Bulletin of the Iranian Mathematical Society Vol. 38 *No. 4* (*2012*), *pp 925-934.*

A COMMON FIXED POINT THEOREM ON ORDERED METRIC SPACES

H. K. NASHINE AND I. ALTUN*

Communicated by Behzad Djafari-Rouhani

Abstract. A common fixed point result for weakly increasing mappings satisfying generalized contractive type of Zhang in ordered metric spaces are derived.

1. **Introduction**

Branciari [8] obtained a fixed point result for a single mapping satisfying an analogue of Banach contraction principle for an integral-type inequality. After that, some generalizations of this interesting result were given. For example, Zhang [22] proved the following theorem:

Theorem 1.1. *Let* $(\mathcal{X}, d)$ *be a complete metric space and let* $\mathcal{S},\mathcal{T} : \mathcal{X} \to \mathcal{X}$ *be two maps such that*

$$F(d(\mathcal{T}x, \mathcal{S}y)) \leq \psi(F(\Phi(x, y)))$$

for every $x, y \in \mathcal{X}$*, where* F*,* ψ *and* $\Phi(x, y)$ *are as in Section 2. Then* $\mathcal{S}$ *and* $\mathcal{T}$ *have a unique common fixed point.*

On the other hand, recently, there has been a trend to weaken the requirement on the contraction by considering metric spaces endowed with partial order. In the context of ordered metric spaces, the usual

MSC(2010): Primary: 54H25; Secondary: 47H10.
Keywords: Fixed point, complete metric space, partially ordered set.
Received: 16 February 2011, Accepted: 7 May 2011.
*Corresponding author

contraction is weakened but at the expense that the operator is monotone. It is of interest to determine if it is still possible to establish the existence of a unique fixed point assuming that the operator considered is monotone in such a setting [9]. The first result in this direction was given by Ran and Reurings [21] who presented its applications to matrix equation:

Theorem 1.2. *Let $(\mathcal{X}, \preceq)$ be a partially ordered set and suppose that there exists a metric d in $\mathcal{X}$ such that $(\mathcal{X}, d)$ is a complete metric space. Let $\mathcal{T} : \mathcal{X} \to \mathcal{X}$ be a continuous and nondecreasing mapping such that there exists $\lambda \in [0, 1)$ with*

$$d(\mathcal{T}x, \mathcal{T}y)) \leq \lambda d(x, y)$$

for all comparable $x, y \in \mathcal{X}$. If there exists $x_0 \in X$ with $x_0 \preceq \mathcal{T}x_0$, then $\mathcal{T}$ has a fixed point.

Subsequently, Nieto and Rodŕiguez-López [18] extended the result of Ran and Reurings for nondecreasing mappings and applied his result to obtain a unique solution for a first order ordinary differential equation with periodic boundary conditions. Furthermore, Harjani and Sadarangani [12, 13] proved the ordered version fixed point theorem for weakly contractive conditions, and, Amini-Harandi and Emami [5] proved the ordered version of Rich type fixed point theorem. One can find further results on fixed points in ordered metric spaces in [1, 2, 3, 6, 7, 9, 10, 11, 14, 15, 16, 17, 19] and [20].

In this paper we prove an ordered version of Theorem 1.1 using the concept of weakly increasingness for two maps. At the end of this paper we give an example which does not satisfy Theorems 1.1, 1.2, but satisfies the result of this paper.

2. **Main Result**

We begin this section by giving the concept of weakly increasing mappings (see [6]).

Definition 2.1. *Let $(\mathcal{X}, \preceq)$ be a partially ordered set. Two mappings $\mathcal{S}, \mathcal{T} : \mathcal{X} \to \mathcal{X}$ are said to be weakly increasing if $\mathcal{S}x \preceq \mathcal{T}\mathcal{S}x$ and $\mathcal{T}x \preceq \mathcal{S}\mathcal{T}x$ for all $x \in \mathcal{X}$.*

Note that, two weakly increasing mappings need not be nondecreasing. There exist some examples to illustrate this fact in [4].

In the sequel, we consider the set of functions by $F, \psi : [0, +\infty) \to [0, +\infty)$ such that

(i) F is nondecreasing, continuous, and $F(0) = 0 < F(t)$ for every $t > 0$;
(ii) ψ is nondecreasing, right continuous, and $\psi(t) < t$ for every $t > 0$.

Define $\mathcal{F} = \{F : F \text{ satisfies (i)}\}$ and $\Psi = \{\psi : \psi \text{ satisfies (ii)}\}$.
Our main result is as follows:

Theorem 2.2. *Let $(\mathcal{X}, \preceq)$ be a partially ordered set and suppose that there exists a metric d in $\mathcal{X}$ such that $(\mathcal{X}, d)$ is a complete metric space. Let $\mathcal{S}$,$\mathcal{T} : \mathcal{X} \to \mathcal{X}$ are two weakly increasing mappings such that*

$$F(d(\mathcal{T}x, \mathcal{S}y)) \leq \psi(F(\Phi(x, y))) \tag{2.1}$$

for all comparable $x, y \in \mathcal{X}$, where $F \in \mathcal{F}$, $\psi \in \Psi$ and

$$\Phi(x, y) = \max\{d(x, y), d(x, \mathcal{T}x), d(y, \mathcal{S}y), \frac{d(x, \mathcal{S}y) + d(y, \mathcal{T}x)}{2}\}. \tag{2.2}$$

If $\mathcal{S}$ or $\mathcal{T}$ is continuous, then $\mathcal{T}$ and $\mathcal{S}$ have a common fixed point.

Proof. First of all we show that, if $\mathcal{S}$ or $\mathcal{T}$ has a fixed point, then it is a common fixed point of $\mathcal{S}$ and $\mathcal{T}$. Indeed, let z be a fixed point of $\mathcal{S}$. Now assume that $d(z, \mathcal{T}z) > 0$, if we use the inequality (2.1), for $x = y = z$, we have

$$F(d(\mathcal{T}z, z)) = F(d(\mathcal{T}z, \mathcal{S}z)) \leq \psi(F(\Phi(x, y))) = \psi(F(d(\mathcal{T}z, z)),$$

which is a contradiction. Thus $d(z, \mathcal{T}z) = 0$ and so z is a common fixed point of $\mathcal{S}$ and $\mathcal{T}$. Similarly, if z is a fixed point of $\mathcal{T}$, then it is also a fixed point of $\mathcal{S}$. Let x_0 be an arbitrary point of $\mathcal{X}$. If $x_0 = \mathcal{S}x_0$ the proof is finished, so assume that $x_0 \neq \mathcal{S}x_0$. We can define a sequence $\{x_n\}$ in $\mathcal{X}$ as follows:

$$x_{2n+1} = \mathcal{S}x_{2n} \text{ and } x_{2n+2} = \mathcal{T}x_{2n+1} \text{ for } n \in \{0, 1, \cdots\}.$$

Without loss of generality, we can suppose that the successive term of $\{x_n\}$ are different. Otherwise we are again finished. Note that, since $\mathcal{S}$ and $\mathcal{T}$ are weakly increasing, we have

$$x_1 = \mathcal{S}x_0 \preceq \mathcal{T}\mathcal{S}x_0 = \mathcal{T}x_1 = x_2 = \mathcal{T}x_1 \preceq \mathcal{S}\mathcal{T}x_1 = \mathcal{S}x_2 = x_3.$$

Continuing this process we get

$$x_1 \preceq x_2 \preceq \cdots \preceq x_n \preceq x_{n+1} \preceq \cdots .$$

We claim that

$$F(d(x_{n+1},x_n)) < F(d(x_n,x_{n-1})). \tag{2.3}$$

Setting $x = x_{2n}$ and $y = x_{2n+1}$ in (2.2), we have

$$\begin{aligned}\Phi(x_{2n+1},x_{2n}) &= \max\{d(x_{2n+1},x_{2n}), d(\mathcal{T}x_{2n+1},x_{2n+1}), d(\mathcal{S}x_{2n},x_{2n}),\\ &\qquad \frac{d(x_{2n+1},\mathcal{S}x_{2n}) + d(\mathcal{T}x_{2n+1},x_{2n})}{2}\}\\ &= \max\{d(x_{2n+1},x_{2n}), d(x_{2n+2},x_{2n+1})\}.\end{aligned}$$

Therefore, from (2.1)

$$\begin{aligned}F(d(x_{2n+2},x_{2n+1})) &= F(d(\mathcal{T}x_{2n+1},\mathcal{S}x_{2n})) \leq \psi(F(\Phi(x_{2n+1},x_{2n})))\\ &= \psi(F(d(x_{2n+1},x_{2n}))) < F(d(x_{2n},x_{2n+1})). \qquad (2.4)\end{aligned}$$

Similarly, we have

$$F(d(x_{2n+1},x_{2n})) < F(d(x_{2n},x_{2n-1})). \tag{2.5}$$

Thus from (2.4) and (2.5), we get

$$F(d(x_{n+1},x_n)) < F(d(x_n,x_{n-1}))$$

for all $n \in \mathbb{N}$. Now, from

$$F(d(x_{n+1},x_n)) < \psi(F(d(x_n,x_{n-1}))) < \cdots < \psi^n(F(d(x_1,x_0)))$$

and property of ψ, we obtain $\lim_{n\to\infty} F(d(x_{n+1},x_n)) = 0$, which implies

$$\lim_{n\to\infty} d(x_{n+1},x_n) = 0. \tag{2.6}$$

Next, we claim that $\{x_n\}$ is a Cauchy sequence. Suppose it is not. Then there exists an $\varepsilon > 0$ such that $d(x_n,x_m) \geq \varepsilon$ for infinite values of m and n with $m < n$. This assures that there exist two sequences $\{m(k)\}, \{n(k)\}$ of natural numbers, with $m(k) < n(k)$, such that for each $k \in \mathbb{N}$

$$d(x_{2m(k)},x_{2n(k)+1}) > \varepsilon\,. \tag{2.7}$$

It is not restrictive to suppose that $n(k)$ is the least positive integer exceeding $m(k)$ and satisfying (2.7). We have

$$\begin{aligned}\varepsilon &< d(x_{2m(k)},x_{2n(k)+1})\\ &\leq d(x_{2m(k)},x_{2n(k)-1}) + d(x_{2n(k)-1},x_{2n(k)}) + d(x_{2n(k)},x_{2n(k)+1})\\ &\leq \varepsilon + d(x_{2n(k)-1},x_{2n(k)}) + d(x_{2n(k)},x_{2n(k)+1}).\end{aligned}$$

Letting $k \to \infty$, we have $d(x_{2m(k)}, x_{2n(k)+1}) \to \varepsilon$. We note that

$$\begin{aligned} & d(x_{2m(k)}, x_{2n(k)+1}) - d(x_{2m(k)}, x_{2m(k)+1}) - d(x_{2n(k)+2}, x_{2n(k)+1}) \\ \leq\ & d(x_{2m(k)+1}, x_{2n(k)+2}) \\ \leq\ & d(x_{2m(k)}, x_{2n(k)+1}) + d(x_{2m(k)}, x_{2m(k)+1}) + d(x_{2n(k)+2}, x_{2n(k)+1}). \end{aligned}$$

Thus $d(x_{2m(k)+1}, x_{2n(k)+2}) \to \varepsilon$ as $k \to \infty$. We have

$$\begin{aligned} \Phi(x_{2n(k)+1}, x_{2m(k)})) &= \max\{d(x_{2n(k)+1}, x_{2m(k)}), \\ & \quad d(x_{2n(k)+1}, x_{2n(k)+2}), d(x_{2m(k)}, x_{2m(k)+1}), \\ & \quad \frac{d(x_{2n(k)+1}, x_{2m(k)+1}) + d(x_{2m(k)}, x_{2n(k)+2})}{2}\} \\ &\leq \max\{d(x_{2n(k)+1}, x_{2m(k)}), d(x_{2n(k)+1}, x_{2n(k)+2}), \\ & \quad d(x_{2m(k)}, x_{2m(k)+1}), d(x_{2n(k)+1}, x_{2m(k)}) + \\ & \quad \frac{d(x_{2m(k)}, x_{2m(k)+1}) + d(x_{2n(k)+1}, x_{2n(k)+2})}{2}\}. \end{aligned}$$

Hence, letting $k \to \infty$ we have $\lim_{k\to\infty} \Phi(x_{2n(k)+1}, x_{2m(k)}) \leq \varepsilon$. Therefore we have

$$\begin{aligned} F(d(x_{2m(k)+1}, x_{2n(k)+2})) &= F(d(\mathcal{S}x_{2m(k)}, \mathcal{T}x_{2n(k)+1})) \\ &\leq \psi(F(\Phi(x_{2n(k)+1}, x_{2m(k)}))) \end{aligned}$$

and letting as $k \to \infty$ in the above equation, while F being continuous and ψ right continuous, we get

$$F(\varepsilon) \leq \psi(F(\varepsilon)) < F(\varepsilon),$$

which is a contradiction. Therefore $\{x_n\}$ is a Cauchy sequence and so there exists a $z \in \mathcal{X}$ with $\lim_{n\to\infty} x_n = z$.

If $\mathcal{S}$ or $\mathcal{T}$ is continuous, then clearly $z = \mathcal{S}z = \mathcal{T}z$. □

Our second result is the following.

Theorem 2.3. *Let $(\mathcal{X}, \preceq)$ be a partially ordered set and suppose that there exists a metric d in $\mathcal{X}$ such that $(\mathcal{X}, d)$ is a complete metric space. Let $\mathcal{S}$,$\mathcal{T} : \mathcal{X} \to \mathcal{X}$ be two weakly increasing mappings such that*

$$F(d(\mathcal{T}x, \mathcal{S}y)) \leq \psi(F(\Phi(x, y))) \tag{2.8}$$

for all comparable $x, y \in \mathcal{X}$, where $F \in \mathcal{F}$, $\psi \in \Psi$ and

$$\Phi(x, y) = \max\{d(x, y), d(\mathcal{T}x, x), d(\mathcal{S}y, y), \frac{d(x, \mathcal{S}y) + d(\mathcal{T}x, y)}{2}\}.$$

If the following holds

$$(2.9)\qquad \begin{cases} \{x_n\} \subset \mathcal{X} \text{ is a nondecreasing sequence with } x_n \to z \text{ in } \mathcal{X}, \\ \text{implies that } x_n \preceq z \text{ for all } n. \end{cases}$$

Then, $\mathcal{T}$ *and* $\mathcal{S}$ *have a common fixed point in* $\mathcal{X}$*.*

Proof. Following the proof of Theorem 2.2, we have $\{x_n\}$ is a Cauchy sequence in $(\mathcal{X}, d)$ which is complete. Then, there exists $z \in \mathcal{X}$ such that

$$\lim_{n\to\infty} x_n = z.$$

Now suppose that (2.9) holds and $d(z, \mathcal{S}z) > 0$. From (2.9), we have $x_{2n} \preceq z$ for all $n \in \mathbb{N}$. Hence, we can apply the condition (2.8). Then by setting $x = x_{2n}$ and $y = z$ in (2.8), we obtain:

$$\begin{aligned} F(d(x_{2n+2}, \mathcal{S}z)) &= F(d(\mathcal{T}x_{2n+1}, \mathcal{S}z)) \\ &\le \psi(F(\max\{d(x_{2n+1}, z), d(x_{2n+1}, \mathcal{T}x_{2n+1}), d(z, \mathcal{S}z), \\ &\quad \frac{d(x_{2n+1}, \mathcal{S}z) + d(z, \mathcal{T}x_{2n+1})}{2}\})) \\ &= \psi(F(\max\{d(x_{2n+1}, z), d(x_{2n+1}, x_{2n+2}), d(z, \mathcal{S}z), \\ &\quad \frac{d(x_{2n+1}, \mathcal{S}z) + d(z, x_{2n+2})}{2}\})). \end{aligned}$$

Letting $n \to \infty$ in the above inequality and using the continuity of F and right continuity of ψ, we have

$$F(d(z, \mathcal{S}z)) \le \psi(F(d(z, \mathcal{S}z))) < F(d(z, \mathcal{S}z))$$

which is a contradiction. Therefore $d(z, \mathcal{S}z)) = 0$ and thus $z = \mathcal{S}z$. Hence, z is a common fixed point of $\mathcal{T}$ and $\mathcal{S}$. □

Putting $\mathcal{S} = \mathcal{T}$ in the above theorems we obtain:

Theorem 2.4. *Let* $(\mathcal{X}, \preceq)$ *be a partially ordered set and suppose that there exists a metric* d *in* $\mathcal{X}$ *such that* $(\mathcal{X}, d)$ *is a complete metric space. Let* $\mathcal{T} : \mathcal{X} \to \mathcal{X}$ *be nondecreasing map such that* $x_0 \preceq \mathcal{T}x_0$ *for some* $x_0 \in \mathcal{X}$*. Suppose that*

$$(2.10)\qquad F(d(\mathcal{T}x, \mathcal{T}y)) \le \psi(F(\Theta(x, y)))$$

for all comparable $x, y \in \mathcal{X}$*, where* $F \in \mathcal{F}$*,* $\psi \in \Psi$ *and*

$$\Theta(x, y) = \max\{d(x, y), d(\mathcal{T}x, x), d(\mathcal{T}y, y), \frac{d(x, \mathcal{T}y) + d(\mathcal{T}x, y)}{2}\}.$$

If $\mathcal{T}$ *is continuous or* (2.9) *holds. Then* $\mathcal{T}$ *has a fixed point in* $\mathcal{X}$*.*

Remark 2.5. *In Theorem 2.4* [21] *it is proved that if*

(2.11) *every pair of elements has a lower bound and upper bound,*

then for every $x \in \mathcal{X}$*,*

$$\lim_{n\to\infty} \mathcal{T}^n(x) = y,$$

where y *is the fixed point of* $\mathcal{T}$ *such that*

$$y = \lim_{n\to\infty} \mathcal{T}^n(x_0),$$

hence $\mathcal{T}$ *has a unique fixed point. If condition* (2.11) *fails, it is possible to find examples of functions* $\mathcal{T}$ *with more than one fixed point. There exist some examples to illustrate this fact in* [18].

Example 2.6. *Let* $A = \{\frac{1}{n} : n \in \mathbb{N}\} \cup \{0\}, B = (1,\infty)$ *and* $\mathcal{X} = A \cup B$*. Define a relation on* $\mathcal{X}$ *as follows: for* $x, y \in \mathcal{X}$

$$x \preceq y \Leftrightarrow \{x = y \text{ or } x, y \in A \text{ with } x \le y\}.$$

It is easy to see that $\preceq$ *is a partial order on* $\mathcal{X}$*. Let* d *be the usual metric on* $\mathcal{X}$*, then* $(\mathcal{X}, d)$ *is complete. Let*

$$F(t) = \begin{cases} 0 & , \quad t = 0 \\ t^{\frac{1}{t}} & , \quad 0 < t < e \\ t + e^{\frac{1}{e}} - e & , \quad e \le t \end{cases},$$

then $F \in \mathcal{F}$*. Let* $\psi(t) = \frac{t}{2}$*, then* $\psi \in \Psi$*. Define a mapping* $\mathcal{T} : \mathcal{X} \to \mathcal{X}$ *as follows:*

$$\mathcal{T}x = \begin{cases} \dfrac{1}{n+1} & , \quad x = \dfrac{1}{n} \\ 0 & , \quad x = 0 \\ 2x & , \quad x \in B \end{cases}.$$

Now $x \preceq y \Rightarrow \{x = y \text{ or } x, y \in A \text{ with } x \le y\} \Rightarrow \{\mathcal{T}x = \mathcal{T}y \text{ or } \mathcal{T}x, \mathcal{T}y \in A \text{ with } \mathcal{T}x \le \mathcal{T}y\} \Rightarrow \mathcal{T}x \preceq \mathcal{T}y$*. That is,* $\mathcal{T}$ *is a nondecreasing map and also* $0 = x_0 \preceq \mathcal{T}x_0 = 0$*. Now we prove that the inequality* (2.10) *is satisfied for all comparable* $x, y \in \mathcal{X}$*. Suppose that* $x \preceq y$*. If* $x = y$*, then it is clear that* (2.10) *holds. If* $x \neq y$*, then* $x, y \in A$ *with* $x \le y$ *and so using Example 2 of* [22] *we again see that* (2.10) *holds. Also the condition* (2.9) *is satisfied on* $\mathcal{X}$*. Therefore all conditions of*

Theorem 2.4 is satisfied. Thus $\mathcal{T}$ has a fixed point in $\mathcal{X}$. But we can not apply Theorem 1.1 of this paper. For example, let $x = 2, y = 1$, then

$$F(d(\mathcal{T}x, \mathcal{T}y)) = F(\frac{7}{2})$$

and

$$F(\Theta(x, y)) = F(\frac{9}{4}).$$

Now it is clear that there are no functions F and ψ as in Theorem 1.1 satisfying

$$F(d(\mathcal{T}x, \mathcal{T}y)) = F(\frac{7}{2}) \leq \psi(F(\Theta(x, y))) = \psi(F(\frac{9}{4})).$$

That is, the contractive condition of the result of [22] *is not satisfied. Also note that the contractive condition*

$$d(\mathcal{T}x, \mathcal{T}y) \leq \lambda\Theta(x, y), \;\; \lambda \in [0, 1)$$

for all comparable $x, y \in \mathcal{X}$ is not satisfied. To see this, let $x_n = \frac{1}{n+1}$, $y_n = \frac{1}{n}$, then $x_n \preceq y_n$ for all $n \in \mathbb{N}$, and

$$\sup_{n\in\mathbb{N}} \frac{d(\mathcal{T}x, \mathcal{T}y)}{d(x, y)} = 1.$$

This shows that the result of [21] *and its linear generalized version cannot be applied to this example.*

Acknowledgments

The authors thank the referees for their appreciation, valuable comments and suggestions.

References

[1] M. Abbas, T. Nazir and S. Radenović, Common fixed point of four maps in partially ordered metric spaces, *Appl. Math. Lett.* (2011), no. 9, 1520–1526.

[2] R. P. Agarwal, M. A. El-Gebeily and D. O'Regan, Generalized contractions in partially ordered metric spaces, *Appl. Anal.* **87** (2008), no. 1, 109–116.

[3] I. Altun, B. Damjanović and D. Djorić, Fixed point and common fixed point theorems on ordered cone metric spaces, *Appl. Math. Lett.* **23** (2010), no. 3, 310–316.

[4] I. Altun and H. Simsek, Some fixed point theorems on ordered metric spaces and application, *Fixed Point Theory and Appl.* (2010), Art. ID 621469, 17 pp.

[5] A. Amini-Harandi and H. Emami, A fixed point theorem for contraction type maps in partially ordered metric spaces and application to ordinary differential equations, *Nonlinear Anal.* **72** (2010), no. 5, 2238–2242.

[6] I. Beg and A. R. Butt, Fixed point for set-valued mappings satisfying an implicit relation in partially ordered metric spaces, *Nonlinear Anal.* **71** (2009), no. 9, 3699–3704.

[7] T. G. Bhaskar and V. Lakshmikantham, Fixed point theorems in partially ordered metric spaces and applications, *Nonlinear Anal.* **65** (2006), no. 7, 1379–1393.

[8] A. Branciari, A fixed point theorem for mappings satisfying a general contractive condition of integral type, *Int. J. Math. Math. Sci.* **29** (2002), no. 9, 531–536.

[9] A. Cabada and J. J. Nieto, Fixed points and approximate solutions for nonlinear operator equations, *J. Comput. Appl. Math.* **113** (2000), no. 1-2, 17–25.

[10] J. Caballero, J. Harjani and K. Sadarangani, Contractive-like mapping principles in ordered metric spaces and application to ordinary differential equations, *Fixed Point Theory and Appl.* (2010), Art. ID 916064, 14 pp.

[11] L. Ćirić, N. Cakić, M. Rajović and J. S. Ume, Monotone generalized nonlinear contractions in partially ordered metric spaces, *Fixed Point Theory and Appl.* (2008), Art. ID 131294, 11 pp.

[12] J. Harjani and K. Sadarangani, Fixed point theorems for weakly contractive mappings in partially ordered sets, *Nonlinear Anal.* **71** (2009), no. 7-8, 3403–3410.

[13] J. Harjani and K. Sadarangani, Generalized contractions in partially ordered metric spaces and applications to ordinary differential equations, *Nonlinear Anal.* **72** (2010), no. 3-4, 1188–1197.

[14] Z. Kadelburg, M. Pavlović and S. Radenović, Common fixed point theorems for ordered contractions and quasicontractions in ordered cone metric spaces, *Comput. Math. Appl.* **59** (2010), no. 9, 3148–3159.

[15] H. K. Nashine and I. Altun, Fixed point theorems for generalized weakly contractive condition in ordered metric spaces, *Fixed Point Theory Appl.* (2011), Art. ID 132367, 20 pp.

[16] H. K. Nashine and B. Samet, Fixed point results for mappings satisfying (ψ, φ)-weakly contractive condition in partially ordered metric spaces, *Nonlinear Anal.* **74** (2011) 2201–2209.

[17] H. K. Nashine, B. Samet and C. Vetro, Monotone generalized nonlinear contractions and fixed point theorems in ordered metric spaces, *Math. Comput. Modelling* **54** (2011), no. 1-2, 712–720.

[18] J. J. Nieto and R. Rodríguez-López, Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations, *Order* **22** (2005), no. 3, 223–239.

[19] S. Radenović and Z. Kadelburg, Generalized weak contractions in partially ordered metric spaces, *Comput. Math. Appl.* **60** (2010), no. 6, 1776–1783.

[20] S. Radenović, Z. Kadelburg, D. Jandrlić and A. Jandrlić, Some results on weakly contractive maps, *Bull. Iranian Math. Soc.* **38** (2012), no. 3, 625–645.

[21] A. C. M. Ran and M. C. B. Reurings, A fixed point theorem in partially ordered sets and some applications to matrix equations, *Proc. Amer. Math. Soc.* **132** (2004), no. 5, 1435–1443.

[22] X. Zhang, Common fixed point theorems for some new generalized contractive type mappings, *J. Math. Anal. Appl.* **333** (2007), no. 2, 780–786.

Hemant Kumar Nashine
Department of Mathematics, Disha Institute of Management and Technology, Satya Vihar, Vidhansabha-Chandrakhuri Marg, Mandir Hasaud, Raipur-492101, Chhattisgarh, India
Email: hemantnashine@rediffmail.com, nashine_09@rediffmail.com

Ishak Altun
Department of Mathematics, Kirikkale University, 71450 Yahsihan, Kirikkale, Turkey
Email: ialtun@kku.edu.tr, ishakaltun@yahoo.com