then by using the argument of Theorem 5.4 in [15] with appropriate modifications, we can see that $LM(A,P)$ is Connes-amenable.

Conversely, suppose $LM(A,P)$ is Connes-amenable and X is a normal A-bimodule. Then, $LM(A,P)$ is unital and hence by Lemma 3.7 in [9], $LM(A,P) \simeq A\widehat{\otimes}M_n$. Since $X\widehat{\otimes}M_n = (X_*\check{\otimes}M_n)^*$, then $X\widehat{\otimes}M_n$ is a dual $A\widehat{\otimes}M_n-$ bimodule. Moreover, by the above remark, elements of $A\widehat{\otimes}M_n$ and $X\widehat{\otimes}M_n$ have finite representations in terms of elementary tensors and action of $X\widehat{\otimes}M_n$ on $X_*\check{\otimes}M_n$ is componentwise. Thus, normality of $X\widehat{\otimes}M_n$ follows from normality of X. Now, suppose $D : A \longrightarrow X$ is a bounded $\omega^* - \omega^*$ continuous derivation. Then, $D \otimes 1 : A\widehat{\otimes}M_n \longrightarrow X\widehat{\otimes}M_n$ is a bounded derivation, and $\omega^* - \omega^*$ continuity of $D \otimes 1$ follows from the fact that elements of $A\widehat{\otimes}M_n$ and $X\widehat{\otimes}M_n$ are finite sums of elementary tensors and action of $X\widehat{\otimes}M_n$ on $X_*\check{\otimes}M_n$ is componentwise. Therefore, by assumption, $D \otimes 1 = \delta_\phi$, for some $\phi \in X\widehat{\otimes}M_n$. By the above remark, ϕ has a unique representation of the form $\phi = \sum_{i,j=1}^n x_{ij}\otimes$

ε_{ij}. Let $a \in A$ and $r \leq n$ be a natural number. Then,

$$\begin{aligned} Da \otimes \varepsilon_{rr} &= (D \otimes 1)(a \otimes \varepsilon_{rr}) = (a \otimes \varepsilon_{rr}).\phi - \phi.(a \otimes \varepsilon_{rr}) \\ &= \sum_{j=1}^{n} a x_{rj} \otimes \varepsilon_{rj} - \sum_{i=1}^{n} x_{ir} a \otimes \varepsilon_{ir}. \end{aligned}$$

Letting $a = 1$, we conclude that for all $i, j \neq r$, $x_{rj} = 0 = x_{ir}$. Thus, $\phi = \sum_{i=1}^{n} x_{ii} \otimes \varepsilon_{ii}$. Now, the identities,

$$Da \otimes \varepsilon_{11} = (a \otimes \varepsilon_{11}).\phi - \phi.(a \otimes \varepsilon_{11}) = (a x_{11} - x_{11} a) \otimes \varepsilon_{11}$$

imply that $D = \delta_{x_{11}}$. □

Theorem 3.2. *Suppose P is a square matrix over G° and $S = M^{\circ}(G, P)$. Then, the following conditions are equivalent.*

i) *G is amenable and $\ell^1(S)$ has a bounded approximate identity.*

ii) *$\ell^1(S)$ is Connes-amenable.*

Proof. Proposition 5.6 and Lemma 5.1(ii) of [9] imply that existence of a bounded approximate identity in $\ell^1(S)$ is equivalent to the existence of a bounded approximate identity in $LM(\ell^1(G), P)$. Also, Theorem 4.4.13 in [18], $\ell^1(G)$ is Connes-amenable if and only if G is amenable. On the other hand, by Lemma 5.1(ii) and Proposition 5.6 in [9], Connes-amenability of $\ell^1(S)$ is equivalent to Connes-amenability of $LM(\ell^1(G), P)$. Therefore, equivalence of (i) and (ii) follows from Theorem 3.1 . □

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