ISSN: 1017-060X (Print)

ISSN: 1735-8515 (Online)

Bulletin of the

Iranian Mathematical Society

Vol. 43 (2017), No. 5, pp. 1313–1321

Title:

Dilations for C^*-dynamical systems with abelian groups on Hilbert C^*-modules

Author(s):

Z. Wang and J. Zhang

Published by the Iranian Mathematical Society
http://bims.ims.ir

Bull. Iranian Math. Soc.
Vol. 43 (2017), No. 5, pp. 1313–1321
Online ISSN: 1735-8515

DILATIONS FOR C^*-DYNAMICAL SYSTEMS WITH ABELIAN GROUPS ON HILBERT C^*-MODULES

Z. WANG* AND J. ZHANG

(Communicated by Ali Ghaffari)

Abstract. In this paper we investigate the dilations of completely positive definite representations of C^*-dynamical systems with abelian groups on Hilbert C^*-modules. We show that if $(\mathcal{A}, G, \alpha)$ is a C^*-dynamical system with G an abelian group, then every completely positive definite covariant representation (π, φ, E) of $(\mathcal{A}, G, \alpha)$ on a Hilbert C^*-module E admits an unitary dilation $(\hat{\pi}, \hat{\varphi}, \hat{E})$.
Keywords: Dilation, covariant representation, C^*-dynamical system, Hilbert C^*-module.
MSC(2010): Primary: 46L05; Secondary: 46L55.

1. Introduction

Hilbert C^*-modules are generalizations of Hilbert spaces by allowing the inner products to take values in C^*-algebras rather than in the field of complex numbers. Hilbert C^*-modules are very useful in the fields of operator theory, operator K-theory, group representation theory, Morita equivalence of C^*-algebras, etc.

Definition 1.1 ([7]). Let $\mathcal{A}$ be a C^*-algebra. An inner product $\mathcal{A}$-module is a linear space E which is a right $\mathcal{A}$-module, together with a map $(x, y) \mapsto \langle x, y \rangle : E \times E \to \mathcal{A}$ such that

(i) $\langle x, \alpha y + \beta z \rangle = \alpha \langle x, y \rangle + \beta \langle x, z \rangle \quad (x, y, z \in E, \alpha, \beta \in \mathbb{C})$,
(ii) $\langle x, ya \rangle = \langle x, y \rangle a \quad (x, y \in E, a \in \mathcal{A})$,
(iii) $\langle y, x \rangle = \langle x, y \rangle^* \quad (x, y \in E)$,
(iv) $\langle x, x \rangle \geq 0$; if $\langle x, x \rangle = 0$ then $x = 0$ $(x \in E)$.

An inner product $\mathcal{A}$-module which is complete with respect to the norm $\| \cdot \|$ defined by $\|x\| = \|\langle x, x \rangle\|^{\frac{1}{2}}$ is called a Hilbert $\mathcal{A}$-module.

Article electronically published on 31 October, 2017.
Received: 10 June 2015, Accepted: 29 May 2016.
*Corresponding author.

Given two Hilbert $\mathcal{A}$-modules E and F, we denote by $\mathcal{L}(E,F)$ the set of all bounded $\mathcal{A}$-linear maps $t : E \to F$ for which there is a map $t^* : F \to E$ such that

$$\langle tx, y\rangle = \langle x, t^*y\rangle, \quad \forall x \in E, y \in F.$$

Namely, $\mathcal{L}(E,F)$ is the set of all adjointable homomorphisms from E to F. For convenience, we write $\mathcal{L}(E)$ for $\mathcal{L}(E,E)$.

Let G be a group, E a Hilbert C^*-module over a C^*-algebra $\mathcal{B}$, then a map $\phi : G \to \mathcal{L}(E)$ is completely positive definite means that for every finite set of elements $g_1, g_2, \cdots, g_n \in G$, the operator matrix $[\phi(g_i^{-1}g_j)]$ is positive.

A C^*-dynamical system is a triple $(\mathcal{A}, G, \alpha)$, where $\mathcal{A}$ is a C^*-algebra, G is a group and $\alpha : G \to \mathrm{Aut}(\mathcal{A})$ is a group homomorphism. Furthermore, if G is a topological group, α is required to be continuous under the point-norm topology, i.e., $g \mapsto \alpha(g)(a)$ from G to $\mathcal{A}$ is continuous for all $a \in \mathcal{A}$. Let E be a Hilbert C^*-module over a C^*-algebra $\mathcal{B}$, a covariant representation of a C^*-dynamical $(\mathcal{A}, G, \alpha)$ on E is a pair (π, ϕ), where $\pi : \mathcal{A} \to \mathcal{L}(E)$ is a representation of $\mathcal{A}$ and $\phi : G \to \mathcal{L}(E)$ is a representation of G, which satisfy

$$\phi(g)\pi(\alpha(g)(a)) = \pi(a)\phi(g) \tag{1.1}$$

for all $a \in \mathcal{A}$ and all $g \in G$. If ϕ is completely positive definite (resp. unitary), (π, ϕ) is called completely positive definite (resp. unitary).

If T is an operator on a Hilbert space $\mathcal{H}$, an operator S on a Hilbert space $\mathcal{K} \supset \mathcal{H}$ is called a dilation of T if $P_{\mathcal{H}}S^k|_{\mathcal{H}} = T^k$ for all $k \geq 0$. By a result of Sarason [14], S is a dilation of T if and only if $\mathcal{H}$ is a semi-invariant subspace of $\mathcal{K}$ and $P_{\mathcal{H}}S|_{\mathcal{H}} = T$. If $\mathcal{A}$ is an algebra and $\phi : \mathcal{A} \to \mathcal{B}(\mathcal{H})$ is a representation of $\mathcal{A}$ on a Hilbert space $\mathcal{H}$, then a representation $\psi : \mathcal{A} \to \mathcal{B}(\mathcal{K})$ of $\mathcal{A}$ on a Hilbert space $\mathcal{K}$ containing $\mathcal{H}$ is a dilation of ϕ, if $\phi(a) = P_{\mathcal{H}}\psi(a)|_{\mathcal{H}}$ for all $a \in \mathcal{A}$.

Dilation theory is a classical theory in operator theory with extensive applications. Sz.-Nagy's dilation theorem [2] is a central result in the dilation theory of contractions, which says that every contraction on a Hilbert space $\mathcal{H}$ has a unitary dilation. Since the work of Sz.-Nagy, many mathematicians did a lot of nice work. Ando [1] generalized Sz.-Nagy's dilation theorem to a pair of commuting contractions, he proved such a pair has a commuting dilation. However, Ando's dilation theorem can not be generalized any further, Varopoulos [20] showed that it fails for a triple of commuting contractions. However, in the case of row contraction $T = [T_1, T_2, \cdots, T_n]$, where $\|T\| \leq 1$, considered as an operator in $\mathcal{B}(H^{(n)}, H)$, the Frazho-Bunce dilation theorem [3, 4] shows that there is a minimal dilation to a row isometry. For more, see also [13].

Stinespring's theorem for completely positive maps on C^*-algebras is another essential result in dilation theory which is a natural generalization of the GNS theorem. Following this way, many mathematicians considered the dilation of a group or semigroup of completely positive maps, i.e., the so called CP-semigroup. For example, Muhly and Solel [8–10], Pandiscia [12], Shalit [15,16], Skalski [17], Skeide [18] and Solel [19]. In [8], Muhly and Solel studied the dilations of covariant representations of C^*-correspondences, and in [10], they studied the dilations of CP-semigroups via the dilations of covariant representations of a C^*-correspondence. They showed there exists a one-to-one correspondence between the set of CP-maps and the set of covariant representations of a C^*-correspondence.

It is interesting to ask whether the dilation of every covariant representation of a C^*-dynamical system is again covariant? In [11], Muhly and Solel studied this problem for C^*-dynamical systems $(\mathcal{A}, \alpha)$, with α a endomorphism of $\mathcal{A}$. They established coisometry and unitary dilations of a contractive covariant representation of $(\mathcal{A}, \alpha)$. Joita, Costache, and Zamfir [6] extended the result of Muhly and Solel [11] to representations on Hilbert C^*-modules.

The purpose of this paper is to establish the unitary dilations of covariant representations of C^*-dynamical systems with groups rather than a single endomorphism. In order to do this we need to focus on the case where the covariant representation is completely positive definite and the group is abelian. Now we give two lemmas we will use, which are fundamental in the theory of Hilbert C^*-modules.

Lemma 1.2 ([7, Lemma 4.1])**.** *Let E be a Hilbert $\mathcal{A}$-module and let t be a bounded $\mathcal{A}$-linear operator on E. The following conditions are equivalent:*

(i) *t is a positive element of $\mathcal{L}(E)$,*
(ii) *$\langle x, tx\rangle \geq 0$ for all x in E.*

Lemma 1.3 ([7, Theorem 3.5])**.** *Let $\mathcal{A}$ be a C^*-algebra, let E, F be Hilbert $\mathcal{A}$-modules and let u be a linear map from E to F. Then the following conditions are equivalent:*

(i) *u is an isometric, surjective $\mathcal{A}$-linear map,*
(ii) *u is a unitary element of $\mathcal{L}(E, F)$.*

2. Main results

Theorem 2.1. *Let $(\mathcal{A}, G, \alpha)$ be a C^*-dynamical system with an abelian group G, and let (π, φ) be a covariant completely positive definite representation of $(\mathcal{A}, G, \alpha)$ on a Hilbert C^*-module E over a C^*-algebra $\mathcal{B}$. Then there exists a Hilbert C^*-module $\hat{E}$ over $\mathcal{B}$, a unitary covariant representation $(\hat{\pi}, \hat{\varphi})$ on $\hat{E}$ and an adjointable isometric operator $W : E \to \hat{E}$ such that $\operatorname{ran}(E)$ is a complemented submodule of $\hat{E}$ and*

$$\pi(a) = W^*\hat{\pi}(a)W, \quad \varphi(g) = W^*\hat{\varphi}(g)W \tag{2.1}$$

for all $a \in \mathcal{A}, g \in G$.

Proof. Firstly, we give the construction of $\hat{E}$ which is similar to the proof of [5, Proposition 3.1]. Let $K(G, E)$ be the vector space of all finitely supported functions from G to E. Define a right module action on $K(G,E)$ by $\mathcal{B}$ as

$$(fb)(g) = f(g)b, \quad \forall f \in K(G,E), g \in G, b \in \mathcal{B}, \tag{2.2}$$

and define a sesquilinear function on $K(G,E)$ as

$$\langle f_1, f_2 \rangle = \sum_{g_1, g_2 \in G} \langle f_1(g_1), \varphi(g_1^{-1}g_2) f_2(g_2) \rangle_E, \quad \forall f_1, f_2 \in K(G,E). \tag{2.3}$$

By the fact that φ is completely positive definite and Lemma 1.2, we obtain that $\langle f, f \rangle \geq 0$ for all $f \in K(G,E)$. And by the definition of completely positive map, for every $g \in G$, $\begin{bmatrix} \varphi(e) & \varphi(g) \\ \varphi(g^{-1}) & \varphi(e) \end{bmatrix}$ is positive, where e is the unit element in G, so $\varphi(g)^* = \varphi(g^{-1})$, and so

$$\begin{aligned}
\langle f_2, f_1 \rangle &= \sum_{g_1, g_2 \in G} \langle f_2(g_1), \varphi(g_1^{-1}g_2) f_1(g_2) \rangle_E \\
&= \sum_{g_1, g_2 \in G} \langle \varphi(g_1^{-1}g_2) f_1(g_2), f_2(g_1) \rangle_E^* \\
&= \sum_{g_1, g_2 \in G} \langle f_1(g_2), \varphi(g_1^{-1}g_2)^* f_2(g_1) \rangle_E^* \\
&= \sum_{g_1, g_2 \in G} \langle f_1(g_2), \varphi((g_1^{-1}g_2)^{-1}) f_2(g_1) \rangle_E^* \\
&= \sum_{g_1, g_2 \in G} \langle f_1(g_2), \varphi(g_2^{-1}g_1) f_2(g_1) \rangle_E^* \\
&= \sum_{g_1, g_2 \in G} \langle f_1(g_1), \varphi(g_1^{-1}g_2) f_2(g_2) \rangle_E^* \\
&= \langle f_1, f_2 \rangle^*.
\end{aligned}$$

Hence $\langle \cdot, \cdot \rangle$ is a pre-inner product on $K(G,E)$. It is apparent that $\mathcal{N} = \{f \in K(G,E) : \langle f, f \rangle = 0\}$ is a submodule of $K(G,E)$. Let $\hat{E}$ be the completion of $K(G,E)/\mathcal{N}$ with respect to the induced inner product, then $\hat{E}$ is a Hilbert $\mathcal{B}$-module. We denote by $\overline{f}$ the equivalent class in $K(G,E)/\mathcal{N}$ with representation element f. Now we proceed the proof in four steps.

Step 1. For each $g \in G$, define $\hat{\varphi}(g)$ on $K(G,E)/\mathcal{N}$ as

$$\hat{\varphi}(g)(\overline{f}) = \overline{\hat{\varphi}(g)_f},$$

where $\hat{\varphi}(g)_f : G \to E$ defined as $\hat{\varphi}(g)_f(g') = f(g^{-1}g')$ for all $g' \in G$. It is not hard to check $\hat{\varphi}(g)$ is $\mathcal{B}$-linear and surjective on $K(G,E)/\mathcal{N}$. Fixed $g \in G$, then

$$\begin{aligned}
\|\hat{\varphi}(g)(\overline{f})\|^2 &= \langle \hat{\varphi}(g)(\overline{f}), \hat{\varphi}(g)(\overline{f})\rangle \\
&= \langle \overline{\hat{\varphi}(g)_f}, \overline{\hat{\varphi}(g)_f}\rangle \\
&= \sum_{g_1,g_2\in G} \langle f(g^{-1}g_1), \varphi(g_1^{-1}g_2)f(g^{-1}g_2)\rangle \\
&= \sum_{g_1,g_2\in G} \langle f(g^{-1}g_1), \varphi((g^{-1}g_1)^{-1}(g^{-1}g_2))f(g^{-1}g_2)\rangle \\
&= \langle f, f\rangle \\
&= \|\overline{f}\|^2.
\end{aligned}$$

So $\hat{\varphi}(g)$ is an isometric surjective $\mathcal{B}$-linear map on $K(G,E)/\mathcal{N}$, and so can be extended to an isometric surjective $\mathcal{B}$-linear map on $\hat{E}$, we denote it still by $\hat{\varphi}(g)$.

By Lemma 1.3, $\hat{\varphi}(g) \in \mathcal{L}(\hat{E})$ is unitary. It is not hard to check that $\hat{\varphi}(g_1g_2) = \hat{\varphi}(g_1)\hat{\varphi}(g_2)$ for all $g_1, g_2 \in G$. Therefore, $\hat{\varphi} : g \mapsto \hat{\varphi}(g)$ is a unitary representation of G on $\hat{E}$.

Step 2. Let $g \in G, x \in E$, denote by $\sigma_{g,x}$ the function in $K(G,E)$ defined as

$$\sigma_{g,x}(g') = \begin{cases} 0 & g' \neq g, \\ x & g' = g, \end{cases}$$

for all $g' \in G$. Recall that every $f \in K(G,E)$ is finitely supported, then there exists a unique set of $g_1, g_2, \cdots, g_n, x_1, x_2, \cdots, x_n$ such that

$$f = \sum_{i=1}^{n} \sigma_{g_i,x_i}, \tag{2.4}$$

where the elements g_i are distinct with each other and we call (2.4) the canonical form of f. For each $a \in A$, define

$$\hat{\pi}(a) : K(G,E)/\mathcal{N} \to K(G,E)/\mathcal{N}$$

by $\hat{\pi}(a)(\overline{f}) = \overline{f_a}$ for all $f = \sum_{i=1}^{n} \sigma_{g_i,x_i} \in K(G,E)$, where

$$f_a(g') = \sum_{i=1}^{n} \pi(\alpha(g_i^{-1}g'^2)(a))(\sigma_{g_i,x_i}(g'))$$

for all $g' \in G$. It is not hard to check that $\hat{\pi}(a)$ is $\mathcal{B}$-linear. Now we show that $\hat{\pi}(a)$ is continuous. For all $\overline{f} \in K(G,E)/\mathcal{N}$ with $f = \sum_{i=1}^{n} \sigma_{gi,xi}$,

$$\begin{aligned}
\|\hat{\pi}(a)(\overline{f})\|^2 &= \|\langle \overline{f_a}, \overline{f_a}\rangle\| \\
&= \|\langle f_a, f_a\rangle\| \\
&= \Big\| \sum_{g',g''\in G} \langle f_a(g'), \varphi(g'^{-1}g'') f_a(g'')\rangle \Big\| \\
&= \Big\| \sum_{g',g''\in G} \Big\langle \sum_{i=1}^{n} \pi(\alpha(g_i^{-1}g'^2)(a))(\sigma_{g_i,x_i}(g')), \varphi(g'^{-1}g'') f_a(g'')\Big\rangle \Big\| \\
&= \Big\| \sum_{i,j=1}^{n} \Big\langle \pi(\alpha(g_i)(a))(x_i), \varphi(g_i^{-1}g_j)\pi(\alpha(g_j)(a))(x_j)\Big\rangle \Big\|.
\end{aligned}$$

Let $A = [\phi(g_i^{-1}g_j)]$,

$$T = \operatorname{diag}(\pi(\alpha(g_1)(a)), \pi(\alpha(g_2)(a)), \cdots, \pi(\alpha(g_n)(a))),$$

and $X = (x_1, x_2, \cdots, x_n)^T$. By (1.1), we obtain that

$$\pi(\alpha(g_i)(a))\varphi(g_i^{-1}g_j) = \varphi(g_i^{-1}g_j)\pi(\alpha(g_j)(a)),$$

from which it follows that $AT = TA$, so $T^*AT = A^{\frac{1}{2}}T^*TA^{\frac{1}{2}} \le \|T\|^2 A$ and so

$$\langle \overline{f_a}, \overline{f_a}\rangle = \langle X, T^*ATX\rangle \le \langle X, \|T\|^2 AX\rangle = \|T\|^2\langle X, AX\rangle = \|T\|^2\langle \overline{f}, \overline{f}\rangle.$$

Hence, $\|\langle \overline{f_a}, \overline{f_a}\rangle\| \le \|T\|^2\|\langle \overline{f}, \overline{f}\rangle\|$, and $\|\hat{\pi}(a)(\overline{f})\| \le \|T\|\|\overline{f}\|$. Therefore, $\hat{\pi}(a)$ is continuous on $K(G,E)$. It can be extended naturally to a continuous $\mathcal{B}$-linear operator on $\hat{E}$, we also denote the extended operator by $\hat{\pi}(a)$. Now we prove that $\hat{\pi} : a \mapsto \hat{\pi}(a)$ is a representation of $\mathcal{A}$ on $\hat{E}$. It is apparent that $\hat{\pi}$ is linear. Put $\overline{\sigma_{g_1,x_1}}, \overline{\sigma_{g_2,x_2}} \in K(G,E)/\mathcal{N}$, then for any $a \in \mathcal{A}$,

$$\begin{aligned}
\langle \overline{\sigma_{g_1,x_1,}}, \hat{\pi}(a)\overline{\sigma_{g_2,x_2}}\rangle &= \langle \sigma_{g_1,x_1,}, \sigma_{g_2,\pi(\alpha(g_2)(a))(x_2)}\rangle \\
&= \langle x_1, \varphi(g_1^{-1}g_2)\pi(\alpha(g_2)(a))(x_2)\rangle \\
&= \langle x_1, \pi(\alpha(g_1)(a))\varphi(g_1^{-1}g_2)(x_2)\rangle \\
&= \langle \pi(\alpha(g_1)(a^*))(x_1), \varphi(g_1^{-1}g_2)(x_2)\rangle \\
&= \langle \hat{\pi}(a^*)\overline{\sigma_{g_1,x_1,}}, \overline{\sigma_{g_2,x_2}}\rangle. \qquad (2.5)
\end{aligned}$$

Since $\hat{\pi}(a)$ and $\hat{\pi}(a^*)$ are linear, it follows from (2.5) that

$$\langle \hat{\pi}(a^*)(\overline{f_1}), \overline{f_2}\rangle = \langle \overline{f_1}, \hat{\pi}(a)(\overline{f_2})\rangle$$

for all $\overline{f_1}, \overline{f_2} \in K(G,E)/\mathcal{N}$. Furthermore, for every $y_1, y_2 \in \hat{E}$, $\langle \hat{\pi}(a^*)(y_1), y_2\rangle = \langle y_1, \hat{\pi}(a)(y_2)\rangle$. Hence, $\hat{\pi}(a^*) = \hat{\pi}(a)^*$. Similarly, $\hat{\pi}(ab) = \hat{\pi}(a)\hat{\pi}(b)$. Therefore, $\hat{\pi}$ is a representation of $\mathcal{A}$ on $\hat{E}$.

Step 3. We show that $(\hat{\pi}, \hat{\varphi})$ is a covariant representation of $(\mathcal{A}, G, \alpha)$. Commutativity of G will be used in this step. Put $a \in \mathcal{A}$ and $g \in G$, then for any $\overline{f} \in K(G, E)/\mathcal{N}$ with $f = \sum_{i=1}^{n} \sigma_{gi,xi}$,

$$\hat{\pi}(a)\hat{\varphi}(g)(\overline{f}) = \hat{\pi}(a)\overline{\hat{\varphi}(g)_f} = \overline{(\hat{\varphi}(g)_f)_a},$$

where $(\hat{\varphi}(g)_f)_a$ maps g' to $\sum_{i=1}^{n} \pi(\alpha(g_i^{-1}g^{-1}g'^2)(a))(\sigma_{gi,xi}(g^{-1}g'))$ for all $g' \in G$. While,

$$\hat{\varphi}(g)\hat{\pi}(\alpha(g)(a))(\overline{f}) = \hat{\varphi}(g)(\overline{f_{\alpha(g)(a)}}) = \overline{\hat{\varphi}(g)_{f_{\alpha(g)(a)}}},$$

where

$$\begin{aligned} \hat{\varphi}(g)_{f_{\alpha(g)(a)}}(g') &= f_{\alpha(g)(a)}(g^{-1}g') \\ &= \sum_{i=1}^{n} \pi(\alpha(g_i^{-1}(g^{-1}g')^2)(\alpha(g)(a)))(\sigma_{g_i,x_i}(g^{-1}g')) \\ &= \sum_{i=1}^{n} \pi(\alpha(g_i^{-1}g^{-1}g'^2)(a))(\sigma_{g_i,x_i}(g^{-1}g')) \end{aligned}$$

for all $g' \in G$, since G is Abelian. Hence, $\hat{\pi}(a)\hat{\varphi}(g) = \hat{\varphi}(g)\hat{\pi}(\alpha(g)(a))$.

Step 4. Define $W : E \to \hat{E}$ as $W(x) = \overline{\sigma_{e,x}}$ for all $x \in E$, where e is the unit element in the group G. Then

$$\|W(x)\|^2 = \langle \sigma_{e,x}, \sigma_{e,x} \rangle = \sum_{g_1,g_2 \in G} \langle \sigma_{e,x}, \varphi(g_1^{-1}g_2)\sigma_{e,x} \rangle = \|x\|^2,$$

so W is an isometry. A direct calculation shows that $W \in \mathcal{L}(E, \hat{E})$ and $W^*W = I$. Since the range $\mathrm{ran}(E)$ of E is closed, then by [7, Theorem 3.2], $\mathrm{ran}(E)$ is complemented. Now for any $a \in \mathcal{A}, x \in E$,

$$\begin{aligned} \hat{\pi}(a)W(x) &= \hat{\pi}(a)\overline{\sigma_{e,h}} \\ &= \overline{\sigma_{e,\pi(a)x}} \\ &= W\pi(a)(x), \end{aligned}$$

so $\pi(a) = W^*\hat{\pi}(a)W$ for all $a \in \mathcal{A}$. It is clear that

$$\langle \sigma_{e,\varphi(g)x} - \sigma_{g,x}, \sigma_{e,\varphi(g)x} - \sigma_{g,x} \rangle = 0,$$

for all $g \in G$. This yields $\overline{\sigma_{e,\varphi(g)x}} = \overline{\sigma_{g,x}}$, so $W\varphi(g)(x) = \hat{\varphi}(g)W(x)$ and so $\varphi(g) = W^*\hat{\varphi}(g)W$.

□

Remark 2.2. If we do not distinguish E and $W(E)$, then E is a submodule of $\hat{E}$, and it is not difficult to see that $\hat{\pi}$ and $\hat{\varphi}$ are dilations of π and φ respectively.

Acknowledgements

This paper was supported by the Fundamental Research Funds for the Central Universities (No. GK201504003) and the National Natural Science Foundation of China(No. 11371233,11471199).

References

[1] T. Ando, On a pair of commuting contractions, *Acta Sci. Math. (Szeged)* **24** (1963), no. 1-2, 88–90.

[2] H. Bercovici, L. Kérchy, B. Sz.-Nagy and C. Foias, Harmonic Analysis of Operators in Hilbert Space, Springer, New York, 2010.

[3] J. Bunce, Models for n-tuples of non-commuting operators, *J. Funct. Anal.* **57** (1984), no. 1, 21–30.

[4] A. Frazho, Models for non-commuting operators, *J. Funct. Anal.* **48** (1982), no. 1, 1–11.

[5] J. Heo, Hilbert C*-module representations on Haagerup tensor product and group systems, *Publ. RIMS, Kyoto Uni.* **35** (1999), no. 5, 757–768.

[6] M. Joita, T.L. Costache and M. Zamfir, Dilations on Hilbert C^*-modules for C^*-dynamical systems, *BSG Proc.* **14** (2007) 81-86.

[7] E.C. Lance, Hilbert C^*-modules, A Toolkit for Operator Algebraists, Cambridge Univ. Press, Cambridge, 1995.

[8] P. Muhly and B. Solel, Tensor algebras over C^*-correspondences: representations, dilations and C^*-envelopes, *J. Funct. Anal.* **158** (1998), no. 2, 389–457.

[9] P. Muhly and B. Solel, Tensor algebras, induced representations, and the Wold decomposition, *Canad. J. Math.* **51** (1999), no. 4, 850–880.

[10] P. Muhly and B. Solel, Quantum Markov processes (correspondences and dilations). *Int. J. Math.* **13** (2002), no. 8, 863–906.

[11] P. Muhly and B. Solel, Extensions and dilations for C^*-dynamical systems, *Contemp. Math.* **414** (2006) 375–381.

[12] C. Pandiscia, Ergodic dilation of a quantum dynamical system, *Confluentes Mathematici* **6** (2014), no. 1, 77–91.

[13] G. Popescu, Isometric dilations for infinite sequences of noncommuting operators, *Trans. Amer. Math. Soc.* **316** (1989), no. 2, 523–536.

[14] D. Sarason, Invariant subspaces and unstarred operator algebras, *Pacific J. Math.* **17** (1966), no. 3, 511–517.

[15] O. Shalit, E_0-dilation of strongly commuting CP_0-semigroups, *J. Funct. Anal.* **255** (2008), no. 1, 46–89.

[16] O. Shalit, E-dilation of strongly commuting CP-semigroups (the nonunital case), *Houston J. Math.* **37** (2011), no. 1, 203–232.

[17] A. Skalski, On isometric dilations of product systems of C^*-correspondences and applications to families of contractions associated to higher-rank graphs, *Indiana Univ. Math. J.* **58** (2009), no. 5, 2227–2252.

[18] M. Skeide, Isometric dilations of representations of product systems via commutants, *Int. J. Math.* **19** (2008), no. 5, 521–540.

[19] B. Solel, Regular dilations of representations of product systems, *Math. Proc. R. Ir. Acad.* **108A** (2008), no. 1, 89–110.

[20] N. Varopoulos, On an inequality of von Neumann and an application of the metric theory of tensor products to operators theory, *J. Funct. Anal.* **16** (1974), no. 1, 83–100.

(Zhonghua Wang) School of Mathematics and Information Science, Shaanxi Normal University, Xi'an, Shaanxi, 710062, P.R. China.
E-mail address: wzhh@snnu.edu.cn

(Jianhua Zhang) School of Mathematics and Information Science, Shaanxi Normal University, Xi'an, Shaanxi, 710062, P.R. China.
E-mail address: jhzhang@snnu.edu.cn