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POLYNOMIAL ÖRE EXTENSIONS OF BAER AND P.P.-RINGS

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Abstract. For a ring endomorphism α and an α-derivation δ, we introduce (α,δ)-compatible rings which generalize α-rigid rings. We study the relationship between the Baer and p.p. properties of a ring and its Öre extensions. These include formal skew power series, skew Laurent polynomials and skew Laurent series. As a consequence we obtain a generalization of [3] and [16].

1. Introduction

Throughout this paper R denotes an associative ring with unity, $\alpha : R \to R$ is an endomorphism, which is not assumed to be surjective, and δ is an α-derivation of R, that is, δ is an additive map such that $\delta(ab) = \delta(a)b + \alpha(a)\delta(b)$, for all $a, b \in R$. We denote by $S = R[x;\alpha,\delta]$ the Öre extension whose elements are polynomials $\Sigma_{i=0}^{n} r_i x^i \in R[x;\alpha,\delta]$, $r_i \in R$, where the addition is defined as usual and the multiplication is given by $xb = \alpha(b)x + \delta(b)$ for any $b \in R$.

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Recall that R is a (quasi-)Baer ring if the right annihilator of every (right ideal) non-empty subset of R is generated (as a right ideal) by an idempotent of R. These definitions are left-right symmetric. The study of Baer rings has its roots in functional analysis. In [28] Rickart studied C^*-algebras with the property that every right annihilator of any element is generated by a projection, i.e., p is a projection if $p = p^2 = p^*$, where $*$ is the involution on the algebra. Using Rickart's work, Kaplansky [20] defined an AW*-algebra as a C*-algebra with the stronger property that the right annihilator of the nonempty subset is generated by a projection. In [10] Clark defined quasi-Baer rings and used them to characterize any finite dimensional algebra with unity over an algebraically closed field which is isomorphic to a twisted matrix units semigroup algebra. Further work on Baer and quasi-Baer rings appears in [4-18], [22] and [26]. A ring satisfying a generalization of Rickart's condition, i.e., a ring for which every right annihilator of any element as a right ideal is generated by an idempotent, has a homological characterization as a right p.p.-ring. A ring R is called a *right (resp. left) p.p.*-ring if every principal right (resp. left) ideal is projective, or, equivalently, if the right (resp. left) annihilator of an element of R is generated, as a right (resp. left) ideal, by an idempotent of R. A ring R is called a *p.p.-ring*, or a Rickart ring, if it is both right and left p.p. The concept of p.p.-ring is not left-right symmetric by a result of Chase [8]. A right p.p.-ring R is Baer (so p.p), if R is orthogonally finite (see Small [29]), and a right p.p.-ring R is p.p if R is abelian (see Endo [11]). Moreover Chatters and Xue [9], proved that in a duo p.p.-ring R, if I is a finitely generated right projective ideal of R, then I is left projective and a direct summand of an invertible ideal. Note that in a *reduced ring* R, i.e., it has no nonzero nilpotent elements, and R is Baer if and only if R is quasi-Baer.

A natural question for a given class of Baer rings is the behavior of a given class with respect to polynomial extensions? In 1974, Armendariz gave the following result on the behavior of a polynomial ring over a Baer ring: Let R be a reduced ring, then $R[x]$ is a Baer ring if and only if R is a Baer ring [3, Theorem B]. Armendariz provided an example to show that the condition to be reduced is not superfluous. Generalizations of Armendariz's result for several types of polynomial extensions over Baer and quasi-Baer rings are obtained by various authors, [3-7],[9],[12-14],[16-17], and [26-27].

Following Krempa [22], an endomorphism α of a ring R is called to be *rigid* if $a\alpha(a) = 0$ implies $a = 0$ for $a \in R$. A ring R is said to be *α-rigid* if there exists a rigid endomorphism α of R. Note that any rigid endomorphism of a ring is a monomorphism, and α-rigid rings are reduced (see Hong et al.[16]). Properties of α-rigid rings has been studied by Krempa [23], Hirano [14] and Hong et al. [16]. In [16] C.Y. Hong et al. studied Öre extensions of Baer and p.p.-rings over α-rigid rings.

Following [2], we say that R is *α-compatible* if for each $a, b \in R$, $ab = 0 \Leftrightarrow a\alpha(b) = 0$. Moreover, R is said to be *δ-compatible* if for each $a, b \in R$, $ab = 0 \Rightarrow a\delta(b) = 0$. If R is both α-compatible and δ-compatible, we say that R is *(α, δ)-compatible*. In this case, clearly the endomorphism α is injective. In light of similarity with the notion of α-rigid rings, we will show that an α-rigid ring is α-compatible and reduced (see Lemma 2.4). Thus α-compatible rings generalize α-rigid rings for the case R is not assumed to be reduced. A ring is called *abelian* if every idempotent element of the ring is central.

Following [27], a ring R is called an *Armendariz ring* if whenever two polynomials $f(x) = \sum_{i=0}^{m} a_i x^i$, $g(x) = \sum_{j=0}^{n} b_j x^j \in R[x]$ satisfy $f(x)g(x) = 0$, we have $a_i b_j = 0$ for every i, j.

Motivated by results of Armendariz [3], Anderson and Camillo [1], Kim and Lee [22], Hong et al. [16] and [17], we investigate a generalization of α-rigid rings and introduce conditions *(SA1)*, *(SA2)* which are skew polynomial versions of Armendariz rings:

Let α be a monomorphism of R and δ an α-derivation of R. We say R satisfies the
(i) *(SA1) condition* if whenever $f(x)g(x) = 0$ for $f(x) = \sum_{i=0}^{m} a_i x^i$

and $g(x) = \sum_{j=0}^{n} b_j x^j \in R[x; \alpha, \delta]$, then $a_i b_j = 0$ for all i,j.
(ii) *(SA2) condition* if whenever $f(x)g(x) = 0$ for $f(x) = \sum_{i=0}^{\infty} a_i x^i$ and $g(x) = \sum_{j=0}^{\infty} b_j x^j \in R[[x; \alpha]]$, then $a_i b_j = 0$ for all i,j.

Note that α-rigid rings satisfy both *(SA1)* and *(SA2)* conditions, by [16, Proposition 6].

In this paper we impose the (α, δ)-compatibility assumption on the ring R and prove the following results which unify and extend some known results non-trivially.

We will show that:
(1) If R satisfies the condition *(SA1)*, then R is a Baer (resp. right p.p.-) ring if and only if $R[x; \alpha, \delta]$ is a Baer (resp. right p.p.-) ring, or equivalently, if and only if $R[x, x^{-1}; \alpha]$ is a Baer (resp. right p.p.-) ring.
Since α-rigid rings satisfy the condition *(SA1)*, this result is a generalization of Hong et al. [16, Theorems 11, 14 and Corollaries 12, 16].
Note that for $\alpha = Id$ and $\delta = 0$, an Armendariz ring satisfies the condition (SA1), hence we obtain [22, Theorems 9 and 10] as an immediate corollary of Theorem 3.4.
Since a reduced ring is Armendariz, Corollary 3.16 is a generalization of a result of Birkenmeier et al. [7, Corollary 1.10] in the following sense:
If R is an Armendariz ring, then R is a Baer (resp. right p.p.-) ring if and only if $R[x, x^{-1}]$ is a Baer (resp. right p.p.-) ring.

(2) If R satisfies the condition *(SA2)*, then R is a Baer ring if and only if $R[[x; \alpha]]$ is a Baer ring, or equivalently, if and only if $R[[x, x^{-1}; \alpha]]$ is a Baer ring.
Since α-rigid rings satisfy the condition *(SA2)*, this result is a generalization of a result of Hong et al.[16, Theorem 21 and Corollary 22].

(3) The ring R satisfies the ascending chain condition on right annihilators if and only if so does $R[x;\alpha,\delta]$.

(4) If R is an α-rigid ring, then R is a Baer ring if and only if $R[x,x^{-1};\alpha]$ is a Baer ring, or equivalently, if and only if $R[[x,x^{-1};\alpha]]$ is a Baer ring.
If R is an α-rigid ring, then R is a p.p.-ring if and only if $R[x,x^{-1};\alpha]$ is a p.p.-ring.

2. (α,δ)-Compatible Rings

The (α,δ)-compatiblility condition on R is an interesting condition from which we will derive a number of useful properties. Furthermore, it will be a basic tool in the proof of the main results in section 3. In this section we will present some results on annihilators in Öre extension rings that can be deduced from the (α,δ)-compatiblility condition on R.

We now provide examples of α-compatible left p.p.-rings which are not α-rigid. Observe that any non-reduced regular ring is semiprime and p.p.

Example 2.1. Let R_1 be a non-reduced left p.p.-ring, D a domain and $R = R_1 \oplus D[y]$. Let $\alpha : D[y] \to D[y]$ be a monomorphism which is not surjective. Then we have:
(1) R is a left p.p.-ring.

Indeed, since R_1 and $D[y]$ are left p.p., $R_1 \oplus D[y]$ is left p.p.

(2) Let $\overline{\alpha} : R \to R$ be an endomorphism defined by $\overline{\alpha}(a \oplus f(y)) = a \oplus \alpha(f(y))$ for each $a \in R_1$ and $f(y) \in D[y]$. Then $\overline{\alpha}$ is a monomorphism of R which is not surjective and R is $\overline{\alpha}$-compatible which is not $\overline{\alpha}$-rigid:

Let $(a \oplus f(y))(b \oplus g(y)) = 0$. Then $ab = 0$ and $f(y)g(y) = 0$. Since $D[y]$ is a domain, $f(y) = 0$ or $\alpha(g(y)) = 0$, hence $(a \oplus f(y))\overline{\alpha}(b \oplus g(y)) = 0$. Now assume that $(a \oplus f(y))\overline{\alpha}(b \oplus g(y)) = 0$. Then $ab = 0$ and $f(y)\alpha(g(y)) = 0$. Since $D[y]$ is a domain and α is a monomorphism, $f(y) = 0$ or $g(y) = 0$. Hence $(a \oplus f(y))(b \oplus g(y)) = 0$, and consequently R is $\overline{\alpha}$-compatible. But R is not reduced, and hence R is not $\overline{\alpha}$-rigid.

Note that by [16, Proposition 6], α-rigid rings satisfy the condition *(SA1)*. The following is an example of a non-α-rigid ring which is (α, δ)-compatible and satisfies the condition *(SA1)*.

Example 2.2. Let δ be an α-derivation of R where R is an α-rigid ring. Let

$$R_3 = \left\{ \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{pmatrix} \mid a, b, c, d \in R \right\}$$

be a subring of the upper triangular matrix ring $T_3(R)$. The endomorphism α of R is extended to the endomorphism $\overline{\alpha} : R_3 \to R_3$ defined by $\overline{\alpha}((a_{ij})) = (\alpha(a_{ij}))$ and the α-derivation δ of R is also extended to $\overline{\delta} : R_3 \to R_3$ defined by $\overline{\delta}((a_{ij})) = (\delta(a_{ij}))$. It follows that $\overline{\delta}$ is an $\overline{\alpha}$-derivation of R_3. We show that: (i) R_3 is an $(\overline{\alpha}, \overline{\delta})$-compatible ring, (ii) R_3 is not $\overline{\alpha}$-rigid, (iii) R_3 satisfies the condition *(SA1)*.

(i) Assume that $\begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & a_1 & d_1 \\ 0 & 0 & a_1 \end{pmatrix} \overline{\alpha}\left(\begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & a_2 & d_2 \\ 0 & 0 & a_2 \end{pmatrix}\right) = 0$. Then we have the following equations:

$$a_1\alpha(a_2) = 0 \qquad (1)$$

$$a_1\alpha(b_2) + b_1\alpha(a_2) = 0 \qquad (2)$$

$$a_1\alpha(c_2) + b_1\alpha(d_2) + c_1\alpha(a_2) = 0 \qquad (3)$$

$a_1\alpha(d_2)+d_1\alpha(a_2)=0.$ (4)

Since R is reduced, from (1) we have $\alpha(a_2)a_1=0$. Multiplying the equation (2) by $\alpha(a_2)$ from the left-hand side, since R is reduced, it follows that $b_1\alpha(a_2)=\alpha(a_2)b_1=0$. Hence $a_1\alpha(b_2)=\alpha(b_2)a_1=0$. Similarly, multiplying equation (3) by $\alpha(a_2)$ from the left-hand side, we get $c_1\alpha(a_2)=\alpha(a_2)c_1=0$. Hence (3) turns to

$a_1\alpha(c_2)+b_1\alpha(d_2)=0$ (5)

Multiplying equation (4) by $\alpha(a_2)$ from the left-hand side, we get $d_1\alpha(a_2)=\alpha(a_2)d_1=0$ and $a_1\alpha(d_2)=\alpha(d_2)a_1=0$. Multiplying equation (5) by a_1 from the right-hand side, we get $a_1\alpha(c_2)=\alpha(c_2)a_1=0$ and $b_1\alpha(d_2)=\alpha(d_2)b_1=0$. Now since R is α-rigid, $a_1a_2=a_1b_2=a_1c_2=a_1d_2=b_1a_2=b_1d_2=c_1a_2=d_1a_2=0$.

Hence $\begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & a_1 & d_1 \\ 0 & 0 & a_1 \end{pmatrix}\begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & a_2 & d_2 \\ 0 & 0 & a_2 \end{pmatrix}=0$. Now assume that

$\begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & a_1 & d_1 \\ 0 & 0 & a_1 \end{pmatrix}\begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & a_2 & d_2 \\ 0 & 0 & a_2 \end{pmatrix}=0$. Then by a similar argument, we have

$a_1\alpha(a_2)=a_1\alpha(b_2)=b_1\alpha(a_2)=c_1\alpha(a_2)=d_1\alpha(a_2)=a_1\alpha(d_2)=a_1\alpha(c_2)=b_1\alpha(d_2)=0.$

Hence $\begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & a_1 & d_1 \\ 0 & 0 & a_1 \end{pmatrix}\overline{\alpha}\left(\begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & a_2 & d_2 \\ 0 & 0 & a_2 \end{pmatrix}\right)=0$. Therefore R_3 is $\overline{\alpha}$-compatible.

Assume that $\begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & a_1 & d_1 \\ 0 & 0 & a_1 \end{pmatrix}\begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & a_2 & d_2 \\ 0 & 0 & a_2 \end{pmatrix}=0$. Then $a_1a_2=a_1b_2=a_1c_2=$

$a_1d_2 = b_1a_2 = b_1d_2 = c_1a_2 = d_1a_2 = 0$. Since R is α-rigid, $a_1\delta(a_2) = a_1\delta(b_2) = a_1\delta(c_2) = a_1\delta(d_2) = b_1\delta(a_2) = b_1\delta(d_2) = c_1\delta(a_2) = d_1\delta(a_2) = 0$. Thus

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & a_1 & d_1 \\ 0 & 0 & a_1 \end{pmatrix} \overline{\delta}\left(\begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & a_2 & d_2 \\ 0 & 0 & a_2 \end{pmatrix}\right) = 0.$$

Therefore R_3 is $\overline{\delta}$-compatible.

(ii) Since R_3 is not reduced, so R_3 is not $\overline{\alpha}$-rigid.

(iii) Assume that

$p = \sum_{i=0}^{m} \begin{pmatrix} a_i & b_i & c_i \\ 0 & a_i & d_i \\ 0 & 0 & a_i \end{pmatrix} x^i$ and $q = \sum_{j=0}^{n} \begin{pmatrix} a_j^{'} & b_j^{'} & c_j^{'} \\ 0 & a_j^{'} & d_j^{'} \\ 0 & 0 & a_j^{'} \end{pmatrix} x^j \in$ $R_3[x;\overline{\alpha},\overline{\delta}]$ such that $pq = 0$. By a result of Hong et al. [16, Proposition 6], $R[x;\alpha,\delta]$ is reduced. So that by another result of Hong et al. [17, Proposition 17],

$\begin{pmatrix} a_i & b_i & c_i \\ 0 & a_i & d_i \\ 0 & 0 & a_i \end{pmatrix} \overline{\alpha}^i\left(\begin{pmatrix} a_j^{'} & b_j^{'} & c_j^{'} \\ 0 & a_j^{'} & d_j^{'} \\ 0 & 0 & a_j^{'} \end{pmatrix}\right) = 0$, for all i, j. Since R_3 is $\overline{\alpha}$-compatible,

$\begin{pmatrix} a_i & b_i & c_i \\ 0 & a_i & d_i \\ 0 & 0 & a_i \end{pmatrix} \begin{pmatrix} a_j^{'} & b_j^{'} & c_j^{'} \\ 0 & a_j^{'} & d_j^{'} \\ 0 & 0 & a_j^{'} \end{pmatrix} = 0$ for all i, j. Therefore, R_3 satisfies the condition *(SA1)*.

By the following example there exists an automorphism α of a ring R such that: (1) R is α-compatible, (2) R is not α-rigid, and (3) R satisfies the conditions *(SA1)* and *(SA2)*:

Example 2.3. Let D be an integral domain and consider the trivial extension of D given by:

$$R = \left\{ \begin{pmatrix} a & d \\ 0 & a \end{pmatrix} \mid a, d \in D \right\}.$$

We see that R is a commutative ring. Some properties of rings of this type have been studied in [17]. Let $\alpha : R \to R$ be an automorphism defined by

$\alpha\left(\begin{pmatrix} a & d \\ 0 & a \end{pmatrix}\right) = \begin{pmatrix} a & ud \\ 0 & a \end{pmatrix}$, where u is a fixed unit element of D.

(1) R is α-compatible:
Assume that $\begin{pmatrix} a & d \\ 0 & a \end{pmatrix}\begin{pmatrix} b & d_1 \\ 0 & b \end{pmatrix} = 0$, so that $ab = 0 = ad_1 + db$. Thus $a = 0$ or $b = 0$. In either case, $aud_1 + db = 0$, hence $\begin{pmatrix} a & d \\ 0 & a \end{pmatrix}\alpha\left(\begin{pmatrix} b & d_1 \\ 0 & b \end{pmatrix}\right) = 0$. If $\begin{pmatrix} a & d \\ 0 & a \end{pmatrix}\alpha\left(\begin{pmatrix} b & d_1 \\ 0 & b \end{pmatrix}\right) = 0$, then by a similar argument $\begin{pmatrix} a & d \\ 0 & a \end{pmatrix}\begin{pmatrix} b & d_1 \\ 0 & b \end{pmatrix} = 0$. Therefore, R is α-compatible.

(2) R is not α-rigid:

$\begin{pmatrix} 0 & d \\ 0 & 0 \end{pmatrix}\alpha\left(\begin{pmatrix} 0 & d \\ 0 & 0 \end{pmatrix}\right) = 0$, but $\begin{pmatrix} 0 & d \\ 0 & 0 \end{pmatrix} \neq 0$ for $d \neq 0$.

(3) R satisfies the conditions *(SA1)* and *(SA2)*:
Let $f(x) = \sum_{i=0}^{\infty} A_i x^i$ and $g(x) = \sum_{j=0}^{\infty} B_j x^j \in R[[x;\alpha]]$, where $A_i = \begin{pmatrix} a_i & c_i \\ 0 & a_i \end{pmatrix}$ and $B_j = \begin{pmatrix} b_j & d_j \\ 0 & b_j \end{pmatrix}$ for $0 \leq i$, $0 \leq j$. Assume that $f(x)g(x) = 0$. Then we have:

$$\sum_{k=0}^{\infty} \sum_{i+j=k} A_i \alpha^i(B_j) x^{i+j} = 0. \tag{1}$$

We claim that $A_i \alpha^i(B_j) = 0$, and hence by α compatibility of R we see that $A_i B_j = 0$ for all i, j.

(i) Suppose that there is $A_k = \begin{pmatrix} a_k & c_k \\ 0 & a_k \end{pmatrix}$ with $a_k \neq 0$ and $A_0 = \cdots = A_{k-1} = 0$ where $0 \leq k$. From the equation (1), $A_0B_k + A_1\alpha(B_{k-1}) + \cdots + A_{k-1}\alpha^{k-1}(B_1) + A_k\alpha^k(B_0) = 0$, so, $A_k\alpha^k(B_0) = 0$. That is,

$$\begin{pmatrix} a_k & c_k \\ 0 & a_k \end{pmatrix} \begin{pmatrix} b_0 & u^k d_0 \\ 0 & b_0 \end{pmatrix} = \begin{pmatrix} a_k b_0 & a_k u^k d_0 + c_k b_0 \\ 0 & a_k b_0 \end{pmatrix} = 0.$$

Thus $a_k b_0 = 0$, and hence, $b_0 = 0$ and $a_k u^k d_0 = 0$. Therefore, $d_0 = 0$, And thus $B_0 = 0$. Since $A_0B_{k+1} + A_1\alpha(B_k) + \cdots + A_k\alpha^k(B_1) + A_{k+1}\alpha^{k+1}(B_0) = 0$, we have $A_k\alpha^k(B_1) = 0$ and so $B_1 = 0$ by the same method as above. Continuing this process, it follows that $B_j = 0$ for all j.

(ii) Assume that there is $B_k = \begin{pmatrix} b_k & d_k \\ 0 & b_k \end{pmatrix}$ with $b_k \neq 0$ and $B_0 = \cdots = B_{k-1} = 0$, where $0 \leq k$. From the equation (1), $A_0B_k + A_1\alpha(B_{k-1}) + \cdots + A_k\alpha^k(B_0) = 0$, so $A_0B_k = 0$ and hence $A_0 = 0$. By the same method as in (i), $A_0B_{k+1} + A_1\alpha(B_k) + \cdots + A_k\alpha^k(B_1) + A_{k+1}\alpha^{k+1}(B_0) = 0$ which implies $A_1\alpha(B_k) = 0$ and hence $A_1 = 0$. Continuing this process, we have $A_i = 0$ for all i.

(iii) Assume that $A_i = \begin{pmatrix} 0 & c_i \\ 0 & 0 \end{pmatrix}$, $B_j = \begin{pmatrix} 0 & d_j \\ 0 & 0 \end{pmatrix}$ for all i, j. Then

$$A_i\alpha^i(B_j) = \begin{pmatrix} 0 & c_i \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & u^i d_j \\ 0 & 0 \end{pmatrix} = 0.$$

Therefore R satisfies the condition *(SA2)* by (i), (ii) and (iii). Similarly one can show that R satisfies the condition *(SA1)*.

The following results exhibit some properties of (α, δ)-compatible Rings.

Lemma 2.4. Let R be an (α, δ)-compatible ring. Then we have:
(i) If $ab = 0$, then $a\alpha^n(b) = \alpha^n(a)b = 0$ for every positive integer n.
(ii) If $\alpha^k(a)b = 0$ for some positive integer k, then $ab = 0$.

(iii) If $ab = 0$, then $\alpha^n(a)\delta^m(b) = 0 = \delta^m(a)\alpha^n(b)$ for every positive integers m, n.

Proof. (i) If $ab = 0$, then $\alpha^n(a)\alpha^n(b) = 0$ and hence by α-compatibility of R $\alpha^n(a)b = 0$ for every positive integer n.
(ii) If $\alpha^k(a)b = 0$ for some positive integer k, then by α-compatibility of R, $\alpha^k(a)\alpha^k(b) = 0$ and hence by injectivity of α, $ab = 0$.
(iii) It is enough to show that $\delta(a)\alpha(b) = 0$. If $ab = 0$, then by (i) and δ-compatibility of R, $\alpha(a)\delta(b) = 0$. Hence $\delta(a)b = \delta(ab) - \alpha(a)\delta(b) = 0$, thus $\delta(a)\alpha(b) = 0$. □

In the trivial case where α is the identity and δ is the zero map, it is clear that every ring R is (α, δ)-compatible. However, Kerr[21] constructed an example of a commutative Goldie ring R whose polynomial ring $R[x]$ has an infinite ascending chain of annihilator ideals. We now impose the skew Armendariz condition (SA1) on R which leads to the following result:

Theorem 2.5. Let R be an (α, δ)-compatible ring which satisfies the condition *(SA1)*. Then R satisfies the ascending chain condition on right annihilators if and only if so does $R[x; \alpha, \delta]$.

Proof. Let $I_1 \subseteq I_2 \subseteq I_3 \cdots$ be a chain of right annihilators of $R[x; \alpha, \delta]$. Then there exists $\emptyset \neq D_i \subseteq R[x; \alpha, \delta]$, such that $r_{R[x;\alpha,\delta]}(D_i) = I_i$ for $i \geq 1$ and $\cdots \subseteq D_2 \subseteq D_1$. Assume that C_i is the set of coefficients of elements of D_i for $i \geq 1$. Since R satisfies the ascending chain condition on right annihilators, there exists $n \in \mathbb{N}$, such that $r_R(C_n) = r_R(C_i)$ for $i \geq n$. We show that $r_{R[x;\alpha,\delta]}(D_n) = r_{R[x;\alpha,\delta]}(D_i)$, for $i \geq n$. Let $i \geq n$ and $g(x) = r_0 + r_1x + \cdots + r_nx^n \in r_{R[x;\alpha,\delta]}(D_i)$. Then, since R satisfies the condition *(SA1)* and it is (α, δ)-compatible, $C_ir_j = 0$ for $0 \leq j \leq m$. Hence $r_j \in r_R(C_n) = r_R(C_i)$ for $0 \leq j \leq m$. Thus $g(x) \in r_{R[x;\alpha,\delta]}(D_n)$. Therefore, $r_{R[x;\alpha,\delta]}(D_n) = r_{R[x;\alpha,\delta]}(D_i)$. Conversely, assume that

$J_1 \subseteq J_2 \subseteq J_3 \cdots$ is a chain of right annihilator of R. Then there exists $\emptyset \neq B_i \subseteq R$, such that $r_R(B_i) = J_i$ for $i \geq 1$ and $\cdots \subseteq B_2 \subseteq B_1$. Then $r_{R[x;\alpha,\delta]}(B_i) = r_{R[x;\alpha,\delta]}(B_n)$, for some n and all $i \geq n$. Hence $r_R(B_i) = R \cap r_R(B_i) = R \cap r_R(B_n) = r_R(B_n)$, for $i \geq n$. Therefore, $J_i = J_n$ for $i \geq n$. $\Box$

3. Polynomial Extensions of Baer and Right p.p.-Rings

In this section we study the relationship between the Baer and p.p properties of a ring R and its Ore extension rings. As a consequence we obtain a generalization of [7, Corollary 1.10].

Recall that for a ring R with an injective ring endomorphism $\alpha : R \to R$, $R[x;\alpha]$ is the Öre extension of R. The set $\{x^j\}_{j\geq 0}$ is easily seen to be a left Öre subset of $R[x;\alpha]$, so that one can localize $R[x;\alpha]$ and form the skew Laurent polynomial ring $R[x, x^{-1};\alpha]$. Elements of $R[x, x^{-1};\alpha]$ are finite sums of elements of the form $x^{-j}rx^i$ where $r \in R$ and i, j are non-negative integers. Multiplication is subject to $xr = \alpha(r)x$ and $rx^{-1} = x^{-1}\alpha(r)$ for all $r \in R$. The skew Laurent series ring $R[[x, x^{-1};\alpha]]$ is defined similarly.

Now we consider D.A. Jordan's construction of the ring $A(R,\alpha)$ (See [19], for more details). Let $A(R,\alpha)$ or A be the subset $\{x^{-i}rx^i \mid r \in R\,,\ i \geq 0\}$ of the skew Laurent polynomial ring $R[x, x^{-1};\alpha]$. For each $j \geq 0$, $x^{-i}rx^i = x^{-(i+j)}\alpha^j(r)x^{(i+j)}$. It follows that the set of all such elements forms a subring of $R[x, x^{-1};\alpha]$ with $x^{-i}rx^i + x^{-j}sx^j = x^{-(i+j)}(\alpha^j(r) + \alpha^i(s))x^{(i+j)}$ and
$(x^{-i}rx^i)(x^{-j}sx^j) = x^{-(i+j)}\alpha^j(r)\alpha^i(s)x^{(i+j)}$ for $r, s \in R$ and $i, j \geq 0$. Note that α is actually an automorphism of $A(R,\alpha)$. We have $R[x, x^{-1};\alpha] \simeq A[x, x^{-1};\alpha]$ via an isomorphism which maps $x^{-i}rx^j$ to $\alpha^{-i}(r)x^{j-i}$.

Note that, since A is a subring of the ring $R[[x, x^{-1};\alpha]]$, each element $f(x) \in R[[x, x^{-1};\alpha]]$ can be written in the form
$f(x) = (x^{-t_p}r_px^{t_p})x^p + \cdots + (x^{-t_{n-1}}r_{n-1}x^{t_{n-1}})x^{n-1} + \sum_{i=m}^{\infty}(x^{-t_n}r_ix^{t_n})x^i$, for some non-negative integer t_n.

Theorem 3.1. Let α be an automorphism of a ring R and let R be α-compatible. If R satisfies the condition *(SA1)*, then R is a Baer ring if and only if $R[x, x^{-1}; \alpha]$ is a Baer ring.

Proof. Let U be a nonempty subset of $R[x, x^{-1}; \alpha]$ and let U_0 be the set of coefficient of elements of U. Since R is a Baer ring, there is an idempotent $e \in R$ such that $r_R(U_0) = eR$. Since R is (α, δ)-compatible, $Ue = 0$. Hence $eR[x, x^{-1}; \alpha] \subseteq r_{R[x,x^{-1};\alpha]}(U)$. Now assume that
$f(x) = a_k x^k + a_{k+1}x^{k+1} + \cdots + a_n x^n \in r_{R[x,x^{-1};\alpha]}(U)$. Then, since R satisfies the condition *(SA1)*, $U_0 a_i = 0$ for $i = k, \cdots, n$. Thus, $a_i = ea_i$ for $i = k, \cdots, n$. Therefore,
$r_{R[x,x^{-1};\alpha]}(U) = eR[x, x^{-1}; \alpha]$. Conversely, assume that $R[x, x^{-1}; \alpha]$ is a Baer ring and L is a non-empty subset of R. There exists an idempotent $e(x) = e_k x^k + e_{k+1}x^{k+1} + \cdots + e_0 + e_1 x + \cdots + e_n x^n \in R[x, x^{-1}; \alpha]$, such that $r_{R[x,x^{-1};\alpha]}(L) = e(x)R[x, x^{-1}; \alpha]$. Hence $Le_0 = 0$. Thus $e_0 = e(x)e_0$ and hence $e_0^2 = e_0$. Then, since R satisfies condition *(SA1)*, $e_0 R \subseteq r_R(L)$. Next let $t \in r_R(L)$. Then $t = e(x)t$. Hence, since R satisfies condition *(SA1)*, $e_0 t = t$. Therefore, $r_R(L) = e_0 R$. □

Theorem 3.2. Let α be an automorphism of a ring R and let R be α-compatible. If R satisfies the condition *(SA1)*, then R is a right p.p.-ring if and only if $R[x, x^{-1}; \alpha]$ is a right p.p.-ring.

Proof. Assume that $R[x, x^{-1}; \alpha]$ is a right p.p.-ring and $a \in R$. There exists an idempotent $e(x) = e_k x^k + e_{k+1}x^{k+1} + \cdots + e_0 + e_1 x + \cdots + e_n x^n \in R[x, x^{-1}; \alpha]$ such that $r_{R[x,x^{-1};\alpha]}(a) = e(x)R[x, x^{-1}; \alpha]$. We show that $r_R(a) = e_0 R$. Since $ae(x) = 0$, $ae_0 = 0$, it follows that $e_0 R \subseteq r_R(a)$. Let $t \in r_R(a)$. Then $t = e(x)t$. Since R satisfies the condition *(SA1)*, $t = e_0 t$. Therefore, $r_R(a) = e_0 R$. Conversely, suppose R is a right p.p.-ring and $f(x) = \sum_{i=k}^{m} r_i x^i \in R[x, x^{-1}; \alpha]$. Note that R is an abelian ring. Let $e^2 = e$ and $r \in R$. Then $(er(1-e))^2 = 0$. Hence by α-compatibility of R, $e\alpha(e) = \alpha(e)e = e$

and $(er(1-e))\alpha(er(1-e)) = 0$. Let $f(x) = e(1-er(1-e)x)$ and $g(x) = (1+er(1-e)x)(1-e)$. Then
$f(x)g(x) = e(1-e) - er(1-e)\alpha(er(1-e))\alpha^2(1-e)x^2 = 0$. Since R satisfies the condition (SA1), $er(1-e) = e(er(1-e))(1-e) = 0$. Thus $er = ere$. Similarly $re = ere$ and hence $er = re$. Hence R is an abelian ring. Thus there is $e^2 = e \in R$ such that $r_R(r_k) \cap \cdots \cap r_R(r_m) = eR$. We show that $r_{R[x,x^{-1};\alpha]}(f(x)) = eR[x,x^{-1};\alpha]$. Since R is α-compatible and $r_ie = 0$ for $k \leq i \leq m$, $f(x)e = 0$. Hence $eR[x,x^{-1};\alpha] \subseteq r_{R[x,x^{-1};\alpha]}(f(x))$. Let $g(x) = \sum_{j=s}^{n} b_jx^j \in r_{R[x,x^{-1};\alpha]}(f(x))$. Then $f(x)g(x) = 0$. Multiply this equation by x^{-s} and x^{-k} from the right-hand and the left-hand side, respectively. Since R is α-compatible and satisfies the condition *(SA1)*, we have $r_ib_j = 0$ for all i, j. Thus $b_j = eb_j$ for all j. Hence $g(x) = eg(x)$. Therefore, $r_{R[x,x^{-1};\alpha]}(f(x)) = eR[x,x^{-1};\alpha]$. □

Lemma 3.3. Let α be an endomorphism of a ring R and let δ be an α-derivation of R. If R is α-rigid, then R is (α,δ)-compatible.

Proof. Let R be α-rigid. Since α-rigid rings are reduced, $ab = 0$ if and only if $ba = 0$. Hence $a\alpha(b)\alpha(a\alpha(b)) = a\alpha(ba)\alpha^2(b) = 0$. Since R is α-rigid, we have $a\alpha(b) = 0$. Similarly, $ba = 0$ implies that $\alpha(a)b = 0$. Thus $0 = \delta(ba) = \delta(b)a + \alpha(b)\delta(a)$, so $(\alpha(b)\delta(a))^2 = -\delta(b)a\alpha(b)\delta(a) = 0$. Since R is reduced, $\alpha(b)\delta(a) = 0$, so $\delta(b)a = 0$ and hence $a\delta(b) = 0$. Now suppose that $a\alpha(b) = 0$, then $ba\alpha(ba) = 0$. Since R is α-rigid, $ab = ba = 0$. Therefore, R is (α,δ)-compatible. □

Note that there are numerous examples (see Examples 2.1, 2.2 and 2.3), which show that the converse of Lemma 3.3 does not hold.

Lemma 3.4. Let R be a ring. Then R is α-compatible (resp. α-rigid) if and only if $A(R,\alpha)$ is α-compatible (resp. α-rigid).

Proof. It is clear that any subring of an α-compatible ring is also α-compatible. Suppose R is α-compatible and $(x^{-i}rx^i)(x^{-j}ax^j) = 0$, where $j, i \geq 0$ and $r, a \in R$. Hence $\alpha^j(r)\alpha^i(a) = 0$ and thus $\alpha^j(r)\alpha^{i+1}(a) = 0$. So $(x^{-i}rx^i)(x^{-j}\alpha(a)x^j) = (x^{-i}rx^i)\alpha(x^{-j}ax^j) = 0$. Therefore, A is α-compatible. □

Lemma 3.5. Let R be an α-compatible ring. Then R is a Baer (resp. right p.p. -) ring if and only if $A(R, \alpha)$ is a Baer (resp. right p.p.-) ring.

Proof. Assume that R is a right p.p.-ring. Let $a = x^{-i}tx^i \in A$ and $x^{-j}bx^j \in r_A(a)$. By Lemma 2.4, $b \in r_R(t)$. Since R is right p.p., $r_R(t) = eR$ for an idempotent $e \in R$. Thus $eb = b$, so by α-compatibility of R, $\alpha^n(e)b = b$ for every positive integer n. Hence $e(x^{-j}bx^j) = x^{-j}bx^j$, thus $r_A(a) \subseteq eA$. Since R is α-compatible, $eA \subseteq r_A(a)$. Hence $r_A(a) = eA$, thus A is a right p.p.-ring. Conversely, suppose A is right p.p, $r \in R$ and $b \in r_R(r)$. Since R is α-compatible and A is right p.p., $b \in r_A(r) = (x^{-j}e_0x^j)A$, where e_0 is an idempotent of R and $j \geq 0$. Since R is α-compatible, $e_0R \subseteq r_R(r)$. Let $b \in r_R(r)$. By Lemma 2.4, $b \in r_A(r)$, hence $b = (x^{-j}e_0x^j)b$. Thus $b = e_0b$ and hence $r_R(r) \subseteq e_0R$. Therefore, R is right p.p.

The other case is similar. □

Since $R[x, x^{-1}; \alpha] \simeq A[x, x^{-1}; \alpha]$, where α is an automorphism of A, using Theorems 3.1, 3.2 and Corollaries 3.4, 3.5 we can extend the above results further to the case where α is not assumed to be surjective:

Theorem 3.6. Let R be an α-compatible ring. If R satisfies the condition *(SA1)*, then R is a Baer (resp. right p.p.-) ring if and only if $R[x, x^{-1}; \alpha]$ is a Baer (resp. right p.p.-) ring.

Proof. First we show that A satisfies the condition *(SA1)*. Let $f(x) = (x^{-t_0}r_0x^{t_0}) + (x^{-t_1}r_1x^{t_1})x + \cdots + (x^{-t_n}r_nx^{t_n})x^n$, $g(x) = (x^{-s_0}k_0x^{s_0}) + (x^{-s_1}k_1x^{s_1})x + \cdots + (x^{-s_m}k_mx^{s_m})x^m \in A[x;\alpha]$ and $f(x)g(x) = 0$, where $t_i, s_j \geq 0$, $r_i, k_j \in R$. Let $w = t_0 + t_1 + \cdots + t_n + s_0 + \cdots + t_m$. Then $x^w f(x)g(x) = 0$. Since R is α-compatible and satisfies the condition *(SA1)*, $r_ik_j = 0$ for all i, j. Hence $(x^{-t_i}r_ix^{t_i})(x^{-s_j}k_jx^{s_j}) = 0$ for all i, j. Therefore, A satisfies the condition *(SA1)*. Since $R[x, x^{-1};\alpha] \simeq A[x, x^{-1};\alpha]$, where α is an automorphism of A, the proof follows from Lemmas 3.4, 3.5 and Theorems 3.1, 3.2. □

Since a reduced ring is Armendariz, Corollary 3.7 is a generalization of Birkenmeier et al. [7, Corollary 1.10] to the following rather general setting :

Corollary 3.7 [7, Corollary 1.10]. Let R be a reduced ring. Then R is a Baer (resp. right p.p.-) ring if and only if $R[x, x^{-1}]$ is a Baer (resp. right p.p.-) ring.

The next results concern skew Laurent and skew Laurent power series ring extensions of α-rigid rings. Since an α-rigid ring is α-compatible and satisfies the condition *(SA1)*, we have the following:

Corollary 3.8. Let R be an α-rigid ring. Then R is a Baer (resp. right p.p.-) ring if and only if $R[x, x^{-1};\alpha]$ is a Baer (resp. right p.p.-) ring.

Theorem 3.9. Let R be an α-compatible ring. If R satisfies the condition *(SA2)*, then R is a Baer ring if and only if $T = R[[x, x^{-1};\alpha]]$ is a Baer ring.

Proof. By Lemma 3.5, R is a Baer ring if and only if $A = A(R, \alpha)$ is a Baer ring. Assume that A is a Baer ring and let L be a non-empty

subset of T. We denote the set of all coefficients of elements of L by L_0. Thus $r_A(L_0) = (x^{-k}ex^k)A$, for some idempotent $e \in R$ and a non-negative integer k. Now we show that $r_T(L) = (x^{-k}ex^k)T$. By Lemma 2.4, $r_T(L) \subseteq (x^{-k}ex^k)T$. Let
$f(x) = (x^{-t_p}r_px^{t_p})x^p + \cdots + (x^{-t_{n-1}}r_{n-1}x^{t_{n-1}})x^{n-1} + \sum_{i=m}^{\infty}(x^{-t_n}r_ix^{t_n})x^i$ $\in L$ and let $g(x) = (x^{-s_l}r_lx^{t_l})x^l + \cdots + (x^{-s_{m-1}}a_{m-1}x^{s_{m-1}})x^{m-1} + \sum_{j=m}^{\infty}(x^{-s_m}a_jx^{s_m})x^j$ be an element of $r_T(L)$. Let $\omega = t_i + \cdots t_n + s_l + \cdots + s_m$. Then we have
$x^{\omega}f(x)g(x) = 0$. Since R satisfies the condition *(SA2)*, by Lemma 2.4 we have $x^{-t_i}r_ix^{t_i}x^{-s_j}a_jx^{s_j} = 0$ for each $i \geq l$, $j \geq p$. Thus for each $i \geq l$, $(x^{-s_j}a_jx^{s_j}) \in (x^{-k}ex^k)A$. Therefore, $r_T(L) = (x^{-k}ex^k)T$.
Conversely, assume that T is a Baer ring and Y is a non-empty subset of the ring A. Then $r_T(Y) = e(x)T$, for some idempotent element $e(x) \in T$. Let $e_0 \in A$ be the constant term of $e(x)$. Since $e(x)(e(x) - 1) = 0$ and $e_0 \in r_A(Y)$, then it is easy to show that $r_A(Y) = e_0A$. □

Corollary 3.10. Let R be an α-rigid ring. Then R is a Baer ring if and only if $R[[x, x^{-1}; \alpha]]$ is a Baer ring.

Proof. Since an α-rigid ring is α-compatible and satisfies the condition *(SA2)*, hence the result follows from Theorem 3.9. □

Theorem 3.11. Let R be an α-compatible ring and satisfy the condition *(SA2)*. Then R is Baer if and only if $R[[x; \alpha]]$ is Baer.

Proof. The proof follows by a similar method used in the proof of Theorem 3.6. □

Corollary 3.12(Hong et al. [16, Theorem 21]). Let R be an α-rigid ring. Then R is a Baer ring if and only if $R[[x; \alpha]]$ is a Baer ring.

Proof. Since an α-rigid ring is α-compatible and satisfies the condition *(SA2)*, the proof follows from Theorem 3.11. □

There is a right p.p.-ring R which satisfies the condition *(SA2)* such that $R[[x;\alpha]]$ is not a right p.p.-ring:

Example 3.13(Birkenmeier et al. [5, Example 3.6]). For a given field F, let

$$R = \{(a_n)_{n=1}^{\infty} \in \Pi_{n=1}^{\infty} F_n \mid a_n \text{ is eventually constant } \}.$$

This is a subring of $\Pi_{n=1}^{\infty} F_n$, where $F_n = F$ for $n = 1, 2, \ldots$. Then the ring R is a commutative von Neumann regular ring, and hence it is a reduced p.p. ring. Let α be the identity map on R. Then R is α-compatible and satisfies the condition *(SA2)*. But $R[[x;\alpha]]$ is not a right p.p. ring.

Theorem 3.14. Let R be an (α,δ)-compatible ring and satisfy the condition *(SA1)*. Then R is a Baer (resp. right p.p.-) ring if and only if $R[x;\alpha,\delta]$ is a Baer (resp. right p.p.-) ring.

Proof. Let X be a nonempty subset of $R[x;\alpha,\delta]$ and let X' be the set of coefficient of elements of X. Since R is a Baer ring, there is an idempotent $e \in R$ such that $r_R(X') = eR$. Since R is (α,δ)-compatible, $Xe = 0$. Hence $eR[x;\alpha,\delta] \subseteq r_{R[x;\alpha,\delta]}(X)$. Now suppose $f(x) = a_0 + a_1x + \cdots + a_nx^n \in r_{R[x;\alpha,\delta]}(X)$. Then, since R satisfies the condition *(SA1)*, $X'a_i = 0$ for $i = 0, \cdots, n$. Thus $a_i = ea_i$ for $i = 0, \cdots, n$. Therefore, $r_{R[x;\alpha,\delta]}(X) = eR[x;\alpha,\delta]$. Conversely, suppose $R[x;\alpha,\delta]$ is a Baer ring and A is a non-empty subset of R. There is an idempotent $e(x)^2 = e(x) = e_0 + e_1x + \cdots + e_nx^n \in R[x;\alpha,\delta]$, such that $r_{R[x;\alpha,\delta]}(A) = e(x)R[x;\alpha,\delta]$. Hence $Ae_0 = 0$. Thus $e_0 = e(x)e_0$ and since R satisfies condition *(SA1)*, $e_0^2 = e_0$. Then $e_0R \subseteq r_R(A)$. Let $t \in r_R(A)$. Then $t = e(x)t$. Hence $e_0t = t$.

Thus $r_R(A) = e_0R$. The rest of the proof is similar to the proof of Theorem 3.2. □

Since an α-rigid ring is (α, δ)-compatible and satisfies the condition *(SA1)*, so we have the following:

Corollary 3.15 (Hong et al. [16, Theorems 11 and 14]) Let R be an α-rigid ring. Then R is a Baer (resp. right p.p.-) ring if and only if $R[x; \alpha, \delta]$ is a Baer (resp. right p.p.-) ring.

Note that for $\alpha = Id$, an Armendariz ring satisfies the condition (SA1), hence we obtain [22, Theorems 9 and 10] as an immediate corollary of Theorem 3.4:

Corollary 3.16. Let R be an Armendariz ring. Then R is a Baer (resp. right p.p.-) ring if and only if $R[x]$ is a Baer (resp. right p.p.-) ring, and equivalently, if and only if $R[x, x^{-1}]$ is a Baer (resp. right p.p.-) ring.

By the following example we show that there exists a non-δ-compatible ring R which is not Baer, but $R[x; \delta]$ is a Baer ring:

Example 3.17. Let $R = Z_2[y]/(y^2)$, where (y^2) is a principal ideal generated by y^2 of the polynomial ring $Z_2[y]$. Note that the only idempotents of R are $0 + (y^2)$ and $1 + (y^2)$. Since $r_R(y + (y^2)) = (y + (y^2))R$ cannot be generated by an idempotent, R is not quasi-Baer and so it is not Baer. Now, let α be the identity map on R and define an α-derivation δ on R by $\delta(y + (y^2)) = 1 + (y^2)$. Then R is not δ-compatible. However, by [4, Example 11]

$$R[x; \alpha, \delta] = R[x; \delta] \cong Mat_2(Z_2[y^2]) \cong Mat_2(Z_2[t]).$$

Note that $Mat_2(Z_2[t])$ is a Baer ring, hence $R[x; \alpha, \delta] = R[x; \delta]$ is a Baer ring.

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