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PRIME HIGHER DERIVATIONS ON ALGEBRAS

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Abstract. Let $\mathcal{A}$ be an algebra. A sequence $\{d_n\}$ of linear mappings from $\mathcal{A}$ into $\mathcal{A}$ is called a higher derivation if $d_n(ab) = \sum_{k=0}^{n} d_k(a)\ d_{n-k}(b)$ for each $a, b \in \mathcal{A}$ and each nonnegative integer n. We say that a sequence $\{d_n\}$ of linear mappings on $\mathcal{A}$ is a prime higher derivation if $d_n(ab) = \sum_{k|n} d_k(a) d_{\frac{n}{k}}(b)$ for each $a, b \in \mathcal{A}$ and each $n \in \mathbb{N}$. Giving some examples of prime higher derivations, we establish a characterization of prime higher derivations in terms of derivations.

1. Introduction

Let $\mathcal{A}$ be an algebra and $\sigma : \mathcal{A} \to \mathcal{A}$ be a linear mapping. A linear mapping $\delta : \mathcal{A} \to \mathcal{A}$ is called a σ-derivation if it satisfies the Leibniz rule $\delta(ab) = \delta(a)\sigma(b) + \sigma(a)\delta(b)$ for all $a, b \in \mathcal{A}$. In the case $\sigma = I_{\mathcal{A}}$, the identity mapping on $\mathcal{A}$, a σ-derivation is called a derivation. (For other approaches to generalized derivations and their applications, see [1, 2, 4, 9, 10] and references therein. In particular, an automatic continuity problem for (σ, τ)-derivations is considered in [8] and an achievement of continuity of (σ, τ)-derivations without linearity is given in [6].)

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A sequence $\{d_n\}$ of linear mappings on $\mathcal{A}$ is called a higher derivation if $d_n(ab) = \sum_{k=0}^{n} d_k(a)d_{n-k}(b)$ for each $a, b \in \mathcal{A}$ and each nonnegative integer n. Higher derivations were introduced by Hasse and Schmidt [5], and algebraists sometimes call them Hasse-Schmidt derivations. For an account on higher derivations the reader is referred to the book [3]. Taking idea from this notion under a number theoretic view, we are motivated to consider all sequences $\{d_n\}$ of linear mappings on $\mathcal{A}$ satisfying the relation $d_n(ab) = \sum_{k|n} d_k(a)d_{\frac{n}{k}}(b)$ for each $a, b \in \mathcal{A}$ and each $n \in \mathbb{N}$. If p is prime then $d_p(ab) = d_1(a)d_p(b) + d_p(a)d_1(b)$ $(a, b \in \mathcal{A})$ which shows that d_p is a d_1-derivation. In the case that d_1 is the identity mapping on $\mathcal{A}$, d_p is a derivation for each prime p. On the other hand, for each prime p, the subsequence $\{d_{p^n}\}$ of $\{d_n\}$ is a higher derivation. These are the reasons for prefering to use the terminology prime higher derivation for such a sequence of linear mappings on $\mathcal{A}$. Let d_p be an arbitrary derivation on $\mathcal{A}$ for each prime p. As a typical example of a prime higher derivation, one can define $d_n : \mathcal{A} \to \mathcal{A}$ by $d_n = \prod_{i=1}^{r} \frac{d_{p_i}^{\alpha_i}}{\alpha_i!}$, where $n = p_1^{\alpha_1} \cdots p_r^{\alpha_r}$ with $p_1 < \ldots < p_r$. However, there are other examples of prime higher derivations on algebras.

In [7], the author gives a characterization of higher derivations on an algebra $\mathcal{A}$ in terms of derivations on $\mathcal{A}$, provided that d_0 is the identity mapping on $\mathcal{A}$. Here, we characterize all prime higher derivations on an algebra $\mathcal{A}$ in terms of derivations on $\mathcal{A}$, provided that d_1 is the identity mapping on $\mathcal{A}$. Though the notion of a prime higher derivation has some interests in its own right, regarding the fact that the subsequence $\{d_{p^n}\}$ of a prime higher derivation $\{d_n\}$ is a higher derivation, we can say that prime higher derivation is a generalization of higher derivation and in fact we generalize the result of [7]. Throughout the paper, all algebras are assumed over the field of complex numbers.

2. **Preliminaries**

Throughout the paper, $\mathbb{P}$ stands for the set of all prime numbers and $I_{\mathcal{A}}$ is the identity mapping on $\mathcal{A}$. We also denote the greatest prime divisor of n by $g(n)$. If $p^\alpha | n$ but $p^{\alpha+1} \not| n$ for a prime p, then we write $p^\alpha \| n$ and we denote α by $e(p, n)$.

Let $\mathcal{A}$ be an algebra and $\sigma : \mathcal{A} \to \mathcal{A}$ be a linear mapping. A linear mapping $\delta : \mathcal{A} \to \mathcal{A}$ is called a σ-derivation if it satisfies the Leibniz rule $\delta(ab) = \delta(a)\sigma(b) + \sigma(a)\delta(b)$ for all $a, b \in \mathcal{A}$. In the case $\sigma = I_{\mathcal{A}}$, a

σ-derivation is called a derivation. A sequence $\{d_n\}$ of linear mappings on $\mathcal{A}$ is called a higher derivation if $d_n(ab) = \sum_{k=0}^{n} d_k(a)d_{n-k}(b)$ for each $a, b \in \mathcal{A}$ and each nonnegative integer n.

Definition 2.1. Let $\mathcal{A}$ be an algebra. We say that a sequence $\{d_n\}$ of linear mappings from $\mathcal{A}$ into $\mathcal{A}$ is a prime higher derivation if $d_n(ab) = \sum_{k|n} d_k(a)d_{\frac{n}{k}}(b)$ for each $a, b \in \mathcal{A}$ and each $n \in \mathbb{N}$.

Lemma 2.2. *Let $\{d_n\}$ be a prime higher derivation on an algebra $\mathcal{A}$. Then, for each $p \in \mathbb{P}$ the sequence $D_n = d_{p^n}$ is a higher derivation on $\mathcal{A}$.*

Proof. We have

$$D_n(ab) = d_{p^n}(ab) = \sum_{k=0}^{n} d_{p^k}(a)d_{p^{n-k}}(b) = \sum_{k=0}^{n} D_k(a)D_{n-k}(b),$$

for each $a, b \in \mathcal{A}$. $\square$

The following lemma guarantees the existence of a notable source of examples of prime higher derivations.

Lemma 2.3. *Let $\mathcal{A}$ be an algebra, $\{d_p\}_{p\in\mathbb{P}}$ be a sequence of derivations on $\mathcal{A}$ and $d_1 = I_{\mathcal{A}}$. For $n \in \mathbb{N}$, define $d_n : \mathcal{A} \to \mathcal{A}$ by $d_n = \frac{1}{e(g(n),n)} d_{\frac{n}{g(n)}} d_{g(n)}$. Then, $\{d_n\}$ is a prime higher derivation.*

Proof. We have to show that $d_n(ab) = \sum_{k|n} d_k(a)d_{\frac{n}{k}}(b)$ for all $a, b \in \mathcal{A}$ and all $n \in \mathbb{N}$. We use strong multiplicative induction. For $n = 1$ and $n \in \mathbb{P}$, the result is obvious. Let the result hold for all proper divisors of n. For $a, b \in \mathcal{A}$ we have

$$\begin{aligned} d_n(ab) &= \frac{1}{e(g(n),n)} d_{\frac{n}{g(n)}} d_{g(n)}(ab) \\ &= \frac{1}{e(g(n),n)} d_{\frac{n}{g(n)}} (d_{g(n)}(a)b + ad_{g(n)}(b)) \\ &= \frac{1}{e(g(n),n)} \sum_{\ell|\frac{n}{g(n)}} [d_\ell d_{g(n)}(a)d_{\frac{n}{\ell g(n)}}(b) + d_\ell(a)d_{\frac{n}{\ell g(n)}} d_{g(n)}(b)] \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{e(g(n),n)}\sum_{\ell|\frac{n}{g(n)}}[e(g(n),\ell g(n))d_{\ell g(n)}(a)d_{\frac{n}{\ell g(n)}}(b)\\
&\quad +e(g(n),\frac{n}{\ell})d_{\ell}(a)d_{\frac{n}{\ell}}(b)]\\
&= \sum_{k|n}d_k(a)d_{\frac{n}{k}}(b). \qquad \square
\end{aligned}$$

Definition 2.4. A prime higher derivation $\{d_n\}$ on an algebra $\mathcal{A}$ is called ordinary if it is of the form specified in Lemma 2.3.

The following result is now obvious.

Proposition 2.5. *A prime higher derivation $\{d_n\}$ on an algebra $\mathcal{A}$ is ordinary if and only if there is a sequence $\{d_p\}_{p\in\mathbb{P}}$ of derivations on $\mathcal{A}$ such that $d_n = \prod_{i=1}^{r}\frac{d_{p_i}^{\alpha_i}}{\alpha_i!}$ for each $n = p_1^{\alpha_1}\cdots p_r^{\alpha_r}$ with $p_1 < \ldots < p_r$.*

Remark 2.6. Let Ψ be a permutation on the prime numbers. Define $\preceq$ on $\mathbb{P}$ by $p \preceq q$ if and only if $\Psi(p) \le \Psi(q)$. Under this order, we assume that $G(n)$ be the greatest prime divisor on n. Then, the proof of Lemma 2.3 is still valid for G instead of g. Note that the only fact used in the proof is that when $\ell|\frac{n}{g(n)}$ we have $g(\ell g(n)) = g(n)$. Using this method, we can find other examples of prime higher derivations.

3. The result

Here, we give a characterization of a prime higher derivation in terms of derivations.

In what follows, we denote the number of not necessarily distinct prime divisors of n by $\Omega(n)$. Note that $\Omega(1) = 0$ and if $k|n$ then $\Omega(n) = \Omega(k) + \Omega(\frac{n}{k})$.

Proposition 3.1. *Let $\mathcal{A}$ be an algebra and $\{d_n\}$ be a prime higher derivation on $\mathcal{A}$ with $d_1 = I_{\mathcal{A}}$. Then, there is a sequence $\{\delta_n\}$ of derivations on $\mathcal{A}$ such that*

$$\Omega(n)d_n = \sum_{k|n,\ k\neq 1}\delta_k d_{\frac{n}{k}},$$

for each $n \geq 2$.

Proof. We use strong multiplicative induction on n. If p is prime, then we put $\delta_p = d_p$. Then, δ_p is a derivation and $\Omega(p)d_p = d_p = \delta_p = \delta_p d_1$.

Now suppose that δ_k is defined and is a derivation for $k|n$ with $k \neq 1, n$. Putting $\delta_n = \Omega(n)d_n - \sum_{k|n,\ k\neq 1,n} \delta_k d_{\frac{n}{k}}$, we show that the well-defined mapping δ_n is a derivation on $\mathcal{A}$. For $a, b \in \mathcal{A}$, we have

$$\begin{aligned}
\delta_n(ab) &= \Omega(n)d_n(ab) - \sum_{k|n,\ k\neq 1,n} \delta_k d_{\frac{n}{k}}(ab) \\
&= \Omega(n)\sum_{k|n} d_k(a)d_{\frac{n}{k}}(b) - \sum_{k|n,\ k\neq 1,n} \delta_k \left(\sum_{\ell|\frac{n}{k}} d_\ell(a)d_{\frac{n}{k\ell}}(b)\right).
\end{aligned}$$

Since the δ_k are derivations, we have

$$\begin{aligned}
\delta_n(ab) &= \sum_{k|n} \Omega(k)d_k(a)d_{\frac{n}{k}}(b) + \sum_{k|n} d_k(a)\Omega(\frac{n}{k})d_{\frac{n}{k}}(b) \\
&\quad - \sum_{k|n,\ k\neq 1,n}\ \sum_{\ell|\frac{n}{k}} \left[\delta_k(d_\ell(a))d_{\frac{n}{k\ell}}(b) + d_\ell(a)\delta_k(d_{\frac{n}{k\ell}}(b))\right].
\end{aligned}$$

Separating the terms $k = 1, n$ from the first and second summation and $\ell = 1, \frac{n}{k}$ from the last one, we have

$$\begin{aligned}
\delta_n(ab) &= \Omega(n)d_n(a)b + a\Omega(n)d_n(b) \\
&\quad - \sum_{k|n,\ k\neq 1,n} \delta_k(d_{\frac{n}{k}}(a))b - \sum_{k|n,\ k\neq 1,n} a\delta_k(d_{\frac{n}{k}}(b)) \\
&\quad + \sum_{k|n,\ k\neq n} \left[\Omega(k)d_k(a) - \sum_{j|k,\ j\neq 1} \delta_j d_{\frac{k}{j}}(a)\right] d_{\frac{n}{k}}(b) \\
&\quad + \sum_{k|n,\ k\neq 1} d_k(a) \left[\Omega(\frac{n}{k})d_{\frac{n}{k}}(b) - \sum_{j|\frac{n}{k},\ j\neq 1} \delta_j d_{\frac{n}{kj}}(b)\right].
\end{aligned}$$

By our assumption,

$$\begin{aligned}
\Omega(k)d_k(a) - \sum_{j|k,\ j\neq 1} \delta_j d_{\frac{k}{j}}(a) = 0, \\
\Omega(\frac{n}{k})d_{\frac{n}{k}}(b) - \sum_{j|\frac{n}{k},\ j\neq 1} \delta_j d_{\frac{n}{kj}}(b) = 0.
\end{aligned}$$

Thus,

$$\begin{aligned}\delta_n(ab) &= \Omega(n)d_n(a)b - \sum_{k|n,\ k\neq 1,n} \delta_k(d_{\frac{n}{k}}(a))b \\ &\quad + a\Omega(n)d_n(b) - \sum_{k|n,\ k\neq 1,n} a\delta_k(d_{\frac{n}{k}}(b)) \\ &= \delta_n(a)b + a\delta_n(b).\end{aligned}$$

hence, δ_n is a derivation on $\mathcal{A}$. □

To illustrate the recursive relation mentioned in Proposition 3.1, let us compute some terms of $\{d_n\}$.

Example 3.2. *Using Proposition 3.1, the first six non-prime terms of $\{d_n\}$ are*

$$\begin{aligned} d_4 &= \frac{1}{2}\delta_2^2 + \frac{1}{2}\delta_4, \\ d_6 &= \frac{1}{2}\delta_2\delta_3 + \frac{1}{2}\delta_3\delta_2 + \frac{1}{2}\delta_6 \\ d_8 &= \frac{1}{6}\delta_2^3 + \frac{1}{6}\delta_2\delta_4 + \frac{1}{3}\delta_8 \\ d_9 &= \frac{1}{2}\delta_3^2 + \frac{1}{2}\delta_9 \\ d_{10} &= \frac{1}{2}\delta_2\delta_5 + \delta_{10} \\ d_{12} &= \frac{1}{6}\delta_2^2\delta_3 + \frac{1}{6}\delta_2\delta_3\delta_2 + \frac{1}{6}\delta_2\delta_6 + \frac{1}{6}\delta_3\delta_2^2 + \frac{1}{6}\delta_3\delta_4 + \frac{1}{3}\delta_4\delta_3 + \frac{1}{3}\delta_6\delta_2 \\ &\quad + \frac{1}{3}\delta_{12} \end{aligned}$$

Theorem 3.3. *Let $\{d_n\}$ be a prime higher derivation on an algebra $\mathcal{A}$ with $d_1 = I_{\mathcal{A}}$. Then, there is a sequence $\{\delta_n\}$ of derivations on $\mathcal{A}$ such that*

$$d_n = \sum_{i=1}^{\Omega(n)} \left(\sum_{n=k_1\cdots k_i} \left(\prod_{j=1}^{i} \frac{1}{\Omega(k_j) + \ldots + \Omega(k_i)} \right) \delta_{k_1} \ldots \delta_{k_i} \right) \quad (n \geq 2),$$

where the inner summation is taken over all representations of n as a multiplication of not necessarily distinct natural numbers greater than 1.

Proof. We show that if d_n is of the above form then it satisfies the recursive relation of Proposition 3.1. Since the solution of the recursive relation is unique, this proves the theorem. Simplifying the notation, we put $a_{k_1,\ldots,k_i} = \prod_{j=1}^{i} \frac{1}{\Omega(k_j)+\ldots+\Omega(k_i)}$. Note that if $k_1 \cdots k_i = n$ then $\Omega(n)a_{k_1,\ldots,k_i} = a_{k_2,\ldots,k_i}$. Moreover, $a_n = \frac{1}{\Omega(n)}$.

Now, we have

$$\begin{aligned}
\Omega(n)d_n &= \sum_{i=2}^{n}\left(\sum_{n=k_1\cdots k_i}\Omega(n)a_{k_1,\ldots,k_i}\delta_{k_1}\ldots\delta_{k_i}\right)+\delta_n\\
&= \sum_{i=2}^{n}\left(\sum_{k_1|n,\ k_1\neq 1,n}\delta_{k_1}\sum_{\frac{n}{k_1}=k_2\cdots k_i}a_{k_2,\ldots,k_i}\delta_{k_2}\ldots\delta_{k_i}\right)+\delta_n\\
&= \sum_{k_1|n,\ k_1\neq 1,n}\delta_{k_1}\sum_{i=2}^{\frac{n}{k_1}}\left(\sum_{\frac{n}{k_1}=k_2\cdots k_i}a_{k_2,\ldots,k_i}\delta_{k_2}\ldots\delta_{k_i}\right)+\delta_n\\
&= \sum_{k_1|n,\ k_1\neq 1,n}\delta_{k_1}d_{\frac{n}{k_1}}+\delta_n\\
&= \sum_{k|n,\ k_1\neq 1}\delta_k d_{\frac{n}{k}}.
\end{aligned}$$

□

Example 3.4. We evaluate the coefficients $a_{k_1,\ldots,k_i}$ for the case $n = 12$. For $n = 12$, we can write

$$12 = 2\cdot 6 = 6\cdot 2 = 3\cdot 4 = 4\cdot 3 = 2\cdot 2\cdot 3 = 2\cdot 3\cdot 2 = 3\cdot 2\cdot 2.$$

By the definition of $a_{k_1,\ldots,k_i}$, we have

$$\begin{aligned}
a_{12} &= \frac{1}{3},\\
a_{2,6} &= \frac{1}{1+2}\cdot\frac{1}{2} = \frac{1}{6},\\
a_{6,2} &= \frac{1}{2+1}\cdot\frac{1}{1} = \frac{1}{3},\\
a_{3,4} &= \frac{1}{1+2}\cdot\frac{1}{2} = \frac{1}{6},\\
a_{4,3} &= \frac{1}{2+1}\cdot\frac{1}{1} = \frac{1}{3},\\
a_{2,2,3} &= a_{2,3,2} = a_{3,2,2} = \frac{1}{1+1+1}\cdot\frac{1}{1+1}\cdot\frac{1}{1} = \frac{1}{6}.
\end{aligned}$$

We can therefore deduce that

$$d_{12} = \frac{1}{3}\delta_{12}+\frac{1}{6}\delta_2\delta_6+\frac{1}{3}\delta_6\delta_2+\frac{1}{6}\delta_3\delta_4+\frac{1}{3}\delta_4\delta_3+\frac{1}{6}\delta_2\delta_2\delta_3+\frac{1}{6}\delta_2\delta_3\delta_2+\frac{1}{6}\delta_3\delta_2\delta_2.$$

Theorem 3.5. *Let $\mathcal{A}$ be an algebra, D be the set of all higher derivations $\{d_n\}$ on $\mathcal{A}$ with $d_1 = I_{\mathcal{A}}$ and Δ be the set of all sequences $\{\delta_n\}$ of derivations on $\mathcal{A}$ with $\delta_1 = 0$. Then, there is a one to one correspondence between D and Δ.*

Proof. Let $\{\delta_n\} \in \Delta$. Define $d_n : \mathcal{A} \to \mathcal{A}$ by $d_1 = I_{\mathcal{A}}$ and

$$d_n = \sum_{i=1}^{\Omega(n)} \left(\sum_{n=k_1\cdots k_i} \left(\prod_{j=1}^{i} \frac{1}{\Omega(k_j)+\ldots+\Omega(k_i)} \right) \delta_{k_1}\ldots\delta_{k_i} \right) \quad (n \geq 2).$$

We show that $\{d_n\} \in D$. By Theorem 3.3, $\{d_n\}$ satisfies the recursive relation

$$\Omega(n)d_n = \sum_{k|n,\ k\neq 1} \delta_k d_{\frac{n}{k}}.$$

To show that $\{d_n\}$ is a prime higher derivation, we use strong multiplicative induction on n. For $n = 1$, we have $d_1(ab) = ab = d_1(a)d_1(b)$ and if p is a prime then $d_p(ab) = \delta_p d_1(ab) = \delta_p(a)b + a\delta_p(b)$. Let us assume that $d_k(ab) = \sum_{i|k} d_i(a)d_{\frac{k}{i}}(b)$ for $k|n$ with $k \neq n$. Thus, we have

$$\begin{aligned}
\Omega(n)d_n(ab) &= \sum_{k|n,\ k\neq 1} \delta_k d_{\frac{n}{k}}(ab) \\
&= \sum_{k|n,\ k\neq 1} \delta_k \sum_{i|\frac{n}{k}} d_i(a)d_{\frac{n}{ki}}(b) \\
&= \sum_{i|n} \left(\sum_{k|\frac{n}{i}} \delta_k d_{\frac{n}{ki}}(a) \right) d_i(b) \\
&\quad + \sum_{i|n} d_i(a) \left(\sum_{k|\frac{n}{i}} \delta_k d_{\frac{n}{ki}}(b) \right).
\end{aligned}$$

Using our assumption, we can write

$$\begin{aligned}\Omega(n)d_n(ab) &= \sum_{i|n}\Omega(\frac{n}{i})d_{\frac{n}{i}}(a)d_i(b)\\ &\quad+\sum_{i|n}d_i(a)\Omega(\frac{n}{i})d_{\frac{n}{i}}(b)\\ &= \sum_{i|n}\Omega(i)d_i(a)d_{\frac{n}{i}}(b)+\sum_{i|n}\Omega(\frac{n}{i})d_i(a)d_{\frac{n}{i}}(b)\\ &= \Omega(n)\sum_{k|n}d_k(a)d_{\frac{n}{k}}(b).\end{aligned}$$

Thus, $\{d_n\} \in D$.

Conversely, suppose that $\{d_n\} \in D$. Define $\delta_n : \mathcal{A} \to \mathcal{A}$ by $\delta_1 = 0$ and

$$\delta_n = \Omega(n)d_n - \sum_{k|n,\ k\neq 1,n} \delta_k d_{\frac{n}{k}}.$$

Then, Proposition 3.1 ensures that $\{\delta_n\} \in \Delta$.

Now, define $\varphi : \Delta \to D$ by $\varphi(\{\delta_n\}) = \{d_n\}$, where,

$$d_n = \sum_{i=1}^{\Omega(n)}\left(\sum_{n=k_1\cdots k_i}\left(\prod_{j=1}^{i}\frac{1}{\Omega(k_j)+\ldots+\Omega(k_i)}\right)\delta_{k_1}\ldots\delta_{k_i}\right).$$

Then, φ is clearly a one to one correspondence. $\square$

Let $\mathcal{A}$ be an algebra and $\{d_n\}$ be a higher derivation on $\mathcal{A}$. If we define $D_n : \mathcal{A} \to \mathcal{A}$ by $D_{2^n} = d_n$ $(n \geq 0)$ and $D_n = 0$ if n is not a power of 2, then $\{D_n\}$ is a prime higher derivation. Evaluating the δ_n of Proposition 3.1 we see that $\delta_n = 0$ if n is not a power of 2. This gives the following result mentioned in [7].

Theorem 3.6. [7, Theorem 2.3] *Let $\{d_n\}$ be a higher derivation on an algebra $\mathcal{A}$ with $d_0 = I$. Then, there is a sequence $\{\delta_n\}$ of derivations on $\mathcal{A}$ such that*

$$d_n = \sum_{i=1}^{n}\left(\sum_{\sum_{j=1}^{i} r_j=n}\left(\prod_{j=1}^{i}\frac{1}{r_j+\ldots+r_i}\right)\delta_{r_1}\ldots\delta_{r_i}\right).$$

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