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ON JORDAN LEFT DERIVATIONS AND GENERALIZED JORDAN LEFT DERIVATIONS OF MATRIX RINGS

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ABSTRACT. Let R be a 2-torsion free ring with identity. In this paper, first we prove that any Jordan left derivation (hence, any left derivation) on the full matrix ring $M_n(R)$ $(n \geq 2)$ is identically zero, and any generalized left derivation on this ring is a right centralizer. Next, we show that if R is also a prime ring and $n \geq 1$, then any Jordan left derivation on the ring $T_n(R)$ of all $n \times n$ upper triangular matrices over R is a left derivation, and any generalized Jordan left derivation on $T_n(R)$ is a generalized left derivation. Moreover, we prove that any generalized left derivation on $T_n(R)$ is decomposed into the sum of a right centralizer and a Jordan left derivation. Some related results are also obtained.

1. **Introduction**

Throughout, R will represent an associative ring with center $Z(R)$. A ring R is *n-torsion free*, where $n > 1$ is an integer, in case $nx = 0, x \in R$ implies $x = 0$. A ring R is *prime* if for $a, b \in R, aRb = 0$ implies that either $a = 0$ or $b = 0$, and is *semiprime* if $aRa = 0$ implies that $a = 0$. An additive mapping $D : R \to R$, with R is an arbitrary ring, is called a *derivation* if $D(xy) = D(x)y + xD(y)$ holds for all pairs $x, y \in R$,

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and is called a *Jordan derivation* in case $D(x^2) = D(x)x + xD(x)$ is fulfilled for all $x \in R$. Obviously, any derivation is a Jordan derivation. The converse is ,in general not true. A classical result of Herstein [14] asserts that any Jordan derivation on a prime ring of characteristic different from two is a derivation. A brief proof of Herstein's result can be found in [8]. Cusack [10] generalized Herstion's theorem to 2-torsion free semiprime rings (see [6] for an alternative proof). It should be mentioned that Beidar, Brešar, Chebotar and Martidale [3] fairly generalized Herstein's theorem. Let R be a ring and let M be a left R-module. An additive mapping $D : R \to M$ is said to be a *left derivation* if $D(xy) = xD(y) + yD(x)$ holds for all pairs $x, y \in R$, and is said to be a *Jordan left derivation* (or *left Jordan derivation*) if $D(x^2) = 2xD(x)$ is fulfilled for all $x \in R$. Obviously, any left derivation is a Jordan left derivation, but in general the converse is not true (see [25], Example 1.1). The concepts of left derivation and Jordan left derivation were introduced by Brešar and Vukman in [9]. One can easily prove that the existence of a nonzero left derivation $D : R \to R$, where R is a prime ring of characteristic different from two, forces the ring R to be commutative. Moreover, any Jordan derivation, which maps a noncommutative prime ring R of characteristic different from two into itself, is zero. This result was first proved by Brešar and Vukman in [9] under the additional assumption that R is also of characteristic different from three. Later on, Deng [11] removed the assumption that R is of characteristic different from three. (See also [17].) Recently, Vukman [21] has proved that in case $D : R \to R$ is a Jordan left derivation, where R is a 2-torsion free semiprime ring, then D is a derivation which maps R into $Z(R)$. For results concerning Jordan left derivations we refer the readers to [9, 11, 15, 16, 17, 18, 19, 21]. An additive mapping $T : R \to R$, where R is an arbitrary ring, is called a *left centralizer* in case $T(xy) = T(x)y$ holds for all pairs $x, y \in R$. In case R has an identity element, $T : R \to R$ is a left centralizer iff T is of the form $T(x) = ax$ for all $x \in R$ and some fixed element $a \in R$. An additive mapping $T : R \to R$ is called a *left Jordan centralizer* in case $T(x^2) = T(x)x$ holds for all $x \in R$. The definitions of *right centralizer* and *right Jordan centralizer* should be self explanatory. An additive mapping $T : R \to R$ is called a *two-sided centralizer* in case T is a left and a right centralizer. Following ideas from [6], Zalar [26] has proved that any left (right) Jordan centralizer on a semiprime ring is a left (right) centralizer. Vukman [18] has proved that if there exists an additive mapping $T : R \to R$, where R is a 2-torsion free semiprime ring, satisfying the relation $2T(x^2) = T(x)x + xT(x)$ for all $x \in R$, then T is a two-sided centralizer. For results concerning centralizers the readers are referred to [4, 5, 12, 18, 22, 23, 24] for more references. An additive mapping F, which maps a ring R into itself, is called a *generalized derivation* in case $F(xy) = F(x)y + xD(y)$ holds for all pairs $x, y \in R$, where $D : R \to R$ is a derivation. Clearly, any generalized derivation is a generalized Jordan derivation, but the converse is not necessarily true. The concept of generalized derivation, which has been introduced by Brešar [7], covers two concepts: the concept of derivation and the concept of left centralizer. Indeed, it is easy to see that generalized derivations are exactly those additive mappings F which can be written in the form $F = D + T$, where D is a derivation and T is a left centralizer (see also Theorems 2.2 and 2.8 below). An additive mapping $F : R \to R$ is called a *generalized Jordan derivation* in case $F(x^2) = F(x)x + xD(x)$ holds for all $x \in R$, where $D : R \to R$ is a Jordan derivation. The concept of generalized Jordan derivation has been introduced by Jing and Lu [16]. They conjectured that any generalized Jordan derivation, which maps a 2-torsion free semiprime ring into itself, is a generalized derivation. This conjecture was proved by Vukman [19]. Let M be a left R-module. An additive mapping $G : R \to M$ is said to be a *generalized left derivation* (resp. *generalized Jordan left derivation*) if there exists a Jordan left derivation $D : R \to M$ such that $G(xy) = xG(y) + yD(x)$ (resp. $G(x^2) = xG(x) + xD(x)$) for all x, y in R. Obviously, any generalized left derivation is a generalized Jordan left derivation, but the converse may not hold in general (see Example 1.1 in [1]).

The main result of this article are as follows. First, we prove that if R is a 2-torsion free ring with identity, then any Jordan left derivation (hence, any left derivation) on the full matrix ring $M_n(R)$ ($n \geq 2$) is identically zero, and any generalized left derivation on this ring is a right centralizer (Theorem 2.1). Next, motivated by a result of M. Ashraf and S. Ali [1], which states that every generalized Jordan left derivation on a prime ring R whose characteristic is different from two, is a generalized left derivation, we prove that any Jordan left derivation on the ring $T_n(R)$ ($n \geq 1$) of all $n \times n$ upper triangular matrices over R is a left derivation (Theorem 2.8), and that any generalized Jordan left derivation on $T_n(R)$ is a generalized left derivation. Moreover, we show that any generalized left derivation of $T_n(R)$ is the sum of a right centralizer and a left derivation (Theorem 2.8). Some other related results are also established.

As usual, I denotes the identity matrix, and E_{ij} denotes the usual matrix unit. Moreover, the zero elements of the rings and modules, zero subrings, and zero submodules are all denoted by 0. Recall that $E_{ij}E_{rs} = \delta_{jr}E_{is}$, where δ is the Kronecker function.

2. Main results and proofs

Theorem 2.1. *Let R be a 2-torsion free ring with identity and let $n \geq 2$. Then*

(i) any Jordan left derivation (hence, any left derivation) D on the ring $M_n(R)$ is identically zero;

(ii) any generalized left derivation on $M_n(R)$ is a right centralizer.

Proof. (i) Linearizing $D(x^2) = 2xD(x)$ and noting that $M_n(R)$ is 2-torsion free, we arrive at an equivalent expression for D which will be used frequently:

(2.1) $D(xy + yx) = 2(xD(y) + yD(x))$ for all $x, y \in M_n(R)$.

Set $N = \{1, \cdots, n\}$. It is easy to observe that for any (a_{rs}) in $M_n(R)$ and $i \in N$, the following conclusion holds:

(2.2) $$\text{if } (a_{rs}) = 2E_{ii}(a_{rs}), \text{ then } (a_{rs}) = 0.$$

Fix $i \in N$. From $E_{ii}^2 = E_{ii}$ we get $D(E_{ii}) = 2E_{ii}D(E_{ii})$, whence, by (2.2), we have

(2.3) $$D(E_{ii}) = 0 \quad \text{for all} \quad 1 \leq i \leq n.$$

Now fix $i \neq j$ in N. From $E_{ij} = E_{ij}E_{jj} + E_{jj}E_{ij}$, (2.1) and (2.3) we obtain

$$D(E_{ij}) = 2(E_{ij}D(E_{jj}) + E_{jj}D(E_{ij})) = 2E_{jj}D(E_{ij}).$$

Thus, by (2.2), $D(E_{ij}) = 0$. Combining the latter result with (2.3), we conclude that

(2.4) $$(E_{ij}) = 0 \quad \text{for all} \quad 0 \leq i, j \leq n.$$

Next, we show that

(2.5) $$\text{for all } r \in R \text{ and } i \neq j \text{ in } N, D(rE_{ij}) = 0.$$

To do this, let r be in R and fix $i \neq j$ in N. Then from $rE_{ij} = (rE_{ij})E_{jj} + E_{jj}(rE_{ij})$, (2.1) and (2.3) we obtain

$$D(rE_{ij}) = 2((rE_{ij})D(E_{jj}) + E_{jj}D(rE_{ij})) = 2E_{jj}D(rE_{ij}),$$

so, by (2.2), (2.5) holds.

In the next step we show that for any $r \in R$ and $i \in N, D(rE_{ii}) = 0$: Fix $i \neq j$ in N and set $E = E_{ii} + E_{jj}$. In view of (2.1), (2.4) and (2.5), we have

$$\begin{aligned} D(rE) &= D(rE_{ii} + rE_{jj}) \\ &= D((rE_{ij})E_{ji} + E_{ji}(rE_{ij})) \\ &= 2((rE_{jj})D(E_{ji}) + E_{ji}D(rE_{ij})) \\ &= 0. \end{aligned}$$

Therefore, from $2rE_{ii} = 2rEE_{ii} = (rE)E_{ii} + E_{ii}(rE)$ and (2.3) we find that

$$2D(rE_{ii}) = (rE)D(E_{ii}) + E_{ii}D(rE) = 0,$$

so that $D(rE_{ii}) = 0$. The latter conclusion together with (2.5) and additivity of D complete the proof of (i).

(ii) Since, by (i), any left derivation on $M_n(R)$ is zero, any generalized left derivation G on this ring satisfies $G(xy) = xG(y)$ for all x, y in $M_n(R)$. Therefore, setting $G(I) = a$, we have $G(x) = xa$ for all x in $M_n(R)$. □

Let R and S be 2-torsion free rings with identity, M be a 2-torsion free (R, S)-bimodule, and T be the upper triangular matrix ring $\begin{pmatrix} R & M \\ 0 & S \end{pmatrix}$ with the usual addition and multiplication of matrices. The following theorem describes the structure of Jordan left derivations of T.

Theorem 2.2. *Let the ring T be as above, and let $D : T \to T$ be a Jordan left derivation. Then there exist Jordan left derivations*

$$\delta : R \to R, \quad \lambda : R \to M, \quad \gamma : S \to S$$

such that $M\gamma(S) = 0$, and for every $\begin{pmatrix} r & m \\ 0 & s \end{pmatrix}$ in T,

$$D\begin{pmatrix} r & m \\ 0 & s \end{pmatrix} = \begin{pmatrix} \delta(r) & \lambda(r) \\ 0 & \gamma(s) \end{pmatrix}.$$

Proof. Linearizing $D(x^2) = 2xD(x)$ and noting that T is 2-torsion free, we arrive at an equivalent expression for D which will be used frequently:

(2.6) $\quad D(xy + yx) = 2(xD(y) + yD(x)) \quad \text{for all} \quad x, y \in T.$

Applying D on $I^2 = I$ and $E_{ii}^2 = E_{ii}$ $(i = 1, 2)$, it is easily observed that

(2.7) $\quad D(E_{11}) = D(E_{22}) = D(I) = 0.$

Let m be in M. From $mE_{12} = E_{11}(mE_{12}) + (mE_{12})E_{11}$, (2.6) and (2.7) we find that

$$D(mE_{12}) = 0 \quad \text{for all} \quad m \in M. \tag{2.8}$$

Now, let s be in S and suppose $D(sE_{22}) = (a_{ij}) \in T$. Applying D on both sides of $2sE_{22} = (sE_{22})E_{22} + E_{22}(sE_{22})$ and using (2.7), we conclude that $2a_{11} = 2a_{12} = 0$, so that $a_{11} = a_{12} = 0$. Therefore, D induces a mapping $\gamma : S \to S$ such that

$$d(sE_{22}) = \gamma(s)E_{22} \quad \text{for all} \quad s \in S. \tag{2.9}$$

Since D is additive, so is γ. Applying D on $s^2E_{22} = (sE_{22})^2$, we observe that $\gamma(s^2) = 2s\gamma(s)$ for all $s \in S$, proving that γ is a Jordan left derivation on S.

Next, let $r \in R$ and assume that $D(rE_{11}) = (b_{ij}) \in T$. Then from $2rE_{11} = (rE_{11})E_{11} + E_{11}(rE_{11})$, (2.6), (2.7) and using the torsion assumption on S, we see that $b_{22} = 0$, whence D induces the mappings $\delta : R \to R$ and $\lambda : R \to M$ such that

$$D(rE_{11}) = \delta(r)E_{11} + \lambda(r)E_{12} \quad \text{for all} \quad r \in R. \tag{2.10}$$

By a similar argument as above, one can show that δ and λ are also Jordan left derivations . Now, in view of (2.8), (2.9) and (2.10), for every $\begin{pmatrix} r & m \\ 0 & s \end{pmatrix}$ in T we have

$$\begin{aligned} D\begin{pmatrix} r & m \\ 0 & s \end{pmatrix} &= D(rE_{11}) + D(mE_{12}) + D(sE_{22}) \\ &= \delta(r)E_{11} + \lambda(r)E_{11} + \gamma(s)E_{22} \\ &= \begin{pmatrix} \delta(r) & \lambda(r) \\ 0 & \gamma(s) \end{pmatrix}. \end{aligned}$$

Finally, to prove that $M\gamma(S) = 0$, let $m \in M$ and $s \in S$ be arbitrary. Then, in view of (2.8) and (2.9), applying D on both sides of $(ms)E_{12} = (mE_{12})(sE_{22}) + (sE_{22})(mE_{12})$, we obtain

$$\begin{aligned} 0 &= 2((mE_{12})D(sE_{22}) + (sE_{22})D(mE_{22})) \\ &= 2(mE_{12})(\gamma(s)E_{22}) \\ &= 2(m\gamma(s))E_{12}, \end{aligned}$$

so that, by the torsion assumption on M, $m\gamma(s) = 0$. $\square$

The following corollary is immediate:

Corollary 2.3. *Let T and D be as above and assume that M is a faithful right S-module. Then $\gamma = 0$.*

Our next goal is to describe Jordan left derivations of $T_n(R)$. To do this, the following lemma is needed.

Lemma 2.4. *Let R be any ring, $n \geq 1$, and let $\delta : R \to R^n$ be a Jordan left derivation. Then, considering R^n as a left R-module, there exist Jordan left derivations $\delta_1, \cdots, \delta_n : R \to R$ such that*

$$\delta(r) = (\delta_1(r), \cdots, \delta_n(r)) \quad \textit{for all} \quad r \in R.$$

Proof. Obviously, δ determines additive mappings $\delta_i : R \to R, 1 \leq i \leq n$, such that for every r in $R, \delta(r) = (\delta_1(r), \cdots, \delta_n(r))$. Now, we have

$$\begin{aligned}(\delta_1(r^2), \cdots, \delta_n(r^2)) &= \delta(r^2) = 2r\delta(r) \\ &= 2r(\delta_1(r), \cdots, \delta_n(r)) \\ &= (2r\delta_1(r), \cdots, 2r\delta_n(r)).\end{aligned}$$

□

In [13], the author has proved that if R is a 2-torsion free ring with identity, $n \geq 2$, and D is a Jordan derivation on $T_n(R)$, then D is a derivation. The following theorem together with the example given below show however that the situation for the case when D is a Jordan left derivation is not much the same.

Theorem 2.5. *Let R be a ring with identity, $n \geq 1$, and assume that D is a Jordan left derivation on $T_n(R)$. Then there exist Jordan left derivations $\delta_i : R \to R, 1 \leq i \leq n$, such that*

$$D(a_{ij}) = \sum_{j=1}^{n} \delta_j(a_{11})E_{1j} \quad \textit{for all} \quad (a_{ij}) \in T_n(R).$$

In particular, if R is a prime ring of characteristic not 2, then D is a left derivation.

Proof. By [1], for $n = 1$ there is nothing to prove. So, let $n \geq 2$. Then we have the obvious ring isomorphism

$$T_n(R) \cong \begin{pmatrix} R & R^{n-1} \\ 0 & T_{n-1}(R) \end{pmatrix},$$

where R^{n-1} is considered as an $(R, T_{n-1}(R))$-bimodule with the obvious scaler multiplications. Since R^{n-1} is a faithful right $T_{n-1}(R)$-module, in view of Theorem 2.2, Corollary 2.3, and upon identifying the matrix rings above, there exist Jordan left derivations $\delta : R \to R$ and $\lambda : R \to R^{n-1}$ such that for every $(a_{ij}) \in T_n(R), D(a_{ij}) = \delta(a_{11})E_{11} + \lambda(a_{11})E_{12}$. By Lemma 2.4, λ decomposes into a product of $n-1$ Jordan left derivations $\lambda_1, \cdots, \lambda_{n-1}$ on R. Now, set $\delta_1 = \delta$ and $\delta_j = \lambda_{j-1}$ for all $j = 2, \cdots, n$.

For the special case when R is prime and char$R \neq 2$, note that, by Theorem 3.2 in [1] , each δ_i (hence D) is a left derivation. □

Remark 2.6. Let R be a prime ring of characteristic not equal to 2 and assume that the ring $T_n(R)$ admits a nonzero Jordan left derivation. Then the theorem above and Corollary 3.2 in [1] imply that R is commutative.

Example 2.7. Let R be a ring and assume that R admits a Jordan left derivation δ that is not a left derivation (see Example 1.1 in [25]), and let $n \geq 2$. Then it can be easily verified that the mapping $D : T_n(R) \to T_n(R)$ given by

$$D(a_{ij}) = \sum_{j=1}^{n} \delta(a_{11})E_{1j} \quad \text{for all} \quad (a_{ij}) \in T_n(R)$$

is a Jordan left derivation that is not a left derivation.

Now we are ready to prove our last result:

Theorem 2.8. *Let R be a prime ring of characteristic not 2, D be a Jordan left derivation on $T_n(R)$ $(n \geq 1)$, and let G be a generalized Jordan left derivation on $T_n(R)$ associated with D. Then G is a generalized left derivation and there exists a (unique) right centralizer F on $T_n(R)$ such that $G = F + D$.*

Proof. Note that by Theorem 2.5, D is a left derivation. Linearizing $G(x^2) = xG(x) + xD(x)$, we find that

$$G(xy + yx) = xG(y) + yG(x) + xD(y) + yD(x) \tag{2.11}$$

for all $x, y \in T_n(R)$. Put $a = G(I)$. So, from (2.11) and the fact that $D(I) = 0$, it follows that, for each x in $T_n(R)$, we have

$$\begin{aligned} 2G(x) &= G(2x) = G(Ix + xI) \\ &= IG(x) + xG(I) + ID(x) + xD(I) \\ &= G(x) + xa + D(x). \end{aligned}$$

That is,

$$G(x) = xa + D(x) \quad \text{for all} \quad x \in T_n(R). \tag{2.12}$$

Therefore noting that D is a left derivation, for every $x, y \in T_n(R)$, we have

$$\begin{aligned} G(xy) &= (xy)a + D(xy) \\ &= x(ya) + xD(y) + yD(x) \\ &= x(ya + D(y)) + yD(x) \\ &= xG(y) + yD(x). \end{aligned}$$

Thus G is a generalized left derivation associated with D. Now, (2.12) shows that $G = F + D$, where F is the right centralizer induced by the matrix $a = G(I)$. The uniqueness of F is evident. □

Remark 2.9. Although any left derivation D on any ring R is a generalized left derivation (associated with D itself), the proof of the theorem above shows that in general the converse is not true : simply let D be a left derivation on $T_n(R)$, and let a be a nonzero matrix in $T_n(R)$. Then the mapping

$$G : T_n(R) \to T_n(R), \quad x \mapsto xa + D(x) \quad (x \in T_n(R))$$

is a generalized left derivation (associated with D) for which $G(I) = a \neq 0$, whence G is not a left derivation.

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