# SOME SEPARATION AXIOMS VIA MODIFIED $\theta ext{-}\text{OPEN SETS}$

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ABSTRACT. In this paper, we introduce some separation axioms by utilizing modified  $\theta$ -open sets and obtain a characterization of Hausdorff spaces due to Dickman and Porter [?] as a corollary of the results.

# 1. Introduction

Veličko [?] introduced the notion of  $\theta$ -open sets and the  $\theta$ -closure operator to study H-closed spaces. Janković [?] investigated several separation axioms by using  $\theta$ -open sets and the  $\theta$ -closure operator. Dickman and Porter [?] investigated the relationships among  $\theta$ -closed sets, Hausdorff spaces and H-closed spaces. Among others, they showed that a topological space X is Hausdorff if and only if for each  $x \in X$  the singleton  $\{x\}$  is  $\theta$ -closed. Recently, the following modifications of  $\theta$ -open sets are introduced and studied: semi- $\theta$ -open [?], pre- $\theta$ -open [?] and semipre- $\theta$ -open [?].

In this paper, by using the notion of m- $\theta$ -open sets [?], we introduce and investigate separation axioms  $m\theta$ - $D_0$ ,  $m\theta$ - $D_1$  and  $m\theta$ - $D_2$  analogous to  $D_0$ ,  $D_1$  and  $D_2$  due to Tong [?]. As a corollary of

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our results, we obtain a characterization of Hausdorff spaces stated above.

#### 2. Preliminaries

In what follows  $(X, \tau)$  and  $(Y, \sigma)$  (or X and Y) denote topological spaces. Let A be a subset of X. We denote the interior, the closure and the complement of a set A by Int(A), Cl(A) and  $X \setminus A$  or  $A^c$  respectively. A point  $x \in X$  is called the  $\theta$ -cluster point of A if  $A \cap Cl(U) \neq \emptyset$  for every open set U of X containing x. The set of all  $\theta$ -cluster points of A is called the  $\theta$ -closure of A, denoted by  $Cl_{\theta}(A)$ . A subset A is called  $\theta$ -closed if  $A = Cl_{\theta}(A)$ . The complement of a  $\theta$ -closed set is called  $\theta$ -open. We denote the collection of all  $\theta$ -open sets of  $(X, \tau)$  by  $\tau_{\theta}$ .

**Definition 2.1.** Let  $(X, \tau)$  be a topological space. A subset A of X is said to be

- (1) semi-open [?] if  $A \subset Cl(Int(A))$ ,
- (2) preopen [?] if  $A \subset Int(Cl(A))$ ,
- (3)  $\beta$ -open [?] or semi-preopen [?] if  $A \subset Cl(Int(Cl(A)))$ .

The family of all semi-open (resp. preopen,  $\beta$ -open, semi-preopen) sets in X is denoted by SO(X) (resp. PO(X),  $\beta(X)$ , SPO(X)).

**Definition 2.2.** The complement of a semi-open (resp. preopen,  $\beta$ -open, semi-preopen) set is said to be semi-closed [?] (resp. preclosed [?],  $\beta$ -closed [?], semi-preclosed [?]).

**Definition 2.3.** The intersection of all semi-closed (resp. preclosed,  $\beta$ -closed, semi-preclosed) sets of X containing A is called the semi-closure [?] (resp. preclosure [?],  $\beta$ -closure [?] or semi-preclosure [?]) of A and is denoted by sCl(A) (resp. pCl(A),  $\beta Cl(A)$ , spCl(A)).

**Definition 2.4.** Let  $(X, \tau)$  be a topological space. A subset A of X is said to be

(1) semi- $\theta$ -open [?] if for each  $x \in A$  there exists a semi-open set U

such that  $x \in U \subset sCl(U) \subset A$ ,

- (2) pre- $\theta$ -open [?] if for each  $x \in A$  there exists a preopen set U such that  $x \in U \subset pCl(U) \subset A$ ,
- (3) semipre- $\theta$ -open [?] for each  $x \in A$  there exists a semipre-open set U such that  $x \in U \subset spCl(U) \subset A$ .

#### 3. m-spaces

**Definition 3.1.** A subfamily m of the power set P(X) of a nonempty set X is called an m-structure on X if m satisfies the following:

- (1)  $\emptyset \in m \text{ and } X \in m$ ,
- (2)  $\bigcup_{\alpha \in \triangle} A_{\alpha} \in m$  whenever  $A_{\alpha} \in m$  for each  $\alpha \in \triangle$ .

We call the pair (X, m) an m-space. Each member of m is said to be m-open and the complement of an m-open set is said to be m-closed.

**Remark 3.2.** It should be noted that condition (2) in Definition 3.1 is called property (B) by Maki et al. in [?]. In this paper, we always assume the property (B) on m-structures.

**Remark 3.3.** Let  $(X, \tau)$  be a topological space. Then the families  $\tau_{\theta}$ ,  $\tau$ , SO(X), PO(X),  $\beta(X)$  are all *m*-structures on X. It is well-known that  $\tau_{\theta}$  is a topology for X.

**Definition 3.4.** Let X be a nonempty set and m an m-structure on X. For a subset A of X, the m-closure of A and the m-interior of A are defined in [?] as follows:

- $(1) \ mCl(A) = \bigcap \{F \mid A \subset F, X \backslash F \in m\},\$
- (2)  $mInt(A) = \bigcup \{U \mid U \subset A, U \in m\}.$

**Remark 3.5.** Let  $(X, \tau)$  be a topological space and A a subset of X. If  $m = \tau$  (resp. SO(X), PO(X),  $\beta(X)$ ), then we have

- (1) mCl(A) = Cl(A) (resp. sCl(A), pCl(A),  $\beta Cl(A)$ ),
- (2) mInt(A) = Int(A) (resp. sInt(A), pInt(A),  $\beta Int(A)$ ).

(cf. Remark 3.10 below for the  $\theta$ -closure  $Cl_{\theta}(A)$  of A)

**Lemma 3.6.** (eg. Maki et al. [?]) Let m be an m-structure on a nonempty set X. For subsets A and B of X, the following properties hold:

- (1)  $mCl(X \setminus A) = X \setminus mInt(A)$  and  $mInt(X \setminus A) = X \setminus mCl(A)$ ,
- (2)  $mCl(\emptyset) = \emptyset$ , mCl(X) = X,  $mInt(\emptyset) = \emptyset$  and mInt(X) = X,
- (3) If  $A \subset B$ , then  $mCl(A) \subset mCl(B)$  and  $mInt(A) \subset mInt(B)$ ,
- (4)  $A \subset mCl(A)$  and  $mInt(A) \subset A$ ,
- (5) mCl(mCl(A)) = mCl(A) and mInt(mInt(A)) = mInt(A).

**Lemma 3.7.** (Popa and Noiri [?]) Let m be an m-structure on a nonempty set X. Then  $x \in mCl(A)$  if and only if  $U \cap A \neq \emptyset$  for every  $U \in m$  containing x.

**Lemma 3.8.** Let m be an m-structure on a nonempty set X. Then for a subset A of X the following properties hold:

- (1)  $A \in m$  if and only if A = mInt(A),
- (2) A is m-closed if and only if A = mCl(A),
- (3) mCl(A) is m-closed and mInt(A) is m-open.

**Proof.** This is an immediate consequence of Lemmas 3.6 and 3.7.  $\square$ 

**Definition 3.9.** Let A be a subset of an m-space (X, m). A point  $x \in X$  is called

- (1) an  $m_{\theta}$ -adherent point of A if  $mcl(U) \cap A \neq \emptyset$  for every  $U \in m$  containing x,
- (2) An  $m_{\theta}$ -interior point of A if  $x \in U \subset mCl(U) \subset A$  for every  $U \in m$  containing x.

The set of all  $m_{\theta}$ -adherent points of A is called the  $m_{\theta}$ -closure [?] of A and is denoted by  $mCl_{\theta}(A)$ . If  $A = mCl_{\theta}(A)$ , then A is called m- $\theta$ -closed. The complement of an m- $\theta$ -closed set is said to be m- $\theta$ -open. The set of all  $m_{\theta}$ -interior points of A is called the  $m_{\theta}$ -interior of A and is denoted by  $mInt_{\theta}(A)$ .

This is clear that union of m- $\theta$ -open sets in X is m- $\theta$ -open.

**Remark 3.10.** (Noiri and Popa [?]). Let A and B be subsets of an m-space (X, m). Then the following properties hold:

- (1)  $X \backslash mCl_{\theta}(A) = mInt_{\theta}(X \backslash A)$  and  $X \backslash mInt_{\theta}(A) = mCl_{\theta}(X \backslash A)$ ,
- (2) A is open if and only if  $A = mInt_{\theta}(A)$ ,
- (3)  $A \subset mCl(A) \subset mCl_{\theta}(A)$  and  $mInt_{\theta}(A) \subset mInt(A) \subset A$ ,
- (4) If  $A \subset B$ , then  $mCl_{\theta}(A) \subset mCl_{\theta}(B)$  and  $mInt_{\theta}(A) \subset mInt_{\theta}(B)$ ,
- (5) If A is m-open, then  $mCl(A) = mCl_{\theta}(A)$ ,
- (6)  $mCl_{\theta}(A)$  is m-closed and  $mInt_{\theta}(A)$  is m-open.

#### 4. Some New Separation Axioms

Tong [?] defined a subset A of a topological space  $(X, \tau)$  to be a D-set if there are two open sets U, V in X such that  $U \neq X$  and  $A = U \setminus V$ . Analogously, we define  $m\theta$ -D-sets as follows:

**Definition 4.1.** Let (X, m) be an m-space. A subset  $A \subset X$  is called an  $m\theta$ -Difference set (in short  $m\theta$ -D-set) if there are two  $m\theta$ -open sets U, V in X such that  $U \neq X$  and  $A = U \setminus V$ .

It is true that every  $m\theta$ -open set  $U \neq X$  is an  $m\theta$ -D-set since  $U = U \setminus \emptyset$ .

#### **Definition 4.2.** An m-space (X, m) is said to be

- (1)  $m\theta$ - $D_0$  (resp.  $m\theta$ - $D_1$ ) if for  $x, y \in X$  such that  $x \neq y$  there exists an  $m\theta$ -D-set of X containing x but not y or (resp. and) an  $m\theta$ -D-set containing y but not x,
- (2)  $m\theta$ - $D_2$  if for  $x, y \in X$  such that  $x \neq y$  there exist disjoint  $m\theta$ -D-sets G and E such that  $x \in G$  and  $y \in E$ ,
- (3)  $m\theta$ - $T_0$  (resp.  $m\theta$ - $T_1$ ) if for  $x, y \in X$  such that  $x \neq y$  there exists an  $m\theta$ -open set U of X containing x but not y or (resp. and) an  $m\theta$ -open set V of X containing y but not x,
- (4)  $m\theta$ - $T_2$  if for  $x, y \in X$  such that  $x \neq y$  there exist disjoint  $m\theta$ -open sets U and V such that  $x \in U$  and  $y \in V$ .

**Remark 4.3.** Let  $(X, \tau)$  be a topological space. If  $m = \tau$ , then  $m\theta - T_1 = \theta - T_1$  [?] and  $m\theta - T_2 = \theta - T_2$  [?].

**Theorem 4.4.** For an m-space (X, m), the following properties hold:

- (1) If (X, m) is  $m\theta T_i$ , then it is  $m\theta T_{i-1}$  for  $i = 1, 2, \dots$
- (2) If (X, m) is  $m\theta$ - $T_i$ , then it is  $m\theta$ - $D_i$  for i = 0, 1, 2,
- (3) If (X, m) is  $m\theta$ - $D_i$ , then it is  $m\theta$ - $D_{i-1}$  for i = 1, 2.

**Proof.** This is obvious from Definition 4.2.  $\square$ 

**Theorem 4.5.** For an m-space (X, m), the following statements are true:

- (1) (X, m) is  $m\theta$ - $D_0$  if and only if (X, m) is  $m\theta$ - $T_0$ ,
- (2) (X, m) is  $m\theta$ - $D_1$  if and only if (X, m) is  $m\theta$ - $D_2$ .

**Proof.** The sufficiency for (1) and (2) follows from Theorem 4.4.  $\square$ 

Necessity for (1). Let (X, m) be  $m\theta$ - $D_0$  so that for any pair of distinct points x and y of X at least one belongs to an  $m\theta$ -D-set O. Therefore, we choose  $x \in O$  and  $y \notin O$ . Suppose  $O = U \setminus V$  for  $U \neq X$  and m- $\theta$ -open sets U and V. This implies that  $x \in U$ . For the case that  $y \notin O$  we have (i)  $y \notin U$ , (ii)  $y \in U$  and  $y \in V$ . For (i), the m-space (X, m) is  $m\theta$ - $T_0$  since  $x \in U$  and  $y \notin U$ . For (ii), the m-space (X, m) is also  $m\theta$ - $T_0$  since  $y \in V$  but  $x \notin V$ .

Necessity for (2). Suppose that (X,m) is  $m\theta$ - $D_1$ . It follows from the definition that for any distinct points x and y in X there exist  $m\theta$ -D-sets G and E such that G containing x but not y and E containing y but not x. Let  $G = U \setminus V$  and  $E = W \setminus D$ , where U, V, W and D are m- $\theta$ -open. By the fact that  $x \notin E$ , we have two cases, i.e. either  $x \notin W$  or both W and D contain x. If  $x \notin W$ , then from  $y \notin G$  either (i)  $y \notin U$  or (ii)  $y \in U$  and  $y \in V$ . If (i) is the case, then it follows from  $x \in U \setminus V$  that  $x \in U \setminus (V \cup W)$ , and also it follows from  $y \in W \setminus D$  that  $y \in W \setminus (U \cup D)$ . Thus we have  $U \setminus (V \cup W)$  and  $W \setminus (U \cup D)$  which are disjoint. If (ii) is the case, it follows that  $x \in U \setminus V$  and  $y \in V$  since  $y \in U$  and  $y \in V$ . Therefore  $(U \setminus V) \cap V = \emptyset$ . If  $x \in W$  and  $x \in D$ , we have  $y \in W \setminus D$  and  $x \in D$ . Hence  $(W \setminus D) \cap D = \emptyset$ . This shows that

X is  $m\theta$ - $D_2$ .

**Definition 4.6.** An m-space (X, m) is said to be m- $T_2$  [?] if for  $x, y \in X$  such that  $x \neq y$  there exist disjoint m-open sets U and V of X such that  $x \in U$  and  $y \in V$ .

**Remark 4.7.** Let  $(X, \tau)$  be a topological space. If  $m = \tau$ , then m- $T_2$  = Hausdorff.

**Theorem 4.8.** For an m-space (X, m), the following properties are equivalent:

- (1) (X, m) is m- $T_2$ ;
- (2) (X, m) is  $m\theta T_1$ ;
- (3) (X, m) is  $m\theta T_0$ .

**Proof.** (1)  $\Rightarrow$  (2): Let x and y be any pair of disjoint points of X. For any  $z \in X \setminus \{y\}$ , by (1) there exist m-open sets  $U_y$  and  $U_z$  such that  $y \in U_y$ ,  $z \in U_z$  and  $U_y \cap U_z = \emptyset$ . We have  $mCl(U_z) \cap U_y = \emptyset$  and  $z \in U_z \subset mCl(U_z) \subset X \setminus U_y \subset X \setminus \{y\}$ . Therefore,  $X \setminus \{y\}$  is an m- $\theta$ -open set containing x. Quite similarly, we can show that  $X \setminus \{x\}$  is an m- $\theta$ -open set containing y. Therefore, (X, m) is  $m\theta$ - $T_1$ .

- $(2) \Rightarrow (3)$ : This is obvious.
- (3)  $\Rightarrow$  (1): Suppose that (X, m) is  $m\theta$ - $T_0$ . For any pair of distinct points x, y, there exists an m- $\theta$ -open set U such that (1)  $x \in U$  and  $y \notin U$  or (2)  $x \notin U$  and  $y \in U$ . In case (1), there exists  $U_x \in m$  such that  $x \in U_x \subset mCl(U_x) \subset U$ . Therefore,  $y \in X \setminus U \subset X \setminus mCl(U_x)$ ,  $X \setminus mCl(U_x)$  is m-open and  $U_x \cap (X \setminus mCl(U_x)) = \emptyset$ . In case (2), similarly, we obtain  $U_y \in m$  such that  $y \in U_y$ ,  $x \in X \setminus mCl(U_y)$  and  $U_y \cap (X \setminus mCl(U_y)) = \emptyset$ . This shows that (X, m) is m- $T_2$ .  $\square$

**Corollary 4.9.** For an m-space (X, m), the following properties are equivalent:

- (1) (X, m) is  $m\theta D_2$ ;
- (2)(X,m) is  $m\theta$ - $D_1$ ;
- (3) (X, m) is  $m\theta D_0$ ;

- (4) (X, m) is  $m\theta$ - $T_0$ ;
- (5) (X, m) is  $m\theta$ - $T_1$ ;
- (6) (X, m) is  $m-T_2$ .

**Proof.** This is an immediate consequence of Theorems 4.4, 4.5 and 4.8.  $\square$ 

**Corollary 4.10.** (Dickman and Porter [?]). For a topological space  $(X, \tau)$ , the following properties are equivalent:

- (1)  $(X, \tau)$  is Hausdorff;
- (2)  $(X, \tau)$  is  $\theta T_1$ ;
- (3) For each  $x \in X$ ,  $Cl_{\theta}(\{x\}) = \{x\}$ .

**Proof.** This is an immediate consequence of Corollary 4.9.  $\square$ 

**Remark 4.11.** For a topological space  $(X, \tau)$ , by Corollary 4.9 we have the following diagram:

$$\begin{array}{cccc} T_2 & \longleftrightarrow & \theta\text{-}D_2 \\ \updownarrow & & \updownarrow \\ \theta\text{-}T_1 & \longleftrightarrow & \theta\text{-}D_1 \\ \updownarrow & & \updownarrow \\ \theta\text{-}T_2 & \longleftrightarrow & \theta\text{-}D_0 \end{array}$$

# 5. Quasi- $\theta$ -continuous functions

**Definition 5.1.** A function  $f:(X, m_X) \to (Y, m_Y)$ , where X and Y are nonempty sets with m-structures  $m_X$  and  $m_Y$ , respectively, is said to be

- (1)  $\theta$ -M-continuous [?] at  $x \in X$  if for each  $V \in m_Y$  containing f(x), there exists  $U \in m_X$  containing x such that  $f(m_X Cl(U)) \subset m_Y Cl(V)$ . A function  $f: (X, m_X) \to (Y, m_Y)$  is said to be  $\theta$ -M-continuous if it has the property at each point  $x \in X$ .
- (2) quasi- $\theta$ -M-continuous if for each  $x \in X$  and each  $m\theta$ -open set V containing f(x), there is an  $m\theta$ -open set U containing x such that  $f(U) \subset V$ , equivalently, if  $f^{-1}(V)$  is  $m\theta$ -open in  $(X, \tau)$  for every  $m\theta$ -open set V of  $(Y, \sigma)$ .

**Remark 5.2.** Let  $f:(X,\tau)\to (Y,\sigma)$  be a function. If  $m_x=\tau$ ,  $m_Y=\sigma$  and  $f:(X,m_X)\to (Y,m_Y)$  is  $\theta$ -M-continuous (resp. quasi- $\theta$ -M-continuous), then f is  $\theta$ -continuous [?] (resp. quasi- $\theta$ -continuous [?]).

**Remark 5.3.** (1) A function  $f:(X,\tau)\to (Y,\sigma)$  is quasi- $\theta$ -continuous if and only if  $f:(X,\tau_{\theta})\to (Y,\sigma_{\theta})$  is continuous. (2) It is shown in [?] that every  $\theta$ -continuous function is quasi- $\theta$ -continuous but the converse is not true.

**Theorem 5.4.** If  $f:(X, m_X) \to (Y, m_Y)$  is a quasi- $\theta$ -M-continuous surjection and A is an  $m\theta$ -D-set in Y, then  $f^{-1}(A)$  is an  $m\theta$ -D-set in X.

**Proof.** Let A be an  $m\theta$ -D-set in Y. Then there are  $m\theta$ -open sets U and V in Y such that  $A = U \setminus V$  and  $U \neq Y$ . By the quasi- $\theta$ -M-continuity of f,  $f^{-1}(U)$  and  $f^{-1}(V)$  are  $m\theta$ -open in X. Since  $U \neq Y$  and f is surjective, we have  $f^{-1}(U) \neq X$ . Hence  $f^{-1}(A) = f^{-1}(U) \setminus f^{-1}(V)$  is an  $m\theta$ -D-set.  $\square$ 

We prove now another characterization of  $m\theta$ - $D_1$  spaces.

**Theorem 5.5.** An m-space  $(X, m_X)$  is  $m\theta$ - $D_1$  if and only if, for each pair of distinct points x and y in X, there exists a quasi- $\theta$ -M-continuous surjection f of  $(X, m_X)$  onto an  $m\theta$ - $D_1$  space  $(Y, m_Y)$  such that  $f(x) \neq f(y)$ .

**Proof.** Necessity. For every pair of distinct points of X, it suffices to take the identity function on X.

Sufficiency. Let x and y be any pair of distinct points in X. By hypothesis, there exists a quasi- $\theta$ -M-continuous surjection f of a space X onto an  $m\theta$ - $D_1$  space Y such that  $f(x) \neq f(y)$ . Therefore, by Theorem 4.5 there exist disjoint  $m\theta$ -D-sets  $A_x$  and  $B_y$  in Y such that  $f(x) \in A_x$  and  $f(y) \in B_y$ . Since f is quasi- $\theta$ -M-continuous and surjective, by Theorem 5.4,  $f^{-1}(A_x)$  and  $f^{-1}(B_y)$ 

are disjoint  $m\theta$ -D-sets in X containing x and y, respectively. Hence by Theorem 4.5, X is  $m\theta$ - $D_1$  space.  $\square$ 

**Theorem 5.6.** If  $(Y, m_Y)$  is  $m\theta$ - $D_1$  and  $f: (X, m_X) \to (Y, m_Y)$  is a quasi- $\theta$ -M continuous injection, then  $(X, m_X)$  is  $m\theta$ - $D_1$ .

**Proof.** Let x be any point of X. Since  $(Y, m_Y)$  is  $m\theta$ - $D_1$ ,  $(Y, m_Y)$  is  $m\theta$ - $T_1$  and hence  $\{f(x)\}$  is  $m\theta$ -closed in  $(Y, \sigma)$  by Corollary 4.9. Since f is a quasi- $\theta$ -M-continuous injection,  $\{x\} = f^{-1}(\{f(x)\})$  is m- $\theta$ -closed in  $(X, m_X)$ . Therefore  $(X, m_X)$  is  $m\theta$ - $D_1$ .  $\square$ 

Corollary 5.7. If  $f:(X, m_X) \to (Y, m_Y)$  is a quasi- $\theta$ -M-continuous injection and  $(Y, m_Y)$  is m- $T_2$ , then  $(X, m_X)$  is m- $T_2$ .

**Proof.** This is an immediate consequence of Corollary 4.9 and Theorem 5.6.  $\square$ 

As corollaries of Theorem 5.5 and 5.6, we obtain the following:

**Corollary 5.8.** A topological space  $(X, \tau)$  is  $\theta$ - $D_1$  if and only if for each pair of distinct points x and y in X, there exists a quasi- $\theta$ -continuous surjection f of X onto a  $\theta$ - $D_1$  space  $(Y, \sigma)$  such that  $f(x) \neq f(y)$ .

**Corollary 5.9.** If  $(Y, \sigma)$  is  $\theta$ - $D_1$  (resp. Hausdorff) and  $f: (X, \tau) \to (Y, \sigma)$  is a quasi- $\theta$ -continuous injection, then  $(X, \tau)$  is  $\theta$ - $D_1$  (resp. Hausdorff).

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