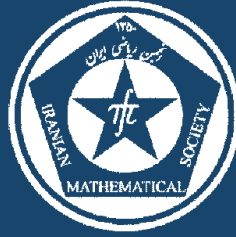


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STOCHASTIC EVOLUTION EQUATIONS WITH MULTIPLICATIVE POISSON NOISE AND MONOTONE NONLINEARITY

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ABSTRACT. Semilinear stochastic evolution equations with multiplicative Poisson noise and monotone nonlinear drift in Hilbert spaces are considered. The coefficients are assumed to have linear growth. We do not impose coercivity conditions on coefficients. A novel method of proof for establishing existence and uniqueness of the mild solution is proposed. Examples on stochastic partial differential equations and stochastic delay differential equations are provided to demonstrate the theory developed.

Keywords: Stochastic evolution equation, monotone operator, Lévy noise, Itô type inequality, stochastic convolution integral.

MSC(2010): Primary: 60H15; Secondary: 60G51, 60H05, 47H05, 47J35.

1. Introduction

Consider the stochastic evolution equation

$$(1.1) \quad dX_t = AX_t dt + f(t, X_t) dt + g(t, X_{t-}) dW_t + \int_E k(t, \xi, X_{t-}) \tilde{N}(dt, d\xi),$$

on a Hilbert space, where W_t is a cylindrical Wiener process and $\tilde{N}(dt, d\xi)$ is a compensated Poisson random measure. We assume f is semi-monotone and g and k are Lipschitz and have linear growth. In section 2 the assumptions on coefficients are stated precisely. The purpose of this article is to prove the existence and uniqueness of the solution of this equation.

The special cases of equation (1.1) have been studied by several authors. For the case that all the coefficients are Lipschitz see [5] for Wiener noise, [10] for general martingale noise, and [1, 15] for the case of jump noise. In the non-Lipschitz case there are two main approaches. The first approach is the variational approach in which the coefficients satisfy certain monotonicity and

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coercivity properties. For this approach see [12,14] and [16] for Wiener noise, and [6] for general martingales. The second approach is the semigroup approach to semilinear stochastic evolution equations with monotone drift. This approach has first appeared in deterministic context in [2] and [9] and has been extended to stochastic evolution equations in [20] and [22]. There are other works with this approach, e.g the exponential asymptotic stability of solutions in the case of Wiener noise has been studied in [8], generalizing the previous results to stochastic functional evolution equations with coefficients depending on the past path of the solution is done in [7], the large deviation principle for the case of Wiener noise is studied in [4]. A limiting problem of such equations arising from random motion of highly elastic strings has been considered in [19]. Finally, the stationarity of a mild solution to a stochastic evolution equation with a monotone nonlinear drift and Wiener noise is studied in [23].

Another related article is [13] which considers monotone nonlinear drift and multiplicative Poisson noise on certain function spaces and proves the existence, uniqueness and regular dependence of the mild solution on initial data. They impose an additional positivity assumption on the semigroup and the drift term is the Nemitsky operator associated with a real monotone function. On the other hand, they don't assume growth conditions on drift term. Their idea is to regularize the monotone nonlinearity f by its Yosida approximation $f_\lambda(x) = \lambda^{-1}(x - (I + \lambda f)^{-1}(x))$.

The semigroup approach to semilinear stochastic evolution equations with monotone nonlinearities has an advantage relative to the variational method since it does not require the coercivity. There are important examples, such as stochastic partial differential equations of hyperbolic type with monotone nonlinear terms, for which the generator does not satisfy the coercivity property and hence the variational method is not directly applicable to these equations. Pardoux [14] has developed a new theory for the application of the variational method to second order hyperbolic equations. But as is shown in Example 4.2, this problem can be treated directly in semigroup setting.

The main contribution of this article is Theorem 3.3 in Section 3 which shows the existence and uniqueness of the mild solution for equation (1.1). In Section 2 the precise assumptions on coefficients are stated. In Section 4 we will provide some concrete examples to which our results apply. These examples consist of semilinear stochastic partial differential equations and a stochastic delay differential equation.

We will use the notion of stochastic integration with respect to cylindrical Wiener process and compensated Poisson random measure. For this definition and properties see [1] and [15].

2. The assumptions

Let H be a separable Hilbert space with inner product $\langle \cdot, \cdot \rangle$. Let S_t be a C_0 semigroup on H with infinitesimal generator $A : D(A) \rightarrow H$. Furthermore we assume the exponential growth condition on S_t holds, i.e. there exists a constant α such that $\|S_t\| \leq e^{\alpha t}$. If $\alpha = 0$, S_t is called a contraction semigroup. We denote by $L_{HS}(K, H)$ the space of Hilbert-Schmidt mappings from a Hilbert space K to H .

Let $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$ be a filtered probability space. Let (E, \mathcal{E}) be a measurable space and $N(dt, d\xi)$ a Poisson random measure on $\mathbb{R}^+ \times E$ with intensity measure $dt\nu(d\xi)$. Our goal is to study equation (1.1) in H , where W_t is a cylindrical Wiener process on a Hilbert space K and $\tilde{N}(dt, d\xi) = N(dt, d\xi) - dt\nu(d\xi)$ is the compensated Poisson random measure corresponding to N . We assume that N and W_t are independent. We also assume the following,

Hypothesis 2.1. **(a):** (Semi-monotonicity) $f(t, x, \omega) : \mathbb{R}^+ \times H \times \Omega \rightarrow H$ is measurable, \mathcal{F}_t -adapted, demicontinuous with respect to x and there exists a constant M such that

$$\langle f(t, x, \omega) - f(t, y, \omega), x - y \rangle \leq M\|x - y\|^2,$$

(b): $g(t, x, \omega) : \mathbb{R}^+ \times H \times \Omega \rightarrow L_{HS}(K, H)$ and $k(t, \xi, x, \omega) : \mathbb{R}^+ \times E \times H \times \Omega \rightarrow H$ are predictable and there exists a constant C such that

$$\|g(t, x, \omega) - g(t, y, \omega)\|_{L_{HS}(K, H)}^2 + \int_E \|k(t, \xi, x) - k(t, \xi, y)\|^2 \nu(d\xi) \leq C\|x - y\|^2,$$

(c): There exists a constant D such that

$$\|f(t, x, \omega)\|^2 + \|g(t, x, \omega)\|_{L_{HS}(K, H)}^2 + \int_E \|k(t, \xi, x)\|^2 \nu(d\xi) \leq D(1 + \|x\|^2),$$

(d): $X_0(\omega)$ is \mathcal{F}_0 measurable and square integrable.

Definition 2.2. By a *mild solution* of equation (1.1) with initial condition X_0 we mean an adapted càdlàg process X_t that satisfies

$$(2.1) \quad X_t = S_t X_0 + \int_0^t S_{t-s} f(s, X_s) ds + \int_0^t S_{t-s} g(s, X_{s-}) dW_s + \int_0^t \int_E S_{t-s} k(s, \xi, X_{s-}) \tilde{N}(ds, d\xi).$$

Because of the presence of monotone nonlinearity in our equation, the usual inequalities for stochastic convolution integrals are not applicable to equation (1.1). For this reason we state the following inequality.

Theorem 2.3 (Itô type inequality, Zangeneh [22]). *Let Z_t be an H -valued càdlàg locally square integrable semimartingale. If*

$$X_t = S_t X_0 + \int_0^t S_{t-s} dZ_s,$$

then

$$\|X_t\|^2 \leq e^{2\alpha t} \|X_0\|^2 + 2 \int_0^t e^{2\alpha(t-s)} \langle X_{s-}, dZ_s \rangle + \int_0^t e^{2\alpha(t-s)} d[Z]_s,$$

where $[Z]_t$ is the quadratic variation process of Z_t .

3. The main result

Our proof for the existence of a mild solution relies on an iterative method which in each step requires solving a deterministic equation, i.e. an equation in which ω appears only as a parameter. The following theorem proved in Zangeneh [20] and [21] guarantees the solvability of such equations and the measurability of the solution with respect to parameter.

Let $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$ be a filtered probability space and assume f satisfies Hypothesis 2.1(a) and there exists a constant D such that $\|f(t, x, \omega)\|^2 \leq D(1 + \|x\|^2)$ and assume $V(t, \omega)$ is an adapted process with càdlàg trajectories and $X_0(\omega)$ is \mathcal{F}_0 measurable.

Theorem 3.1 (Zangeneh, [20] and [21]). *With assumptions made above, the equation*

$$X_t = S_t X_0 + \int_0^t S_{t-s} f(s, X_s, \omega) ds + V(t, \omega)$$

has a unique measurable adapted càdlàg solution $X_t(\omega)$. Furthermore

$$\|X_t\| \leq \|X_0\| + \|V(t)\| + \int_0^t e^{(\alpha+M)(t-s)} \|f(s, S_s X_0 + V(s))\| ds.$$

Remark 3.2. Note that the original theorem is stated for evolution operators and requires some additional assumptions, but those are automatically satisfied for C_0 semigroups. (See Curtain and Pritchard [3, page 29, Theorem 2.21]).

Theorem 3.3 (Existence and Uniqueness of the Mild Solution). *Under the assumptions of Hypothesis 2.1, equation (1.1) has a unique square integrable càdlàg mild solution with initial condition X_0 .*

This theorem has been stated without proof in [18].

Lemma 3.4. *It suffices to prove Theorem 3.3 for the case that $\alpha = 0$.*

Proof. Define

$$\begin{aligned} \tilde{S}_t &= e^{-\alpha t} S_t, & \tilde{f}(t, x, \omega) &= e^{-\alpha t} f(t, e^{\alpha t} x, \omega), & \tilde{g}(t, x, \omega) &= e^{-\alpha t} g(t, e^{\alpha t} x, \omega), \\ & & \tilde{k}(t, \xi, x, \omega) &= e^{-\alpha t} k(t, \xi, e^{\alpha t} x, \omega). \end{aligned}$$

Note that \tilde{S}_t is a contraction semigroup. It is easy to see that X_t is a mild solution of equation (1.1) if and only if $\tilde{X}_t = e^{-\alpha t} X_t$ is a mild solution of the equation with coefficients $\tilde{S}, \tilde{f}, \tilde{g}, \tilde{k}$. □

Proof of Theorem 3.3. Uniqueness. According to the lemma, we can assume $\alpha = 0$. Assume that X_t and Y_t are two mild solutions with same initial conditions. Subtracting them we find

$$X_t - Y_t = \int_0^t S_{t-s} dZ_s,$$

where

$$\begin{aligned} dZ_t &= (f(t, X_t) - f(t, Y_t))dt + (g(t, X_{t-}) - g(t, Y_{t-}))dW_t \\ &\quad + \int_E (k(t, \xi, X_{t-}) - k(t, \xi, Y_{t-}))d\tilde{N}. \end{aligned}$$

Applying Itô type inequality (Theorem 2.3) for $\alpha = 0$ to $X_t - Y_t$ we find

$$\|X_t - Y_t\|^2 \leq 2 \int_0^t \langle X_{s-} - Y_{s-}, dZ_s \rangle + [Z]_t.$$

Taking expectations and noting that integrals with respect to cylindrical Wiener processes and compensated Poisson random measures are martingales, we find that

$$\mathbb{E}\|X_t - Y_t\|^2 \leq 2 \int_0^t \mathbb{E}\langle X_{s-} - Y_{s-}, f(s, X_s) - f(s, Y_s) \rangle ds + \mathbb{E}[Z]_t,$$

where

$$\mathbb{E}[Z]_t = \int_0^t \mathbb{E}\|g(s, X_s) - g(s, Y_s)\|^2 ds + \int_0^t \int_E \mathbb{E}\|k(s, \xi, X_s) - k(s, \xi, Y_s)\|^2 \nu(d\xi) ds.$$

Note that for a càdlàg function the set of discontinuity points is countable, hence when integrating with respect to Lebesgue measure, they can be neglected. We therefore neglect the left limits in integrals with respect to the Lebesgue measure henceforth. Using assumptions of Hypothesis 2.1(a) and 2.1(b) we find that

$$\mathbb{E}\|X_t - Y_t\|^2 \leq (2M + C) \int_0^t \mathbb{E}\|X_s - Y_s\|^2 ds.$$

Using Gronwall's lemma we conclude that $X_t = Y_t$, almost surely.

Existence. It suffices to prove the existence of a solution on a finite interval $[0, T]$. Then one can show easily that these solutions are consistent and give a global solution. We define adapted càdlàg processes X_t^n recursively as follows. Let $X_t^0 = S_t X_0$. Assume X_t^{n-1} is defined. Theorem 3.1 implies that there exists an adapted càdlàg solution X_t^n of

$$(3.1) \quad X_t^n = S_t X_0 + \int_0^t S_{t-s} f(s, X_s^n) ds + V_t^n,$$

where

$$V_t^n = \int_0^t S_{t-s}g(s, X_{s-}^{n-1})dW_s + \int_0^t \int_E S_{t-s}k(s, \xi, X_{s-}^{n-1})\tilde{N}(ds, d\xi).$$

We wish to show that $\{X^n\}$ converges and the limit is the desired mild solution. This is done by the following lemmas.

Lemma 3.5.

$$\mathbb{E} \sup_{0 \leq t \leq T} \|X_t^n\|^2 < \infty.$$

Proof. We prove by induction on n . By Theorem 3.1 we have the following estimate,

$$\|X_t^n\| \leq \|X_0\| + \|V_t^n\| + \int_0^t e^{M(t-s)}\|f(s, S_s X_0 + V_s^n)\|ds.$$

Hence,

$$\sup_{0 \leq t \leq T} \|X_t^n\|^2 \leq 3\|X_0\|^2 + 3 \sup_{0 \leq t \leq T} \|V_t^n\|^2 + 3 \sup_{0 \leq t \leq T} \left(\int_0^t e^{M(t-s)}\|f(s, S_s X_0 + V_s^n)\|ds\right)^2,$$

where by Cauchy-Schwartz inequality we find

$$\leq 3\|X_0\|^2 + 3 \sup_{0 \leq t \leq T} \|V_t^n\|^2 + 3Te^{2MT} \int_0^T \|f(s, S_s X_0 + V_s^n)\|^2 ds,$$

and by Hypothesis 2.1(c) we have

$$\begin{aligned} &\leq 3\|X_0\|^2 + 3 \sup_{0 \leq t \leq T} \|V_t^n\|^2 + 3Te^{2MT} \int_0^T D(1 + \|S_s X_0 + V_s^n\|^2)ds \\ &\leq 3\|X_0\|^2 + 3 \sup_{0 \leq t \leq T} \|V_t^n\|^2 + 3DTe^{2MT} \int_0^T (1 + 2\|X_0\|^2 + 2\|V_s^n\|^2)ds \\ &= 3DT^2e^{2MT} + (3 + 6DT^2e^{2MT})\|X_0\|^2 + (3 + 6DT^2e^{2MT}) \sup_{0 \leq t \leq T} \|V_t^n\|^2. \end{aligned}$$

Hence for completing the proof it suffices to show that $\mathbb{E} \sup_{0 \leq t \leq T} \|V_t^n\|^2 < \infty$.

Applying [10, Theorem 1.1] we find,

$$\mathbb{E} \sup_{0 \leq t \leq T} \|V_t^n\|^2 \leq \mathbf{CE} \left(\int_0^T \|g(s, X_s^{n-1})\|_{HS}^2 ds + \int_0^T \int_E \|k(s, \xi, X_s^{n-1})\|^2 \nu(d\xi) ds \right),$$

where by Hypothesis 2.1(c),

$$\leq \mathbf{CE} \int_0^T D(1 + \|X_s^{n-1}\|^2) ds$$

which is finite by induction Hypothesis. The basis of induction follows directly from Hypothesis 2.1(d). \square

Lemma 3.6. For $0 \leq t \leq T$ we have,

$$(3.2) \quad \mathbb{E} \sup_{0 \leq s \leq t} \|X_s^{n+1} - X_s^n\|^2 \leq C_0 C_1^n \frac{t^n}{n!},$$

where $C_1 = 2C(1 + 2C_1^2)e^{4MT}$ and $C_0 = \mathbb{E} \sup_{0 \leq s \leq T} \|X_s^1 - X_s^0\|^2$. (Note that by Lemma 3.5, $C_0 < \infty$.)

Proof. We prove by induction on n . The statement is obvious for $n = 0$. Assume that the statement is proved for $n - 1$. We have,

$$(3.3) \quad X_t^{n+1} - X_t^n = \int_0^t S_{t-s}(f(s, X_s^{n+1}) - f(s, X_s^n))ds + \int_0^t S_{t-s}dM_s,$$

where

$$\begin{aligned} M_t &= \int_0^t (g(s, X_{s-}^n) - g(s, X_{s-}^{n-1}))dW_s \\ &\quad + \int_0^t \int_E (k(s, \xi, X_{s-}^n) - k(s, \xi, X_{s-}^{n-1}))\tilde{N}(ds, d\xi). \end{aligned}$$

Applying Itô type inequality (Theorem 2.3), for $\alpha = 0$, we have

$$(3.4) \quad \|X_t^{n+1} - X_t^n\|^2 \leq 2 \underbrace{\int_0^t \langle X_{s-}^{n+1} - X_{s-}^n, f(s, X_s^{n+1}) - f(s, X_s^n) \rangle ds}_{A_t} + 2 \underbrace{\int_0^t \langle X_{s-}^{n+1} - X_{s-}^n, dM_s \rangle}_{B_t} + [M]_t.$$

For the term A_t , the semi-monotonicity assumption on f implies

$$(3.5) \quad A_t \leq M \int_0^t \|X_s^{n+1} - X_s^n\|^2 ds$$

We also have

$$\begin{aligned} \mathbb{E}[M]_t &= \int_0^t \mathbb{E} \|g(s, X_s^n) - g(s, X_s^{n-1})\|^2 ds \\ &\quad + \int_0^t \int_E \mathbb{E} \|k(s, \xi, X_s^n) - k(s, \xi, X_s^{n-1})\|^2 \nu(d\xi) ds, \end{aligned}$$

where by Hypothesis 2.1(b),

$$(3.6) \quad \leq C \int_0^t \mathbb{E} \|X_s^n - X_s^{n-1}\|^2 ds.$$

Applying Burkholder-Davies-Gundy inequality ([15, Theorem 3.50]), for $p = 1$, to term B_t we find,

$$\begin{aligned} \mathbb{E} \sup_{0 \leq s \leq t} |B_s| &\leq C_1 \mathbb{E} \left([B]_t^{\frac{1}{2}} \right) \\ &\leq C_1 \mathbb{E} \left(\sup_{0 \leq s \leq t} (\|X_s^{n+1} - X_s^n\|) [M]_t^{\frac{1}{2}} \right), \end{aligned}$$

where C_1 is the universal constant in the Burkholder-Davies-Gundy inequality. Applying Cauchy-Schwartz inequality we find,

$$(3.7) \quad \leq \frac{1}{4} \mathbb{E} \sup_{0 \leq s \leq t} \|X_s^{n+1} - X_s^n\|^2 + C_1^2 \mathbb{E}[M]_t.$$

Now, taking supremums and then expectation on both sides of (3.4) and substituting (3.5), (3.6) and (3.7), we find

$$\begin{aligned} (3.8) \quad \mathbb{E} \sup_{0 \leq s \leq t} \|X_s^{n+1} - X_s^n\|^2 &\leq 2M \int_0^t \mathbb{E} \|X_s^{n+1} - X_s^n\|^2 ds \\ &\quad + C(1 + 2C_1^2) \int_0^t \mathbb{E} \|X_s^n - X_s^{n-1}\|^2 ds \\ &\quad + \frac{1}{2} \mathbb{E} \left(\sup_{0 \leq s \leq t} \|X_s^{n+1} - X_s^n\| \right)^2. \end{aligned}$$

The last term in the right hand side could be subtracted from the left hand side but for this subtraction to be valid it should be finite which is guaranteed by Lemma 3.5. After subtraction we find,

$$\mathbb{E} \sup_{0 \leq s \leq t} \|X_s^{n+1} - X_s^n\|^2 \leq 4M \int_0^t \mathbb{E} \|X_s^{n+1} - X_s^n\|^2 ds + 2C(1 + 2C_1^2) \int_0^t \mathbb{E} \|X_s^n - X_s^{n-1}\|^2 ds.$$

Now let $h^n(t) = \mathbb{E} \sup_{0 \leq s \leq t} \|X_s^{n+1} - X_s^n\|^2$. Hence,

$$h^n(t) \leq 4M \int_0^t h^n(s) ds + 2C(1 + 2C_1^2) \int_0^t h^{n-1}(s) ds.$$

Note that by Lemma 3.5, $h^n(t)$ is bounded on $[0, T]$. Hence we can use Gronwall's inequality for $h^n(t)$ and find

$$h^n(t) \leq C_1 \int_0^t h^{n-1}(s) ds,$$

where by induction hypothesis,

$$\leq C_1 \int_0^t C_0 C_1^{n-1} \frac{s^{n-1}}{(n-1)!} ds = C_0 C_1^n \frac{t^n}{n!},$$

which completes the proof. □

Returning to the proof of Theorem 3.3, we see that since the right hand side of (3.2) is a convergent series, $\{X^n\}$ is a Cauchy sequence in $L^2(\Omega, \mathcal{F}, \mathbb{P}; L^\infty([0, T]; H))$ and hence converges to a process $X_t(\omega)$. By choosing a subsequence they converge almost surely uniformly with respect to t , and since $\{X_t^n\}$ are adapted càdlàg, so is X_t .

It remains to show that X_t is a solution of (2.1). It suffices to show that the terms on both sides of equation (3.1) converge to the corresponding terms of (2.1). We know already that $X_t^n \rightarrow X_t$ in $L^2([0, T] \times \Omega; H)$. Moreover, by [10, Theorem 1.1] we have,

$$\begin{aligned} \mathbb{E} \left\| \int_0^t S_{t-s} g(s, X_{s-}^n) dW_s - \int_0^t S_{t-s} g(s, X_{s-}) dW_s \right\|^2 \\ \leq \mathbf{C} \mathbb{E} \int_0^t \|g(s, X_s^n) - g(s, X_s)\|^2 ds \\ \leq \mathbf{C} \mathbf{C} \int_0^t \mathbb{E} \|X_s^n - X_s\| ds \rightarrow 0, \end{aligned}$$

and

$$\begin{aligned} \mathbb{E} \left\| \int_0^t \int_E S_{t-s} k(s, \xi, X_{s-}^n) d\tilde{N} - \int_0^t \int_E S_{t-s} k(s, \xi, X_{s-}) d\tilde{N} \right\|^2 \\ \leq \mathbf{C} \mathbb{E} \int_0^t \int_E \|k(s, \xi, X_s^n) - k(s, \xi, X_s)\|^2 \nu(d\xi) ds \\ \leq \mathbf{C} \mathbf{C} \int_0^t \mathbb{E} \|X_s^n - X_s\| ds \rightarrow 0. \end{aligned}$$

Hence the terms of V_t^n converge to the corresponding terms of (2.1). Finally we show that the term containing f in (3.1) converges in the weak sense to corresponding term in (2.1). If $x \in H$,

$$(3.9) \quad \mathbb{E} \left\langle x, \int_0^t S_{t-s} (f(s, X_s^n) - f(s, X_s)) ds \right\rangle = \mathbb{E} \int_0^t \langle S_{t-s}^* x, f(s, X_s^n) - f(s, X_s) \rangle ds.$$

By demicontinuity of f , the integrand on the right hand side converges to 0 for almost every $(s, \omega) \in [0, t] \times \Omega$. On the other hand, by Hypothesis 2.1(c), the integrand is dominated by a constant multiple of $\|x\|(1 + \|X_s\| + \|X_s^n\|)$ where $\|X_s^n\| \rightarrow \|X_s\|$ pointwise almost everywhere and in $L^1([0, T] \times \Omega)$, hence by dominated convergence theorem we conclude that right hand side of (3.9) tends to 0. Hence X_t is a mild solution of (1.1). \square

4. Some examples

In this section we provide some concrete examples of semilinear stochastic evolution equations with monotone nonlinearity and multiplicative Poisson

noise. The examples consist of stochastic partial differential equations of parabolic and hyperbolic type and a stochastic delay differential equation. We show that these examples satisfy the assumptions of equation (1.1) and hence one can apply Theorem 3.3 to them.

Example 4.1 (Stochastic reaction-diffusion equations with multiplicative Poisson noise). In this example we consider a class of semilinear stochastic evolution equations with multiplicative Poisson noise. Let \mathcal{D} be a bounded domain with a smooth boundary in \mathbb{R}^d . Consider the equation,

$$(4.1) \quad \begin{cases} du(t) &= Au(t)dt + f(u(t, x))dt + \eta u(t)dt + \int_E k(t, \xi, u(t^-, x))\tilde{N}(dt, d\xi) \\ u(0) &= u_0, \end{cases}$$

where A is the generator of a C_0 semigroup on $L^2(\mathcal{D})$, $f : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous decreasing function with linear growth and $k : [0, T] \times E \times \mathbb{R} \times \Omega \rightarrow \mathbb{R}$ is measurable and satisfies the Lipschitz condition

$$\mathbb{E} \int_E |k(s, \xi, u) - k(s, \xi, v)|^2 \mu(d\xi) \leq C|u - v|^2,$$

and the linear growth condition

$$\mathbb{E} \int_E |k(s, \xi, u)|^2 \mu(d\xi) \leq D(1 + |u|^2),$$

and $u_0 \in L^2(\mathcal{D})$. We show that equation (4.1) satisfies the assumptions of equation (1.1). Let $H = L^2(\mathcal{D})$. We denote the Nemitsky operator associated with a function $f : \mathbb{R} \rightarrow \mathbb{R}$ by the same symbol. Since f and k are continuous and have linear growth, by [17, Theorem 10.58], the associated Nemitsky operators define continuous operators from $L^2(\mathcal{D})$ to $L^2(\mathcal{D})$ and have linear growth. Verifying the other assumptions is straight forward.

Example 4.2 (Second order stochastic hyperbolic equations with Lévy noise). In this example we consider a hyperbolic SPDE with Lévy noise. Let \mathcal{D} be a bounded domain with a smooth boundary in \mathbb{R}^d . Consider the initial boundary value problem,

$$(4.2) \quad \begin{cases} \frac{\partial^2 u}{\partial t^2} = \Delta u - \sqrt[3]{\frac{\partial u}{\partial t}} + u(t^-, x) \frac{\partial Z}{\partial t} & \text{on } [0, \infty) \times \mathcal{D} \\ u = 0 & \text{on } [0, \infty) \times \partial \mathcal{D} \\ u(0, x) = u_0(x) & \text{on } \mathcal{D}. \\ \frac{\partial u}{\partial t}(0, x) = 0 & \text{on } \mathcal{D}, \end{cases}$$

where $Z(t)$ is a real valued square integrable Lévy process and $u_0(x) \in L^2(\mathcal{D})$ is the initial condition. One can replace $-\sqrt[3]{x}$ by any continuous decreasing real function with linear growth.

Let $H^1(\mathcal{D})$ be the Sobolev space of weakly differentiable functions on \mathcal{D} with derivative in $L^2(\mathcal{D})$ and let $H = H^1(\mathcal{D}) \times L^2(\mathcal{D})$.

Note that Δ is self adjoint and negative definite on L^2 . Moreover, we have

$$D((-\Delta)^{\frac{1}{2}}) = H^1(\mathcal{D}).$$

Hence by [15, Lemma B.3], the operator

$$\mathcal{A} = \begin{pmatrix} 0 & I \\ \Delta & 0 \end{pmatrix}$$

generates a C_0 semigroup of contractions on H .

Let $K = E = \mathbb{R}$. We also define for $(u, v) \in H$ and $\phi \in K$ and $\xi \in E$,

$$f(u, v) = \begin{pmatrix} 0 \\ -\sqrt[3]{v(x)} \end{pmatrix}, \quad g(u, v) = 0, \quad k(\xi, u, v) = \begin{pmatrix} 0 \\ u(x)\xi \end{pmatrix}.$$

Hence equation (4.2) can be written as

$$dX(t) = \mathcal{A}X(t)dt + f(X(t))dt + g(X(t^-))dW_t + \int_E k(\xi, X(t^-))\tilde{N}(dt, d\xi).$$

We claim that f , g and k satisfy Hypothesis 2.1. The continuity of f , g and k follow as in example 4.1. The other conditions are straightforward.

Example 4.3 (Stochastic delay equations). In this example we consider a stochastic delay differential equation in \mathbb{R} . The case of Lipschitz coefficients, have been studied before in [15]. We have replaced Lipschitzness of f by the weaker assumption of semi-monotonicity.

Consider the following equation,

$$(4.3) \quad \begin{cases} dx(t) = \left(\int_{-1}^0 x(t+\theta)d\theta \right) dt - \sqrt[3]{x(t)}dt + x(t)dZ_t \\ x(\theta) = \sin(\pi\theta), \quad \theta \in (-1, 0], \end{cases}$$

where Z_t is a real valued square integrable Lévy process. We show that this equation satisfies the assumptions of equation (1.1). The term $-\sqrt[3]{x}$ can be replaced by any continuous decreasing real function with linear growth and the initial condition can be replaced with any function in $L^2((-1, 0])$.

Let $H = \mathbb{R} \times L^2((-1, 0])$ and define the operator A on H by

$$A \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \int_{-1}^0 v(\theta)d\theta \\ \frac{\partial v}{\partial \theta} \end{pmatrix}.$$

According to Da Prato and Zabczyk [5, Proposition A.25], the operator A with domain

$$D(A) = \left\{ \begin{pmatrix} u \\ v \end{pmatrix} \in H : v \in W^{1,2}(-1, 0), v(0) = u \right\}$$

generates a C_0 semigroup S_t on H . Let $K = E = \mathbb{R}$ and let \tilde{N} be the compensated Poisson random measure associated with Z_t . Define for $\begin{pmatrix} u \\ v \end{pmatrix} \in H$ and

$\xi \in \mathbb{R}$,

$$f(u, v) = \begin{pmatrix} -\sqrt[3]{u} \\ 0 \end{pmatrix}, g(u, v) = 0, k(\xi, u, v) = \begin{pmatrix} \xi u \\ 0 \end{pmatrix}.$$

It is easy to verify that f , g and k satisfy Hypothesis 2.1. Now, if we let

$$X(t) = \begin{pmatrix} x(t) \\ x_t \end{pmatrix}$$

where $x_t(\theta) = x(t + \theta)$ for $\theta \in (-1, 0]$, then equation (4.3) can be written as

$$dX(t) = AX(t)dt + f(X(t))dt + g(X(t^-))dW_t + \int_E k(\xi, X(t^-))\tilde{N}(dt, d\xi)$$

with initial condition

$$X(0) = \begin{pmatrix} \psi(0) \\ \psi \end{pmatrix}.$$

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