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THE FISCHER-CLIFFORD MATRICES AND CHARACTER TABLE OF THE MAXIMAL SUBGROUP $2^9{:}(L_3(4){:}S_3)$ OF $U_6(2){:}S_3$

A. L. PRINS

(Communicated by Ali Reza Ashrafi)

Abstract. The full automorphism group of $U_6(2)$ is a group of the form $U_6(2){:}S_3$. The group $U_6(2){:}S_3$ has a maximal subgroup $2^9{:}(L_3(4){:}S_3)$ of order 61931520. In the present paper, we determine the Fischer-Clifford matrices (which are not known yet) and hence compute the character table of the split extension $2^9{:}(L_3(4){:}S_3)$.
Keywords: Coset analysis, Fischer-Clifford matrices, permutation character, fusion map.
MSC(2010): Primary: 20C15; Secondary: 20C40.

1. Introduction

The outer automorphism of the the unitary group $U_6(2)$, often referred to as Fi_{21}, is the symmetric group S_3 and thus the full automorphism group of $U_6(2)$ is a group of the form $U_6(2){:}S_3$. We found in the ATLAS [7] that the group $U_6(2){:}S_3$ has a maximal subgroup of the form $2^9{:}(L_3(4){:}S_3)$ and has order 61931520. We should add here that $U_6(2){:}S_3$ is a maximal subgroup of the Conway group Co_1.

The Fischer-Clifford matrices method [8] seems generally to have been used to calculate character tables of many complicated maximal subgroups of sporadic simple groups and their automorphism groups; see for example [2, 3, 16, 17] and [19]. Recently, the technique of Fischer-Clifford matrices to compute the character tables of some split group extensions, has been used by Ali [4] and others, including the author (see [9, 10, 11] and [20]). The Fischer-Clifford matrices method relies on the fact that every irreducible character of an extension group $\overline{G} = N \cdot G$ can be obtained by induction from the inertia groups of $\overline{G}$. Since the character tables of these inertia subgroups are usually much

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larger and more complicated to compute than the character table of $\overline{G}$, it suffices to use only the ordinary or projective character tables of the inertia factor groups H_i of $\overline{G}$ and arithmetical properties of the Fischer-Clifford matrices to assemble the character table of $\overline{G}$.

In the present paper, the author computes the Fischer-Clifford matrices of $2^9{:}(L_3(4){:}S_3)$ using the properties of these matrices as discussed in [18]. Consequently, the associated character table of $2^9{:}(L_3(4){:}S_3)$ is determined. Although the computation of the character table of the group $2^9{:}(L_3(4){:}S_3)$ can be done in the computer algebra systems MAGMA [5] and GAP [21], the process of constructing the character table of $2^9{:}(L_3(4){:}S_3)$ using the method of Fischer-Clifford matrices reveals interesting facts about the group itself.

The method of coset-analysis (see [14, 15] and [18]) is used to compute the conjugacy classes of elements of $2^9{:}(L_3(4){:}S_3)$. Let $\overline{G} = N \cdot G$ be an extension of N by G, where N is abelian. Then for $g \in G$, we write $\overline{g}$ for a lifting of g in $\overline{G}$ under the natural homomorphism $\overline{G} \longrightarrow G$. We consider a coset $N\overline{g}$ for each class representative g of G, writing k for number of orbits of N acting by conjugation on the coset $N\overline{g}$, and f_j for the number of these fused by the action of $\left\{\overline{h} : h \in C_G(g)\right\}$. Note if $\overline{G}$ is a split extension then $\overline{g}$ becomes g since $G \leq \overline{G}$. The order of the centralizer $C_{\overline{G}}(x)$ for each element $x \in \overline{G}$ in a conjugacy class $[x]_{\overline{G}}$ is given by $|C_{\overline{G}}(x)| = \frac{k|C_G(g)|}{f_j}$. Most of our computations were carried out with the aid of the computer algebra systems MAGMA and GAP. Our notation is standard and readers may refer to the ATLAS.

2. **Theory of Fischer-Clifford matrices**

Since the character table of $2^9{:}(L_3(4){:}S_3)$ will be constructed by means of its Fischer-Clifford matrices the author will give a brief theoretical background of this technique.

Let $\overline{G} = N{\cdot}G$ be an extension of N by G and $\theta \in Irr(N)$, where $Irr(N)$ denotes the irreducible characters of N. Define θ^g by $\theta^g(n) = \theta(gng^{-1})$ for $g \in \overline{G}$ and $n \in N$ and $\theta^g \in Irr(N)$. Let $\overline{H} = \left\{x \in \overline{G} | \theta^x = \theta\right\} = I_{\overline{G}}(\theta)$ be the inertia group of θ in $\overline{G}$ then N is normal in $\overline{H}$. We say that θ is extendible to $\overline{H}$ if there exists $\phi \in Irr(\overline{H})$ such that $\phi\downarrow_N = \theta$. If θ is extendible to $\overline{H}$, then by Gallagher [13], we have

$$\left\{\phi | \phi \in Irr(\overline{H}), < \phi\downarrow_N, \theta > \neq 0\right\} = \left\{\beta\phi | \beta \in Irr(\overline{H}/N)\right\}.$$

Let $\overline{G}$ have the property that every irreducible character of N can be extended to its inertia group. Now let $\theta_1 = 1_N, \theta_2, \cdots, \theta_t$ be representatives of the orbits of $\overline{G}$ on $Irr(N)$, $\overline{H}_i = I_{\overline{G}}(\phi_i)$, $1 \leq i \leq t$, $\phi_i \in Irr(\overline{H}_i)$ be an extension of θ_i to

$\overline{H}_i$ and $\beta \in Irr(\overline{H}_i)$ such that $N \subseteq ker(\beta)$. Then it can be shown that

$$\begin{aligned} Irr(\overline{G}) &= \bigcup_{i=1}^{t} \{(\beta\,\phi_i)^{\overline{G}} \,|\, \beta \in Irr(\overline{H}_i), N \subseteq ker(\beta)\} \\ &= \bigcup_{i=1}^{t} \{(\beta\,\phi_i)^{\overline{G}} \,|\, \beta \in Irr(\overline{H}_i/N)\} \end{aligned}$$

Hence the irreducible characters of $\overline{G}$ will be divided into blocks, where each block corresponds to an inertia group $\overline{H}_i$.

Let H_i be the inertia factor group and ϕ_i be an extension of θ_i to $\overline{H}_i$. Take $\theta_1 = 1_N$ as the identity character of N, then $\overline{H}_1 = \overline{G}$ and $H_1 \cong G$. Let $X(g) = \{x_1, x_2, \cdots, x_{c(g)}\}$ be a set of representatives of the conjugacy classes of $\overline{G}$ from the coset $N\overline{g}$ whose images under the natural homomorphism $\overline{G} \longrightarrow G$ are in $[g]$ and we take $x_1 = \bar{g}$. We define $R(g) = \{(i, y_k) \,|\, 1 \le i \le t, H_i \cap [g] \ne \emptyset, 1 \le k \le r\}$ and we note that y_k runs over representatives of the conjugacy classes of elements of H_i which fuse into $[g]$ in G. Let $\{y_{l_k}\}$ be the representatives of conjugacy classes of $\overline{H}_i$ that contain y_k. Then we define the Fischer-Clifford matrix $M(g)$ by $M(g) = (a^j_{(i,y_k)})$, where $a^j_{(i,y_k)} = \sum_l' \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H_i}}(y_{l_k})|} \phi_i(y_{l_k})$, with columns indexed by $X(g)$ and rows indexed by $R(g)$ and where $\sum_l'$ is the summation over all l for which $y_{l_k} \sim x_j$ in $\overline{G}$. Then the partial character table of $\overline{G}$ on the classes $\{x_1, x_2, \cdots, x_{c(g)}\}$ is given by $\begin{bmatrix} C_1(g)\,M_1(g) \\ C_2(g)\,M_2(g) \\ \vdots \\ C_t(g)\,M_t(g) \end{bmatrix}$ where the Fischer-Clifford matrix $M(g) = \begin{bmatrix} M_1(g) \\ M_2(g) \\ \vdots \\ M_t(g) \end{bmatrix}$ is divided into blocks $M_i(g)$ with each block corresponding to an inertia group $\overline{H}_i$ and $C_i(g)$ is the partial character table of H_i consisting of the columns corresponding to the classes that fuse into $[g]$ in G. Hence the full character table of $\overline{G}$ will be $\begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_t \end{bmatrix}$, where $\Delta_i = [C_i(1)M_i(1)|C_i(g_2)M_i(g_2)|...|C_i(g_k)M_i(g_k)]$ with $\{1, g_1, g_2, ..., g_k\}$ the representatives of conjugacy classes of G. We can also observe that $|Irr(\overline{G})|$ $= |Irr(H_1)| + |Irr(H_2)| +...+ |Irr(H_t)|$.

Let $x_j \in X(g)$ and define $m_j = [C_{\overline{g}}{:}C_{\overline{G}}(x_j)]$, where $C_{\overline{g}} = \{x \in \overline{G} | x(N\overline{g}) = (N\overline{g})x\}$ is the set stabilizer of $N\overline{g}$ in $\overline{G}$ under the action by conjugation of $\overline{G}$ on $N\overline{g}$. Hence $C_{\overline{g}} \leq \overline{G}$ and it can be shown that N is normal in $C_{\overline{g}}$. The Fischer-Clifford matrix $M(g)$ is partitioned row–wise into blocks, where each block corresponds to an inertia group. The columns of $M(g)$ are indexed by $X(g)$ and for each $x_j \in X(g)$, at the top of the columns of $M(g)$, we write $|C_{\overline{G}}(x_j)|$ and at the bottom we write m_j. The rows of $M(g)$ are indexed by $R(g)$ and on the left of each row we write $|C_{H_i}(y_k)|$, where y_k fuses into $[g]$ in G. Then in general we can write $M(g)$ with corresponding weights for rows and columns as follows, where blocks corresponding to the inertia groups are separated by horizontal lines.

$$
\begin{array}{c}
\\
|C_G(g)| \\
|C_{H_2}(y_1)| \\
|C_{H_2}(y_2)| \\
\vdots \\
|C_{H_i}(y_1)| \\
|C_{H_i}(y_2)| \\
\vdots \\
|C_{H_t}(y_1)| \\
|C_{H_t}(y_2)| \\
\vdots \\
\\
\end{array}
\begin{array}{cccc}
|C_{\overline{G}}(x_1)| & |C_{\overline{G}}(x_2)| & \cdots & |C_{\overline{G}}(x_{c(g)})| \\
\left(\begin{array}{cccc}
a^1_{(1,g)} & a^2_{(1,g)} & \cdots & a^{c(g)}_{(1,g)} \\
\hline
a^1_{(2,y_1)} & a^2_{(2,y_1)} & \cdots & a^{c(g)}_{(2,y_1)} \\
a^1_{(2,y_2)} & a^2_{(2,y_2)} & \cdots & a^{c(g)}_{(2,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\
\hline
a^1_{(i,y_1)} & a^2_{(i,y_1)} & \cdots & a^{c(g)}_{(i,y_1)} \\
a^1_{(i,y_2)} & a^2_{(i,y_2)} & \cdots & a^{c(g)}_{(i,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\
\hline
a^1_{(t,y_1)} & a^2_{(t,y_1)} & \cdots & a^{c(g)}_{(t,y_1)} \\
a^1_{(t,y_2)} & a^2_{(t,y_2)} & \cdots & a^{c(g)}_{(t,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\
\hline
\end{array}\right) \\
m_1 & m_2 & \cdots & m_{c(g)}
\end{array}
$$

The Fischer-Clifford matrix $M(g)$ satisfies the following properties:
(a) $a^j_{(1,g)} = 1$ for all $j = \{1, 2, .., c(g)\}$.
(b)$|X(g)| = |R(g)|$.
(c) $\sum_{j=1}^{c(g)} m_j \, a^j_{(i,y_k)} \, \overline{a^j_{(i',y'_k)}} = \delta_{(i,y_k),(i',y'_k)} \, \frac{|C_G(g)|}{|C_{H_i}(y_k)|} \, |N|$.
(d) $\sum_{(i,y_k)\in R(g)} a^j_{(i,y_k)} \, \overline{a^{j'}_{(i,y_k)}} \, |C_{H_i}(y_k)| = \delta_{jj'} \, |C_{\overline{G}}(x_j)|$.
(e) $M(g)$ is square and nonsingular.
If N is elementary abelian, then we obtain the following additional properties of $M(g)$:
(f) $a^1_{(i,y_k)} = \frac{|C_G(g)|}{|C_{H_i}(y_k)|}$.
(g) $|a^1_{(i,y_k)}| \geq |a^j_{(i,y_k)}|$.
(h) $a^j_{(i,y_k)} \equiv a^1_{(i,y_k)} (\text{mod } p)$, if $|N| = p^n$, for p a prime and $n \in \mathbb{N}$.

The group $\overline{G} = 2^9{:}(L_3(4){:}S_3)$ is a split extension with 2^9 abelian and therefore by Mackey's theorem [13] each irreducible character of 2^9 can be extended to its inertia group in $\overline{G}$. Hence by the above theoretical outline we can fully determine the character table of $\overline{G}$.

3. The conjugacy classes of $2^9{:}(L_3(4){:}S_3)$

In this section, we apply the method of coset analysis, as discussed in [18], to determine the conjugacy classes of elements of $2^9{:}(L_3(4){:}S_3)$. Firstly, the group $\overline{G} = 2^9{:}(L_3(4){:}S_3)$ is represented as a permutation group acting on 693 points inside $U = U_6(2){:}S_3$ using Wilson's online ATLAS of Group Representations [23]. Also, with the aid of MAGMA, we determined the specification $\overline{G} = N_U(2^9) = N(2A_{21}B_{210}C_{280})$. Note that of the 511 cyclic subgroups of 2^9 (neglecting the identity subgroup), 21 contain the class $2A$, 210 contain the class $2B$, and 280 contain the class $2C$ of U.

Since $2^9{:}(L_3(4){:}S_3)$ is represented as a permutation group, the MAGMA commands "M:= GModule($\overline{G}$,2^9)" and "M:Maximal" are used to represent $L_3(4){:}S_3$ as a matrix group of dimension 9 over the Galois field GF(2). The generators g_1 and g_2 of $L_3(4){:}S_3$, with respective orders of 4 and 6, are as follows:

$$g_1 = \begin{pmatrix} 0&0&1&1&0&1&0&1&0 \\ 0&0&1&1&0&0&0&0&1 \\ 1&0&0&1&0&0&1&0&1 \\ 0&0&0&1&0&0&0&0&0 \\ 0&1&1&1&1&1&1&1&0 \\ 1&0&1&1&0&0&1&1&0 \\ 1&1&1&1&1&0&1&0&0 \\ 1&0&1&0&1&1&1&1&0 \\ 1&1&0&0&0&0&0&0&1 \end{pmatrix} \quad g_2 = \begin{pmatrix} 1&0&1&0&0&0&0&0&0 \\ 1&1&0&1&0&1&0&1&0 \\ 0&1&0&1&1&0&0&1&0 \\ 1&1&1&1&1&0&1&0&0 \\ 1&1&0&0&1&0&0&1&0 \\ 0&0&1&1&0&0&0&1&0 \\ 0&0&0&1&0&1&1&1&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&1&1&1&0&0&1 \end{pmatrix}.$$

Throughout this paper, let $\overline{G} = 2^9{:}(L_3(4){:}S_3)$ be the split extension of $N = 2^9$ by $G = L_3(4){:}S_3$. Having obtained G as a matrix group, we act G on the conjugacy classes of $N \cong V_9(2)$, where $V_9(2)$ is the vector space of dimension 9 over the Galois field $GF(2)$. As a result of this action, we obtain that the elements of N are partitioned into 4 orbits with respective lengths of 1, 21, 210 and 280. With the aid of MAGMA and the ATLAS, we are able to identify the structures of the stabilizers corresponding to the 4 orbits of elements of N. The point stabilizers, which are subgroups of G, are identified as $P_1 = L_3(4){:}S_3$, $P_2 = 2^4{:}(3 \times A_5){:}2$, $P_3 = 2^4{:}(S_3 \times S_3)$ and $P_4 = 3^2{:}2S_4$, where P_2 and P_4 are maximal in P_1. We should note here that the group $L_3(4){:}S_3$ has two non-conjugate isomorphic maximal subgroups $L_1 = P_2$ and L_2, having the same structure $2^4{:}(3 \times A_5){:}2$. The stablizer P_3 sits maximal in L_2.

Let $\chi(L_3(4){:}S_3|2^9)$ be the permutation character of $L_3(4){:}S_3$ on the classes of 2^9. We obtain that $\chi(L_3(4){:}S_3|2^9) = I_{P_1}^{P_1} + I_{P_2}^{P_1} + I_{P_3}^{P_1} + I_{P_4}^{P_1} = 4 \times 1a + 4 \times 20a + 45a + 45b + 2 \times 64a + 2 \times 105a$,
where $I_{P_1}^{P_1}$, $I_{P_2}^{P_1}$, $I_{P_3}^{P_1}$ and $I_{P_4}^{P_1}$ are the identity characters of the point stabilizers P_i, $i = 1, 2, 3, 4$, induced to G. Note that the identity characters $I_{P_i}^{P_1}$ are

identified with the permutation characters $\chi(L_3(4){:}S_3|P_i)$ of $L_3(4){:}S_3$ acting on the classes of the point stabilizers P_i. We found that $\chi(L_3(4){:}S_3|P_1) = 1a$, $\chi(L_3(4){:}S_3|P_2) = 1a + 20a$, $\chi(L_3(4){:}S_3|P_3) = 1a + 2 \times 20a + 64a + 105a$ and $\chi(L_3(4){:}S_3|P_4) = 1a+20a+45a+45b+64a+105a$. The values of $\chi(L_3(4){:}S_3|2^9)$ on the different classes of G determine the number k of fixed points of each $g \in G$ in 2^9. The values of k are found in the second column of Table 1. All the computations involved in obtaining $\chi(L_3(4){:}S_3|2^9)$ were carried out in MAGMA.

The values of k enabled us to determine the number f_j of orbits Q_i's, $1 \leq i \leq k$ which have fused together under the action of $C_G(g)$, for each class representative $g \in G$, to form one orbit $\triangle_f$. Mpono in [18] used the technique of coset analysis to develop Programmes A and B (see [18]) in CAYLEY [6] for the computation of the conjucacy classes of a split extension $\overline{G} = N{:}G$, where N is an elementary abelian p-group, for a prime p, on which a linear group G acts. Ali in [1] adapted Programmes A and B for MAGMA and these computer programmes are used by the author to compute the conjugacy classes of $\overline{G}$. Programme A computes the values of the $f_j's$, whereas Programme B determines the order of the elements for each conjugacy class $[x]$ in $\overline{G}$. We obtain that $\overline{G}$ has exactly 64 conjugacy classes. All the information involving the conjugacy classes of $\overline{G}$ are listed in Table 1.

4. **The inertia groups of** $2^9{:}(L_3(4){:}S_3)$

Since G has four orbits on N, then by Brauer's Theorem [12] G acts on $Irr(N)$ with the same number of orbits. The lengths of the 4 orbits will be 1 ,r, s and t where $r + s + t = 511$, with corresponding point stabilizers H_1, H_2, H_3 and H_4 as subgroups of G such that $[G : H_1]$ =1, $[G : H_2]$ =r, $[G : H_3]$= s and $[G : H_4] = t$. We generate G as a permutation group on a set of cardinality 693 within MAGMA. Then the maximal subgroups of G, as well as their maximal subgroups are computed. Now, considering the indices of these subgroups in G, the number of the classes of these subgroups, and also the fact that $\overline{G}$ has 64 conjugacy classes, we deduce that the action of G on N has orbits of lengths 1, $r = 21$, $s = 210$ and $t = 280$ with respective point stabilizers $H_1 = L_3(4){:}S_3$, $H_2 = 2^4{:}(3 \times A_5){:}2$, $H_3 = 2^4{:}(S_3 \times S_3)$ and $H_4 = 3^2{:}2S_4$. Thus we obtain four inertia groups $\overline{H}_i = 2^{9\cdot}H_i$, $i \in \{1, 2, 3, 4\}$, in $2^9{:}(L_3(4){:}S_3)$. The structures of H_2 and H_4 have been identified by checking the indices of the maximal subgroups of $L_3(4){:}S_3 \cong L_3(4).3.2_2$ in the ATLAS. The structure of H_3 was determined by direct computations in MAGMA. The groups H_2, H_3 and H_4 are constructed from elements within G and the generators are as follows:

- $H_2 = \langle \alpha_1, \alpha_2 \rangle$, $\alpha_1 \in 3A$, $\alpha_2 \in 6B$ where

$$\alpha_1=\begin{pmatrix} 0&0&0&0&1&1&1&1&0\\ 0&0&0&1&0&1&1&0&0\\ 1&1&1&1&1&0&0&1&0\\ 1&1&0&1&0&1&0&1&1\\ 0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0&0&0\\ 1&0&0&0&1&1&1&1&0\\ 0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&1 \end{pmatrix}, \alpha_2=\begin{pmatrix} 1&0&0&0&0&0&0&1&0\\ 0&1&0&0&1&0&0&0&1\\ 1&1&0&1&1&1&1&0&1\\ 0&1&1&1&1&1&0&0&0\\ 1&0&0&0&1&1&0&1&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&0&1&1&1&1&0\\ 1&0&0&0&1&1&1&1&0\\ 0&1&1&0&1&0&1&1&1 \end{pmatrix}$$

- $H_3 = \langle\beta_1, \beta_2\rangle$, $\beta_1 \in 2B$, $\beta_2 \in 6B$ where

$$\beta_1=\begin{pmatrix} 1&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&1&0&0\\ 0&1&0&1&0&0&0&1&1\\ 0&1&0&0&1&0&0&1&1\\ 0&0&1&0&0&1&1&1&1\\ 0&1&1&0&0&0&0&0&0\\ 0&0&0&1&0&1&1&1&0\\ 0&0&0&1&0&1&1&0&1 \end{pmatrix}, \beta_2=\begin{pmatrix} 1&0&0&0&0&0&0&0&0\\ 1&1&0&1&0&1&0&1&0\\ 0&1&0&1&0&1&1&0&0\\ 1&1&0&0&0&1&1&0&1\\ 0&0&1&1&0&1&0&1&1\\ 1&0&1&0&0&0&1&1&1\\ 1&0&0&1&1&1&0&0&0\\ 1&1&1&1&1&1&1&1&0\\ 1&0&0&1&0&0&1&0&1 \end{pmatrix}$$

- $H_4 = \langle\gamma_1, \gamma_2\rangle$, $\gamma_1 \in 3B$, $\gamma_2 \in 8A$ where

Table 1. The conjugacy classes of elements of 2^9:($L_3(4)$:S_3)

$[g]_G$	k	f_j	$[x]_{\overline{G}}$	$\lvert C_{\overline{G}}(x)\rvert$	$[g]_G$	k	f_j	$[x]_{\overline{G}}$	$\lvert C_{\overline{G}}(x)\rvert$
$1A$	512	$f_1 = 1$	$1A$	61931520	$2A$	32	$f_1 = 1$	$2D$	12288
		$f_2 = 21$	$2A$	2949120			$f_2 = 1$	$2E$	12288
		$f_3 = 210$	$2B$	294912			$f_3 = 1$	$4A$	12288
		$f_4 = 280$	$2C$	221184			$f_4 = 1$	$4B$	12288
							$f_5 = 8$	$4C$	1536
							$f_6 = 8$	$4D$	1536
							$f_7 = 12$	$4E$	1024
$2B$	64	$f_1 = 1$	$2F$	21504	$3A$	32	$f_1 = 1$	$3A$	5760
		$f_2 = 7$	$2G$	3072			$f_2 = 1$	$6A$	5760
		$f_3 = 7$	$4F$	3072			$f_3 = 5$	$6B$	1152
		$f_4 = 21$	$4G$	1024			$f_4 = 5$	$6C$	1152
		$f_5 = 28$	$4H$	768			$f_5 = 10$	$6D$	576
							$f_6 = 10$	$6E$	576
$3B$	8	$f_1 = 1$	$3B$	504	$3C$	8	$f_1 = 1$	$3C$	432
		$f_2 = 7$	$6F$	72			$f_2 = 1$	$6G$	432
							$f_3 = 3$	$6H$	144
							$f_4 = 3$	$6I$	144
$4A$	8	$f_1 = 1$	$4I$	256	$4B$	8	$f_1 = 1$	$4K$	128
		$f_2 = 1$	$4J$	256			$f_2 = 1$	$4L$	128
		$f_3 = 2$	$8A$	128			$f_3 = 1$	$8C$	128
		$f_4 = 4$	$8B$	64			$f_4 = 1$	$8D$	128
							$f_5 = 2$	$8E$	64
							$f_6 = 2$	$8F$	64
$5A$	2	$f_1 = 1$	$5A$	30	$6A$	8	$f_1 = 1$	$6J$	96
		$f_2 = 1$	$10A$	30			$f_2 = 1$	$12A$	96
							$f_3 = 1$	$12B$	96
							$f_4 = 1$	$6K$	96
							$f_5 = 2$	$12C$	48
							$f_6 = 2$	$12D$	48
$6B$	4	$f_1 = 1$	$6L$	24	$7A$	1	$f_1 = 1$	$7A$	42
		$f_2 = 1$	$6M$	24					
		$f_3 = 1$	$12E$	24					
		$f_4 = 1$	$12F$	24					
$7B$	1	$f_1 = 1$	$7B$	42	$8A$	4	$f_1 = 1$	$8G$	32
							$f_2 = 1$	$8H$	32
							$f_3 = 1$	$16A$	32
							$f_4 = 1$	$16B$	32
$14A$	1	$f_1 = 1$	$14A$	14	$14B$		$f_1 = 1$	$14B$	14
$15A$	2	$f_1 = 1$	$15A$	30	$15B$	2	$f_1 = 1$	$15B$	30
		$f_2 = 1$	$30A$	30			$f_2 = 1$	$30B$	30
$21A$	1	$f_1 = 1$	$21A$	21	$21B$	1	$f_1 = 1$	$21B$	21

$$\gamma_1=\begin{pmatrix} 1&1&0&0&0&1&0&1&0\\ 0&0&0&1&0&1&1&0&0\\ 1&0&0&1&0&0&0&1&0\\ 0&1&0&1&0&1&0&1&0\\ 1&1&0&1&1&1&0&0&1\\ 0&0&0&0&1&1&0&0&1\\ 1&1&0&0&0&1&0&1&1\\ 1&1&0&0&0&0&0&0&1\\ 0&1&1&0&0&0&1&1&0 \end{pmatrix}, \gamma_2=\begin{pmatrix} 0&0&0&0&1&0&0&1&1\\ 0&1&1&1&1&1&1&0&0\\ 0&1&1&0&0&0&0&0&0\\ 1&0&0&1&1&0&0&1&1\\ 0&1&0&1&1&0&0&0&0\\ 0&0&0&0&0&1&0&0&1\\ 0&1&0&1&0&1&0&0&0\\ 1&0&0&1&0&1&1&0&0\\ 1&0&1&0&0&1&1&1&0 \end{pmatrix}$$

We obtain the fusions of the inertia factors H_2, H_3 and H_4 into G by using direct matrix conjugation in G and their permutation characters in G of degrees 21, 210 and 280, respectively. MAGMA was used for the various computations. The fusion maps of H_2, H_3 and H_4 into G are shown in Tables 2, 3 and 4.

TABLE 2. **The fusion of H_2 into $L_3(4){:}S_3$**

$[h]_{H_2}$ →	$[g]_{L_3(4):S_3}$	$[h]_{H_2}$ →	$[g]_{L_3(4):S_3}$	$[h]_{H_2}$ →	$[g]_{L_3(4):S_3}$	$[h]_{H_2}$ →	$[g]_{L_3(4):S_3}$
$1A$	$1A$	$3B$	$3A$	$5A$	$5A$	$15A$	$15B$
$2A$	$2A$	$3C$	$3C$	$6A$	$6A$	$15B$	$15A$
$2B$	$2A$	$4A$	$4A$	$6B$	$6A$		
$2C$	$2B$	$4B$	$4B$	$6C$	$6B$		
$3A$	$3A$	$4C$	$4B$	$8A$	$8A$		

TABLE 3. **The fusion of H_3 into $L_3(4){:}S_3$**

$[h]_{H_3}$ →	$[g]_{L_3(4):S_3}$	$[h]_{H_3}$ →	$[g]_{L_3(4):S_3}$	$[h]_{H_3}$ →	$[g]_{L_3(4):S_3}$	$[h]_{H_3}$ →	$[g]_{L_3(4):S_3}$
$1A$	$1A$	$2D$	$2A$	$3C$	$3C$	$4D$	$4B$
$2A$	$2A$	$2E$	$2B$	$4A$	$4A$	$6A$	$6A$
$2B$	$2A$	$3A$	$3A$	$4B$	$4B$	$6B$	$6B$
$2C$	$2B$	$3B$	$3A$	$4C$	$4B$	$6C$	$6A$

TABLE 4. **The fusion of H_4 into $L_3(4){:}S_3$**

$[h]_{H_4}$ →	$[g]_{L_3(4):S_3}$	$[h]_{H_4}$ →	$[g]_{L_3(4):S_3}$	$[h]_{H_4}$ →	$[g]_{L_3(4):S_3}$	$[h]_{H_4}$ →	$[g]_{L_3(4):S_3}$
$1A$	$1A$	$3A$	$3C$	$4A$	$4A$	$8A$	$8A$
$2A$	$2A$	$3B$	$3A$	$6A$	$6A$	$8B$	$8A$
$2B$	$2B$	$3C$	$3B$	$6B$	$6B$		

5. **The Fischer-Clifford matrices of $2^9{:}(L_3(4){:}S_3)$**

Having obtained the fusions of the inertia factors into $L_3(4){:}S_3$ and the conjugacy classes of $L_3(4){:}S_3$ displayed in the format of Table 1, we can proceed to use the theory and properties discussed in Section 2 to help us in the construction of the Fischer-Clifford Matrices of $2^9{:}(L_3(4){:}S_3)$. Note that all the relations hold since 2^9 is an elementary abelian group.

For example, consider the conjugacy class $2B$ of $L_3(4){:}S_3$. Then we obtain that $M(2B)$ has the following form with corresponding weights attached to the rows and columns:

$$M(2B) = \begin{array}{c} \\ 336 \\ 48 \\ 48 \\ 16 \\ 12 \\ \\ \end{array} \begin{array}{c} 21504 \quad 3072 \quad 3072 \quad 1024 \quad 768 \\ \left(\begin{array}{ccccc} a & f & k & p & u \\ b & g & l & q & v \\ c & h & m & r & w \\ d & i & n & s & x \\ e & j & o & t & y \end{array} \right) \\ 8 \quad 56 \quad 56 \quad 168 \quad 224 \end{array}.$$

By properties (a) and (f) of the Fischer-Clifford matrix $M(g)$ in Section 2, we have $a = f = k = p = u = 1$, $b = c = 7$, $d = 21$ and $e = 28$. Thus we get the following form

$$M(2B) = \begin{array}{c} \\ 336 \\ 48 \\ 48 \\ 16 \\ 12 \\ \\ \end{array} \begin{array}{c} 21504 \quad 3072 \quad 3072 \quad 1024 \quad 768 \\ \left(\begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ 7 & g & l & q & v \\ 7 & h & m & r & w \\ 21 & i & n & s & x \\ 28 & j & o & t & y \end{array} \right) \\ 8 \quad 56 \quad 56 \quad 168 \quad 224 \end{array}.$$

By the orthogonality relations for columns and rows (properties (c) and (d) in Section 2) and the remaining properties discussed in Section 2, we obtain the desired Fischer-Clifford matrix $M(2B)$ of $\overline{G}$ as follows:

$$M(2B) = \left(\begin{array}{rrrrr} 1 & 1 & 1 & 1 & 1 \\ 7 & -1 & -5 & 3 & -1 \\ 7 & 7 & -1 & -1 & -1 \\ 21 & -3 & 9 & 1 & -3 \\ 28 & -4 & -4 & -4 & 4 \end{array} \right).$$

For each class representative $g \in L_3(4){:}S_3$, we construct a Fischer-Clifford matrix $M(g)$ which are listed in Table 5 .

6. Character table of $2^9{:}(L_3(4){:}S_3)$

Having obtained the Fischer-Clifford matrices, the fusion maps of the H_i's into $L_3(4){:}S_3$, and the character tables of the inertia factors H_i, we construct the character table of $2^9{:}(L_3(4){:}S_3)$ using the same procedure as explained in Section 2. The character tables of the inertia factors were obtained by direct computations in GAP.

The character table of $\overline{G}$ will be partitioned row-wise into 4 blocks Δ_1, Δ_2, Δ_3 and Δ_4 where each block corresponds to an inertia group $\overline{H}_i = 2^9{:}H_i$. Therefore $Irr(2^9{:}(L_3(4){:}S_3)) = \bigcup_{i=1}^{4} \Delta_i$, where $\Delta_1 = \{\chi_j | 1 \leq j \leq 20\}$, $\Delta_2 = \{\chi_j | 21 \leq j \leq 37\}$, $\Delta_3 = \{\chi_j | 38 \leq j \leq 53\}$ and $\Delta_4 = \{\chi_j | 54 \leq j \leq 64\}$. The character table of $2^9{:}(L_3(4){:}S_3)$ is shown in Table 6. The consistency and accuracy of the character table of $2^9{:}(L_3(4){:}S_3)$ have been tested by using the GAP codes labelled as Programme E in [22].

We can use GAP to compute possible power maps from the character table of $\overline{G}$. Programme E in [22] produces the unique p-power maps listed in Table 7 for our Table 6. Alternatively, the information about the conjugacy classes found in Table 1 can be used to compute the power maps for the elements of $\overline{G}$.

TABLE 5. The Fischer-Clifford matrices of $2^9{:}(L_3(4){:}S_3)$

$M(g)$	$M(g)$
$M(1A)=\begin{pmatrix} 1 & 1 & 1 & 1 \\ 21 & -11 & 5 & -3 \\ 210 & 50 & 2 & -6 \\ 280 & -40 & -8 & 8 \end{pmatrix}$	$M(2A)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & 1 \\ 4 & -4 & 4 & -4 & -2 & 2 & 0 \\ 4 & -4 & 4 & -4 & 2 & -2 & 0 \\ 6 & 6 & 6 & 6 & 0 & 0 & -2 \\ 8 & 8 & -8 & -8 & 0 & 0 & 0 \\ 8 & -8 & -8 & 8 & 0 & 0 & 0 \end{pmatrix}$
$M(2B)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 7 & -1 & -5 & 3 & -1 \\ 7 & 7 & -1 & -1 & -1 \\ 21 & -3 & 9 & 1 & -3 \\ 28 & -4 & -4 & -4 & 4 \end{pmatrix}$	$M(3A)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 \\ 5 & 5 & -3 & -3 & 1 & 1 \\ 5 & -5 & -3 & 3 & -1 & 1 \\ 10 & 10 & 2 & 2 & -2 & -2 \\ 10 & -10 & 2 & -2 & 2 & -2 \end{pmatrix}$
$M(3B)=\begin{pmatrix} 1 & 1 \\ 7 & -1 \end{pmatrix}$	$M(3C)=\begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & -3 & 1 & -1 \\ 3 & 3 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$
$M(4A)=\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 4 & -4 & 0 & 0 \end{pmatrix}$	$M(4B)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & 1 \\ 2 & -2 & 2 & -2 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & -1 \\ 2 & -2 & -2 & 2 & 0 & 0 \end{pmatrix}$
$M(5A)=\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(6A)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 \\ 2 & 2 & -2 & -2 & 0 & 0 \\ 2 & -2 & -2 & 2 & 0 & 0 \end{pmatrix}$
$M(6B)=\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$	$M(7A)=\begin{pmatrix} 1 \end{pmatrix}$
$M(7B)=\begin{pmatrix} 1 \end{pmatrix}$	$M(8A)=\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$
$M(14A)=\begin{pmatrix} 1 \end{pmatrix}$	$M(14B)=\begin{pmatrix} 1 \end{pmatrix}$
$M(15A)=\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(15B)=\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(21A)=\begin{pmatrix} 1 \end{pmatrix}$	$M(21B)=\begin{pmatrix} 1 \end{pmatrix}$

TABLE 6. The Character table of $2^9{:}(L_3(4){:}S_3)$

	1A				$2A$							$2B$				
	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$4A$	$4B$	$4C$	$4D$	$4E$	$2F$	$2G$	$4F$	$4G$	$4H$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1
χ_3	2	2	2	2	2	2	2	2	2	2	2	0	0	0	0	0
χ_4	20	20	20	20	4	4	4	4	4	4	4	-6	-6	-6	-6	-6
χ_5	20	20	20	20	4	4	4	4	4	4	4	6	6	6	6	6
χ_6	40	40	40	40	8	8	8	8	8	8	8	0	0	0	0	0
χ_7	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
χ_8	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
χ_9	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	3	3	3	3	3
χ_{10}	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	3	3	3	3	3
χ_{11}	64	64	64	64	0	0	0	0	0	0	0	8	8	8	8	8
χ_{12}	64	64	64	64	0	0	0	0	0	0	0	-8	-8	-8	-8	-8
χ_{13}	90	90	90	90	-6	-6	-6	-6	-6	-6	-6	0	0	0	0	0
χ_{14}	90	90	90	90	-6	-6	-6	-6	-6	-6	-6	0	0	0	0	0
χ_{15}	105	105	105	105	9	9	9	9	9	9	9	-7	-7	-7	-7	-7
χ_{16}	105	105	105	105	9	9	9	9	9	9	9	7	7	7	7	7
χ_{17}	126	126	126	126	-2	-2	-2	-2	-2	-2	-2	0	0	0	0	0
χ_{18}	126	126	126	126	-2	-2	-2	-2	-2	-2	-2	0	0	0	0	0
χ_{19}	126	126	126	126	-2	-2	-2	-2	-2	-2	-2	0	0	0	0	0
χ_{20}	128	128	128	128	0	0	0	0	0	0	0	0	0	0	0	0
χ_{21}	21	-11	5	-3	5	-3	5	-3	-3	1	1	7	-1	-5	3	-1
χ_{22}	21	-11	5	-3	5	-3	5	-3	-3	1	1	-7	1	5	-3	1
χ_{23}	42	-22	10	-6	10	-6	10	-6	-6	2	2	0	0	0	0	0
χ_{24}	84	-44	20	-12	4	4	4	4	-4	-4	4	-14	2	10	-6	2
χ_{25}	84	-44	20	-12	4	4	4	4	-4	-4	4	14	-2	-10	6	-2
χ_{26}	105	-55	25	-15	9	1	9	1	-7	-3	5	-7	1	5	-3	1
χ_{27}	105	-55	25	-15	9	1	9	1	-7	-3	5	7	-1	-5	3	-1
χ_{28}	126	-66	30	-18	-2	14	-2	14	-2	-10	6	0	0	0	0	0
χ_{29}	126	-66	30	-18	-2	14	-2	14	-2	-10	6	0	0	0	0	0
χ_{30}	126	-66	30	-18	-2	14	-2	14	-2	-10	6	0	0	0	0	0
χ_{31}	168	-88	40	-24	8	8	8	8	-8	-8	8	0	0	0	0	0
χ_{32}	210	-110	50	-30	18	2	18	2	-14	-6	10	0	0	0	0	0
χ_{33}	315	-165	75	-45	11	-13	11	-13	-5	7	-1	-21	3	15	-9	3
χ_{34}	315	-165	75	-45	11	-13	11	-13	-5	7	-1	21	-3	-15	9	-3
χ_{35}	630	-330	150	-90	22	-26	22	-26	-10	14	-2	0	0	0	0	0
χ_{36}	945	-495	225	-135	-15	9	-15	9	9	-3	-3	21	-3	-15	9	-3
χ_{37}	945	-495	225	-135	-15	9	-15	9	9	-3	-3	-21	3	15	-9	3
χ_{38}	210	50	2	-6	18	10	2	-6	2	-2	-2	28	4	8	0	-4
χ_{39}	210	50	2	-6	18	10	2	-6	2	-2	-2	-28	-4	-8	0	4
χ_{40}	210	50	2	-6	2	-6	18	10	2	-2	-2	-14	10	-10	-2	2
χ_{41}	210	50	2	-6	2	-6	18	10	2	-2	-2	14	-10	10	2	-2
χ_{42}	420	100	4	-12	36	20	4	-12	4	-4	-4	0	0	0	0	0
χ_{43}	420	100	4	-12	4	-12	36	20	4	-4	-4	0	0	0	0	0
χ_{44}	420	100	4	-12	20	4	20	4	4	-4	-4	-14	-14	2	2	2
χ_{45}	420	100	4	-12	20	4	20	4	4	-4	-4	14	14	-2	-2	-2
χ_{46}	840	200	8	-24	40	8	40	8	8	-8	-8	0	0	0	0	0
χ_{47}	1260	300	12	-36	-4	-20	-4	-20	4	-4	4	-42	6	-18	-2	6
χ_{48}	1260	300	12	-36	-4	-20	-4	-20	4	-4	4	42	-6	18	2	-6
χ_{49}	1890	450	18	-54	18	42	-30	-6	-6	6	-2	-42	-18	-6	2	6
χ_{50}	1890	450	18	-54	-30	-6	18	42	-6	6	-2	0	24	-12	-4	0
χ_{51}	1890	450	18	-54	-30	-6	18	42	-6	6	-2	0	-24	12	4	0
χ_{52}	1890	450	18	-54	18	42	-30	-6	-6	6	-2	42	18	6	-2	-6
χ_{53}	2520	600	24	-72	-8	-40	-8	-40	8	-8	8	0	0	0	0	0
χ_{54}	280	-40	-8	8	8	-8	-8	8	0	0	0	28	-4	-4	-4	4
χ_{55}	280	-40	-8	8	8	-8	-8	8	0	0	0	-28	4	4	4	-4
χ_{56}	560	-80	-16	16	16	-16	-16	16	0	0	0	0	0	0	0	0
χ_{57}	560	-80	-16	16	-16	16	16	-16	0	0	0	0	0	0	0	0
χ_{58}	560	-80	-16	16	-16	16	16	-16	0	0	0	0	0	0	0	0
χ_{59}	840	-120	-24	24	24	-24	-24	24	0	0	0	-28	4	4	4	-4
χ_{60}	840	-120	-24	24	24	-24	-24	24	0	0	0	28	-4	-4	-4	4
χ_{61}	1120	-160	-32	32	-32	32	32	-32	0	0	0	0	0	0	0	0
χ_{62}	2240	-320	-64	64	0	0	0	0	0	0	0	-56	8	8	8	-8
χ_{63}	2240	-320	-64	64	0	0	0	0	0	0	0	56	-8	-8	-8	8
χ_{64}	4480	-640	-128	128	0	0	0	0	0	0	0	0	0	0	0	0

Table 6 (continued)

	3A						3B		3C				4A			
	$3A$	$6A$	$6B$	$6C$	$6D$	$6E$	$3B$	$6F$	$3C$	$6G$	$6H$	$6I$	$4I$	$4J$	$8A$	$8B$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	-1	-1	-1	-1	-1	-1	-1	-1	2	2	2	2	2	2	2	2
χ_4	5	5	5	5	5	5	-1	-1	2	2	2	2	0	0	0	0
χ_5	5	5	5	5	5	5	-1	-1	2	2	2	2	0	0	0	0
χ_6	-5	-5	-5	-5	-5	-5	1	1	4	4	4	4	0	0	0	0
χ_7	0	0	0	0	0	0	3	3	0	0	0	0	1	1	1	1
χ_8	0	0	0	0	0	0	3	3	0	0	0	0	1	1	1	1
χ_9	0	0	0	0	0	0	3	3	0	0	0	0	1	1	1	1
χ_{10}	0	0	0	0	0	0	3	3	0	0	0	0	1	1	1	1
χ_{11}	4	4	4	4	4	4	1	1	1	1	1	1	0	0	0	0
χ_{12}	4	4	4	4	4	4	1	1	1	1	1	1	0	0	0	0
χ_{13}	0	0	0	0	0	0	-3	-3	0	0	0	0	2	2	2	2
χ_{14}	0	0	0	0	0	0	-3	-3	0	0	0	0	2	2	2	2
χ_{15}	0	0	0	0	0	0	0	0	-3	-3	-3	-3	1	1	1	1
χ_{16}	0	0	0	0	0	0	0	0	-3	-3	-3	-3	1	1	1	1
χ_{17}	6	6	6	6	6	6	0	0	0	0	0	0	-2	-2	-2	-2
χ_{18}	-3	-3	-3	-3	-3	-3	0	0	0	0	0	0	-2	-2	-2	-2
χ_{19}	-3	-3	-3	-3	-3	-3	0	0	0	0	0	0	-2	-2	-2	-2
χ_{20}	-4	-4	-4	-4	-4	-4	-1	-1	2	2	2	2	0	0	0	0
χ_{21}	6	4	-2	-4	0	2	0	0	3	-3	1	-1	1	1	1	-1
χ_{22}	6	4	-2	-4	0	2	0	0	3	-3	1	-1	1	1	1	-1
χ_{23}	-6	-4	2	4	0	-2	0	0	6	-6	2	-2	2	2	2	-2
χ_{24}	9	1	1	-7	-3	5	0	0	3	-3	1	-1	0	0	0	0
χ_{25}	9	1	1	-7	-3	5	0	0	3	-3	1	-1	0	0	0	0
χ_{26}	0	-10	8	-2	-6	4	0	0	-3	3	-1	1	1	1	1	-1
χ_{27}	0	-10	8	-2	-6	4	0	0	-3	3	-1	1	1	1	1	-1
χ_{28}	6	-6	6	-6	-6	6	0	0	0	0	0	0	-2	-2	-2	2
χ_{29}	-3	3	-3	3	3	-3	0	0	0	0	0	0	-2	-2	-2	2
χ_{30}	-3	3	-3	3	3	-3	0	0	0	0	0	0	-2	-2	-2	2
χ_{31}	-9	-1	-1	7	3	-5	0	0	6	-6	2	-2	0	0	0	0
χ_{32}	0	10	-8	2	6	-4	0	0	-6	6	-2	2	2	2	2	-2
χ_{33}	15	15	-9	-9	3	3	0	0	0	0	0	0	-1	-1	-1	1
χ_{34}	15	15	-9	-9	3	3	0	0	0	0	0	0	-1	-1	-1	1
χ_{35}	-15	-15	9	9	-3	-3	0	0	0	0	0	0	-2	-2	-2	2
χ_{36}	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-1
χ_{37}	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-1
χ_{38}	15	5	-1	5	-3	-1	0	0	3	3	-1	-1	2	2	-2	0
χ_{39}	15	5	-1	5	-3	-1	0	0	3	3	-1	-1	2	2	-2	0
χ_{40}	15	5	-1	5	-3	-1	0	0	3	3	-1	-1	-2	-2	2	0
χ_{41}	15	5	-1	5	-3	-1	0	0	3	3	-1	-1	-2	-2	2	0
χ_{42}	-15	-5	1	-5	3	1	0	0	6	6	-2	-2	4	4	-4	0
χ_{43}	-15	-5	1	-5	3	1	0	0	6	6	-2	-2	-4	-4	4	0
χ_{44}	15	25	7	1	-3	-5	0	0	-3	-3	1	1	0	0	0	0
χ_{45}	15	25	7	1	-3	-5	0	0	-3	-3	1	1	0	0	0	0
χ_{46}	-15	-25	-7	-1	3	5	0	0	-6	-6	2	2	0	0	0	0
χ_{47}	15	-15	-9	9	-3	3	0	0	0	0	0	0	0	0	0	0
χ_{48}	15	-15	-9	9	-3	3	0	0	0	0	0	0	0	0	0	0
χ_{49}	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	2	0
χ_{50}	0	0	0	0	0	0	0	0	0	0	0	0	2	2	-2	0
χ_{51}	0	0	0	0	0	0	0	0	0	0	0	0	2	2	-2	0
χ_{52}	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	2	0
χ_{53}	-15	15	9	-9	3	-3	0	0	0	0	0	0	0	0	0	0
χ_{54}	10	-10	2	-2	2	-2	7	-1	1	-1	-1	1	4	-4	0	0
χ_{55}	10	-10	2	-2	2	-2	7	-1	1	-1	-1	1	4	-4	0	0
χ_{56}	-10	10	-2	2	-2	2	-7	1	2	-2	-2	2	8	-8	0	0
χ_{57}	-10	10	-2	2	-2	2	-7	1	2	-2	-2	2	0	0	0	0
χ_{58}	-10	10	-2	2	-2	2	-7	1	2	-2	-2	2	0	0	0	0
χ_{59}	0	0	0	0	0	0	0	0	3	-3	-3	3	-4	4	0	0
χ_{60}	0	0	0	0	0	0	0	0	3	-3	-3	3	-4	4	0	0
χ_{61}	10	-10	2	-2	2	-2	7	-1	4	-4	-4	4	0	0	0	0
χ_{62}	20	-20	4	-4	4	-4	-7	1	-1	1	1	-1	0	0	0	0
χ_{63}	20	-20	4	-4	4	-4	-7	1	-1	1	1	-1	0	0	0	0
χ_{64}	-20	20	-4	4	-4	4	7	-1	-2	2	2	-2	0	0	0	0

Table 6 (continued)

	4B						$5A$		$6A$					
	$4K$	$4L$	$8C$	$8D$	$8E$	$8F$	$5A$	$10A$	$6J$	$12A$	$12B$	$6K$	$12C$	$12D$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
χ_3	0	0	0	0	0	0	2	2	-1	-1	-1	-1	-1	-1
χ_4	-2	-2	-2	-2	-2	-2	0	0	1	1	1	1	1	1
χ_5	2	2	2	2	2	2	0	0	1	1	1	1	1	1
χ_6	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_7	1	1	1	1	1	1	0	0	0	0	0	0	0	0
χ_8	1	1	1	1	1	1	0	0	0	0	0	0	0	0
χ_9	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0
χ_{10}	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0
χ_{11}	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0
χ_{12}	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0
χ_{13}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{15}	1	1	1	1	1	1	0	0	0	0	0	0	0	0
χ_{16}	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0
χ_{17}	0	0	0	0	0	0	1	1	-2	-2	-2	-2	-2	-2
χ_{18}	0	0	0	0	0	0	1	1	1	1	1	1	1	1
χ_{19}	0	0	0	0	0	0	1	1	1	1	1	1	1	1
χ_{20}	0	0	0	0	0	0	-2	-2	0	0	0	0	0	0
χ_{21}	3	-1	1	-3	-1	1	1	-1	2	0	2	0	0	-2
χ_{22}	-3	1	-1	3	1	-1	1	-1	2	0	2	0	0	-2
χ_{23}	0	0	0	0	0	0	2	-2	-2	0	-2	0	0	2
χ_{24}	-2	-2	2	2	2	-2	-1	1	1	1	1	1	-1	-1
χ_{25}	2	2	-2	-2	-2	2	-1	1	1	1	1	1	-1	-1
χ_{26}	1	-3	3	-1	1	-1	0	0	0	-2	0	-2	2	0
χ_{27}	-1	3	-3	1	-1	1	0	0	0	-2	0	-2	2	0
χ_{28}	0	0	0	0	0	0	1	-1	-2	2	-2	2	-2	2
χ_{29}	0	0	0	0	0	0	1	-1	1	-1	1	-1	1	-1
χ_{30}	0	0	0	0	0	0	1	-1	1	-1	1	-1	1	-1
χ_{31}	0	0	0	0	0	0	-2	2	-1	-1	-1	-1	1	1
χ_{32}	0	0	0	0	0	0	0	0	0	2	0	2	-2	0
χ_{33}	-1	3	-3	1	-1	1	0	0	-1	-1	-1	-1	1	1
χ_{34}	1	-3	3	-1	1	-1	0	0	-1	-1	-1	-1	1	1
χ_{35}	0	0	0	0	0	0	0	0	1	1	1	1	-1	-1
χ_{36}	-3	1	-1	3	1	-1	0	0	0	0	0	0	0	0
χ_{37}	3	-1	1	-3	-1	1	0	0	0	0	0	0	0	0
χ_{38}	4	0	-2	2	0	-2	0	0	3	-3	-1	1	-1	1
χ_{39}	-4	0	2	-2	0	2	0	0	3	-3	-1	1	-1	1
χ_{40}	-2	2	0	-4	2	0	0	0	-1	1	3	-3	-1	1
χ_{41}	2	-2	0	4	-2	0	0	0	-1	1	3	-3	-1	1
χ_{42}	0	0	0	0	0	0	0	0	-3	3	1	-1	1	-1
χ_{43}	0	0	0	0	0	0	0	0	1	-1	-3	3	1	-1
χ_{44}	-2	-2	2	2	-2	2	0	0	-1	1	-1	1	1	-1
χ_{45}	2	2	-2	-2	2	-2	0	0	-1	1	-1	1	1	-1
χ_{46}	0	0	0	0	0	0	0	0	1	-1	1	-1	-1	1
χ_{47}	2	2	2	2	-2	-2	0	0	-1	1	-1	1	1	-1
χ_{48}	-2	-2	-2	-2	2	2	0	0	-1	1	-1	1	1	-1
χ_{49}	2	-2	-4	0	2	0	0	0	0	0	0	0	0	0
χ_{50}	0	-4	-2	2	0	2	0	0	0	0	0	0	0	0
χ_{51}	0	4	2	-2	0	-2	0	0	0	0	0	0	0	0
χ_{52}	-2	2	4	0	-2	0	0	0	0	0	0	0	0	0
χ_{53}	0	0	0	0	0	0	0	0	1	-1	1	-1	-1	1
χ_{54}	0	0	0	0	0	0	0	0	2	2	-2	-2	0	0
χ_{55}	0	0	0	0	0	0	0	0	2	2	-2	-2	0	0
χ_{56}	0	0	0	0	0	0	0	0	-2	-2	2	2	0	0
χ_{57}	0	0	0	0	0	0	0	0	2	2	-2	-2	0	0
χ_{58}	0	0	0	0	0	0	0	0	2	2	-2	-2	0	0
χ_{59}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{60}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{61}	0	0	0	0	0	0	0	0	-2	-2	2	2	0	0
χ_{62}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{64}	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 6 (continued)

	6B				$7A$	7B	8A				14A	$14B$	$15A$		15B		21A	21B
	$6L$	$6M$	$12E$	$12F$	$7A$	$7B$	$8G$	$8H$	$16A$	$16B$	$14A$	$14B$	$15A$	$30A$	$15B$	$30B$	$21A$	$21B$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1
χ_3	0	0	0	0	2	2	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_4	0	0	0	0	-1	-1	0	0	0	0	1	1	0	0	0	0	-1	-1
χ_5	0	0	0	0	-1	-1	0	0	0	0	-1	-1	0	0	0	0	-1	-1
χ_6	0	0	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	1	1
χ_7	0	0	0	0	A	$\bar{A}$	-1	-1	-1	-1	$-A$	$-\bar{A}$	0	0	0	0	A	$\bar{A}$
χ_8	0	0	0	0	$\bar{A}$	A	-1	-1	-1	-1	$-\bar{A}$	$-A$	0	0	0	0	$\bar{A}$	A
χ_9	0	0	0	0	A	$\bar{A}$	1	1	1	1	A	$\bar{A}$	0	0	0	0	A	$\bar{A}$
χ_{10}	0	0	0	0	$\bar{A}$	A	1	1	1	1	$\bar{A}$	A	0	0	0	0	$\bar{A}$	A
χ_{11}	-1	-1	-1	-1	1	1	0	0	0	0	1	1	-1	-1	-1	-1	1	1
χ_{12}	1	1	1	1	1	1	0	0	0	0	-1	-1	-1	-1	-1	-1	1	1
χ_{13}	0	0	0	0	B	$\bar{B}$	0	0	0	0	0	0	0	0	0	0	$-A$	$-\bar{A}$
χ_{14}	0	0	0	0	$\bar{B}$	B	0	0	0	0	0	0	0	0	0	0	$-\bar{A}$	$-A$
χ_{15}	-1	-1	-1	-1	0	0	1	1	1	1	0	0	0	0	0	0	0	0
χ_{16}	1	1	1	1	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0
χ_{17}	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0
χ_{18}	0	0	0	0	0	0	0	0	0	0	0	0	D	D	$\bar{D}$	$\bar{D}$	0	0
χ_{19}	0	0	0	0	0	0	0	0	0	0	0	0	$\bar{D}$	$\bar{D}$	D	D	0	0
χ_{20}	0	0	0	0	2	2	0	0	0	0	0	0	1	1	1	1	-1	-1
χ_{21}	1	-1	-1	1	0	0	1	1	-1	-1	0	0	1	-1	1	-1	0	0
χ_{22}	-1	1	1	-1	0	0	-1	-1	1	1	0	0	1	-1	1	-1	0	0
χ_{23}	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	-1	1	0	0
χ_{24}	1	-1	-1	1	0	0	0	0	0	0	0	0	-1	1	-1	1	0	0
χ_{25}	-1	1	1	-1	0	0	0	0	0	0	0	0	-1	1	-1	1	0	0
χ_{26}	-1	1	1	-1	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{27}	1	-1	-1	1	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{28}	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	-1	0	0
χ_{29}	0	0	0	0	0	0	0	0	0	0	0	0	$\bar{D}$	$-\bar{D}$	D	$-D$	0	0
χ_{30}	0	0	0	0	0	0	0	0	0	0	0	0	D	$-D$	$\bar{D}$	$-\bar{D}$	0	0
χ_{31}	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	-1	0	0
χ_{32}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	0	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{35}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{37}	0	0	0	0	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{38}	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{42}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{43}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{45}	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{48}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{49}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{50}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{51}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{53}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	1	-1	1	-1	0	0	2	-2	0	0	0	0	0	0	0	0	0	0
χ_{55}	-1	1	-1	1	0	0	-2	2	0	0	0	0	0	0	0	0	0	0
χ_{56}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{57}	0	0	0	0	0	0	0	0	C	$-C$	0	0	0	0	0	0	0	0
χ_{58}	0	0	0	0	0	0	0	0	$-C$	C	0	0	0	0	0	0	0	0
χ_{59}	-1	1	-1	1	0	0	2	-2	0	0	0	0	0	0	0	0	0	0
χ_{60}	1	-1	1	-1	0	0	-2	2	0	0	0	0	0	0	0	0	0	0
χ_{61}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{62}	1	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{64}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

where $A = \frac{-1-\sqrt{7}i}{2}$, $B = -1-\sqrt{7}i$, $C = -2\sqrt{2}i$, and $D = \frac{-1-\sqrt{15}i}{2}$

7. The fusion of 2^9:($L_3(4)$:S_3) into $U_6(2)$:S_3

$\overline{G}$ is a maximal subgroup of $U_6(2)$:S_3 of index 891. Hence the action of $U_6(2)$:S_3 on the cosets of $\overline{G}$ gives rise to a permutation character $\chi(U_6(2){:}S_3|\overline{G})$

Table 7. The power maps of the elements of $2^9{:}(L_3(4){:}S_3)$

$[g]_G$	$[x]_{\overline{G}}$	2	3	5	7	$[g]_G$	$[x]_{\overline{G}}$	2	3	5	7
1A	1A					2A	2D	1A			
	2A	1A					2E	1A			
	2B	1A					4A	2A			
	2C	1A					4B	2A			
							4C	2B			
							4D	2B			
							4E	2B			
2B	2F	1A				3A	3A		1A		
	2G	1A					6A	3A	2A		
	4F	2B					6B	3A	2A		
	4G	2B					6C	3A	2B		
	4H	2B					6D	3A	2C		
							6E	3A	2B		
3B	3B		1A			3C	3C		1A		
	6F	3B	2C				6G	3C	2C		
							6H	3C	2A		
							6I	3C	2B		
4A	4I	2D				4B	4K	2D			
	4J	2D					4L	2D			
	8A	4B					8C	4E			
	8B	4E					8D	4E			
							8E	4A			
							8F	4E			
5A	5A			1A		6A	6J	3A	2D		
	10A	5A		2A			12A	6B	4B		
							12B	6B	4A		
							6K	3A	2E		
							12C	6E	4C		
							12D	6E	4D		
6B	6L	3C	2F			7A	7A				1A
	6M	3C	2G								
	12E	6I	4H								
	12F	6I	4F								
7B	7B				1A	8A	8G	4I			
							8H	4I			
							16A	8A			
							16B	8A			
14A	14A	7A			2F	14B	14B	7B			2F
15A	15A		5A	3A		15B	15B		5A	3A	
	30A	15A	10A	6A			30B	15B	10A	6A	
21A	21A		7B		3B	21B	21B		7A		3B

of degree 891. We deduce from the character table of $U_6(2){:}S_3$ found in GAP that $\chi(U_6(2){:}S_3|\overline{G}) = 1a + 22b + 252b + 616b$, where $1a$, $22b$, $252b$ and $616b$ are irreducible characters of $U_6(2){:}S_3$ of degrees 1, 22, 252 and 616, respectively.

The technique of set intersections for characters (see [14],[15] and [16]) was used to restrict the irreducible characters $22a, 22b, 44a, 231a, 231b, 385a, 385b$ and $462a$ of $U_6(2){:}S_3$ to $\overline{G}$. We obtain that $(22a)_{\overline{G}} = \chi_2 + \chi_{22}$, $(22b)_{\overline{G}} = \chi_1 + \chi_{21}$, $(44a)_{\overline{G}} = \chi_3 + \chi_{23}$, $(231a)_{\overline{G}} = \chi_{22} + \chi_{40}$, $(231b)_{\overline{G}} = \chi_{21} + \chi_{41}$, $(385a)_{\overline{G}} = \chi_{26} + \chi_{55}$, $(385b)_{\overline{G}} = \chi_{27} + \chi_{54}$ and $(462a)_{\overline{G}} = \chi_{23} + \chi_{43}$.

Using the values of $\chi(U_6(2){:}S_3|\overline{G})$ on the classes of $\overline{G}$, the power maps of the classes of $U_6(2){:}S_3$ and $\overline{G}$, and the technique of set intersections for characters to restrict the irreducible characters $22a, 22b, 44a, 231a, 231b, 385a, 385b$ and $462a$ of $U_6(2){:}S_3$ to $\overline{G}$, we are able to determine completely the fusion of classes of $\overline{G}$ into $U_6(2){:}S_3$. The fusion of classes of $\overline{G}$ into $U_6(2){:}S_3$ is given in Table 8.

TABLE 8. The fusion of $2^9{:}(L_3(4){:}S_3)$ into $U_6(2){:}S_3$

$[g]_{L_3(4):S_3}$	$[x]_{2^9:(L_3(4):S_3)}$ $\longrightarrow$	$[y]_{U_6(2):S_3}$	$[g]_{L_3(4):S_3}$	$[x]_{2^9:(L_3(4):S_3)}$ $\longrightarrow$	$[y]_{U_6(2):S_3}$
$1A$	$1A$	$1A$	$2A$	$2D$	$2B$
	$2A$	$2A$		$2E$	$2C$
	$2B$	$2B$		$4A$	$4A$
	$2C$	$2C$		$4B$	$4B$
				$4C$	$4D$
				$4D$	$4E$
				$4E$	$4C$
$2B$	$2F$	$2D$	$3A$	$3A$	$3E$
	$2G$	$2E$		$6A$	$6I$
	$4F$	$4F$		$6B$	$6K$
	$4G$	$4G$		$6C$	$6M$
	$4H$	$4H$		$6D$	$6P$
				$6E$	$6N$
$3B$	$3B$	$3G$	$3C$	$3C$	$3C$
	$6F$	$6R$		$6G$	$6G$
				$6H$	$6E$
				$6I$	$6F$
$4A$	$4I$	$4C$	$4B$	$4K$	$4G$
	$4J$	$4E$		$4L$	$4H$
	$8A$	$8A$		$8C$	$8E$
	$8B$	$8B$		$8D$	$8F$
				$8E$	$8C$
				$8F$	$8D$
$5A$	$5A$	$5A$	$6A$	$6J$	$6N$
	$10A$	$10A$		$12A$	$12J$
				$12B$	$12H$
				$6K$	$6P$
				$12C$	$12L$
				$12D$	$12N$
$6B$	$6L$	$6V$	$7A$	$7A$	$7A$
	$6M$	$6W$			
	$12E$	$12S$			
	$12F$	$12R$			
$7B$	$7B$	$7A$	$8A$	$8G$	$8D$
				$8H$	$8E$
				$16A$	$16A$
				$16B$	$16B$
$14A$	$14A$	$14A$	$14B$	$14B$	$14A$
$15A$	$15A$	$15B$	$15B$	$15B$	$15B$
	$30A$	$30A$		30B	30A
$21A$	$21A$	$21A$	$21B$	$21B$	$21A$

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