ERRATUM: TOPOLOGICAL CENTRES OF CERTAIN BANACH MODULE ACTIONS

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For a normed space \( \mathcal{X} \), let \( J_\mathcal{X} : \mathcal{X} \to \mathcal{X}^{**} \) denote the canonical embedding of \( \mathcal{X} \) into \( \mathcal{X}^{**} \), with the second adjoint \((J_\mathcal{X})^{**} : \mathcal{X}^{**} \to \mathcal{X}^{****}\).

We defined \( \mathfrak{M}_\mathcal{X} \) (in Section 3 of the paper [1]) by

\[
\mathfrak{M}_\mathcal{X} = \{ x^{**} \in \mathcal{X}^{**} : J_\mathcal{X}(x^{**}) = (J_\mathcal{X})^{**}(x^{**}) \}.
\]

It is routine to verify that \( \mathfrak{M}_\mathcal{X} \) is a closed subspace of \( \mathcal{X}^{**} \) containing \( \mathcal{X} \). We have claimed that

“it would be desirable to characterize those \( \mathcal{X} \) for which \( \mathfrak{M}_\mathcal{X} = \mathcal{X} \).”

We have earlier surprisingly noticed that this is always true! Indeed: Let \( x^{**} \in \mathfrak{M}_\mathcal{X} \) and let \( \{x_\alpha\} \) be a bounded net in \( \mathcal{X} \) such that \( \{J_\mathcal{X}(x_\alpha)\} \) is \( w^* \)-convergent to \( x^{**} \). Then, for each \( x^{***} \in \mathcal{X}^{***} \),

\[
\lim_\alpha \langle x^{***}, J_\mathcal{X}(x_\alpha) \rangle = \lim_\alpha \langle (J_\mathcal{X})^*(x^{***}), x_\alpha \rangle = \lim_\alpha \langle J_\mathcal{X}(x_\alpha), (J_\mathcal{X})^*(x^{***}) \rangle \\
= \langle x^{**}, (J_\mathcal{X})^*(x^{***}) \rangle = \langle (J_\mathcal{X})^{**}(x^{**}), x^{***} \rangle \\
= \langle J_\mathcal{X}^{**}(x^{**}), x^{***} \rangle \\
= \langle x^{***}, x^{**} \rangle.
\]

Therefore, \( \{J_\mathcal{X}(x_\alpha)\} \) converges weakly to \( x^{**} \). As \( \mathcal{X} \) is a weakly closed subspace of \( \mathcal{X}^{**} \), we get \( x^{**} \in \mathcal{X} \), so that \( \mathfrak{M}_\mathcal{X} = \mathcal{X} \), as claimed.


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By this erratum we acknowledge that Example 3.2 (consequently Corollary 3.3) of the paper, in which we have claimed that $\mathfrak{M}_{c_0} \neq c_0$, is not correct and its proof is flawed. The actual error occurs when we use the decomposition $\ell_\infty^* = c^* \oplus c_0^\perp$. Indeed, the right decomposition is $\ell_\infty^* = c_0^* \oplus c_0^\perp = \ell_1 \oplus c_0^\perp$, which is known and holds even in more general situations. It is now more desirable that the reader replace $\mathfrak{M}_X$ with $X$ throughout the paper.

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References