

n -CYCLICIZER GROUPS

L. MOUSAVI

Communicated by Jamshid Moori

ABSTRACT. The cyclicizer of an element x of a group G is defined as $Cyc_G(x) = \{y \in G \mid \langle x, y \rangle \text{ is cyclic}\}$. Here, we introduce an n -cyclicizer group and show that there is no finite n -cyclicizer group for $n = 2, 3$. We prove that for any positive integer $n \neq 2, 3$, there exists a finite n -cyclicizer group and determine the structure of finite 4 and 6-cyclicizer groups. Also, we characterize finite 5, 7 and 8-cyclicizer groups.

1. Introduction

Let G be a group. We know that the centralizer of an element $x \in G$ is defined as follows:

$$C_G(x) = \{y \in G \mid \langle x, y \rangle \text{ is abelian}\}.$$

If, in this definition, we replace the word abelian by the word cyclic, we get a subset of the centralizer of x . This subset is called the cyclicizer of x in G and it is denoted by $Cyc_G(x)$ [9, 10]. Thus,

$$Cyc_G(x) = \{y \in G \mid \langle x, y \rangle \text{ is cyclic}\}.$$

MSC(2010): Primary: 20D60; Secondary: 20D99.

Keywords: Centralizer, cyclicizer, n -cyclicizer group.

Received: 30 July 2009, Accepted: 21 December 2009.

*Corresponding author

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Also, $Cyc(G)$, the cyclicizer of G , is defined as follows:

$$\begin{aligned} Cyc(G) &= \{y \in G \mid \langle x, y \rangle \text{ is cyclic for all } x \in G\} \\ &= \bigcap_{x \in G} Cyc_G(x). \end{aligned}$$

In general, for an element x of a group G , $Cyc_G(x)$ is not a subgroup of G . For example, in the group $G = \mathbb{Z}_2 \oplus \mathbb{Z}_4$, we have

$$Cyc_G((0, 2)) = \{(0, 0), (0, 1), (0, 2), (0, 3), (1, 1), (1, 3)\},$$

which is not a subgroup of G .

In [1] and [2], the cyclicizers of a group are studied via a graph which is called the non-cyclic graph of the group.

For any non-cyclic group G , $Cyc(G)$ is a subgroup, central, cyclic, normal and contained in all maximal cyclic subgroups of G . It is clear that for a nontrivial element x of G , $|Cyc_G(x)| \geq 2$ and $G = \bigcup_{1 \neq x \in G} Cyc_G(x)$. Also, for any group G and $x \in G$, if $\bar{G} = G/Cyc(G)$, then $Cyc_{\bar{G}}(xCyc(G)) = Cyc_G(x)/Cyc(G)$ [2] and it easily follows that $Cyc(\bar{G}) = 1$ (see also [10]).

For a finite group G , let $\text{Cent}(G)$ denote the set of the centralizers of single elements of G . G is called an n -centralizer group if $|\text{Cent}(G)| = n$. We know that there is no n -centralizer group for $n = 2, 3$. Let $Z(G)$ denote the center of a group G . Then, $|\text{Cent}(G)| = 4$ if and only if $G/Z(G) \cong C_2 \times C_2$ and $|\text{Cent}(G)| = 5$ if and only if $G/Z(G) \cong C_3 \times C_3$ or S_3 [7], where C_2 is a cyclic group of size two and S_3 is a symmetric group on three letters .

Moreover, if $|\text{Cent}(G)| = 6$, then $G/Z(G)$ is isomorphic to one of the groups $(C_2)^3$, $(C_2)^4$, A_4 or D_8 [6], where A_4 is an alternating group on four letters and D_8 is a dihedral group of size eight.

Also, $|\text{Cent}(G)| = 7$ if and only if $G/Z(G)$ is isomorphic to one of the groups $C_5 \times C_5$, D_{10} or $\langle x, y \mid x^5 = y^4 = 1, y^{-1}xy = x^3 \rangle$ and if $|\text{Cent}(G)| = 8$, then $G/Z(G)$ is isomorphic to one of the groups D_{12} , $(C_2)^3$ or A_4 [5].

Similarly, we can define an n -cyclicizer group, where n is a positive integer.

Definition 1.1. For a positive integer n , we say that G is an n -cyclicizer group if $|\{Cyc_G(x) \mid x \in G\}| = n$ and in this case, we write $Cycl(G) = n$.

It is obvious that G is a 1-cyclicizer group if and only if G is cyclic. Here, we show that there is no finite n -cyclicizer group for $n = 2, 3$ and prove that for any positive integer $n \neq 2, 3$, there exists a finite group

G such that $Cycl(G) = n$. We also study finite n -cyclicizer groups for $n = 4, 5, 6, 7$ and 8 .

2. n -Cyclicizer Groups for $n = 4, 5, 6, 7$ and 8

The following theorem is proved in [2].

Theorem 2.1. *Let G be a finite non-cyclic group. Then, $|G/Cyc(G)| \leq \max\{(s - 1)^2(s - 3)!, (s - 2)^3(s - 3)!\}$, where s is the number of maximal cyclic subgroups of G .*

It is clear that if G has n maximal cyclic subgroups, then $Cycl(G) \geq n$.

Lemma 2.2. *Let G be a finite non-cyclic group such that $Cycl(G) = n$. Then, G has at most $n - 1$ maximal cyclic subgroups.*

Proof. Assume that $\langle x \rangle$ is a maximal cyclic subgroup of G . Then, $Cyc_G(x) = \langle x \rangle$. Let $Cycl(G) = n$, and $\langle x_1 \rangle, \langle x_2 \rangle, \dots, \langle x_r \rangle$ be distinct maximal cyclic subgroups of G . Since for any i , $1 \leq i \leq r$, $Cyc_G(x_i) = \langle x_i \rangle$, then $r \leq n$. It is clear that $r \neq n$, since $Cyc_G(1) = G$. This completes the proof. \square

Lemma 2.3. *Let G be a finite group. Then, $Cycl(\bar{G}) = n$ if and only if $Cycl(G) = n$.*

Proof. Let $Cycl(\bar{G}) = n$ and $C = Cyc(G)$. The key point of our proof is that $Cyc_G(x) \rightarrow Cyc_{\bar{G}}(\bar{x})$ is a one-to-one correspondence between the set of cyclicizers of G and those of \bar{G} (induced by the natural homomorphism $G \rightarrow \bar{G} = G/C$). For an element x of G , $\bar{X} = Cyc_G(x)/C$ and $\bar{x} = xC$. We know that $Cyc_{\bar{G}}(\bar{x}) = \bar{X}$. Assume that $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n$ be distinct cyclicizers of $\bar{x}_1 = C, \bar{x}_2, \dots, \bar{x}_n$, respectively. It is clear that $Cycl(G) \geq n$. Without loss of generality, we can assume that X_1, X_2, \dots, X_n are distinct cyclicizers of x_1, x_2, \dots, x_n , respectively. Suppose that $Y = Cyc_G(g)$ is different from X_i , for any i , $1 \leq i \leq n$. Then, $\bar{Y} = \bar{X}_i$, for some i , $1 \leq i \leq n$. Thus, $Cyc_G(g)C = Cyc_G(x_i)C$. Therefore, for any $h_i \in Cyc_G(g)$, there exist c_i and $z_i \in C$ such that $h_i c_i = k_i z_i$, where $k_i \in Cyc_G(x_i)$, and so $h_i = k_i c_i$, for some $c_i \in C$. Since $k_i \in Cyc_G(x_i)$, it is not hard to see that $\langle h_i, x_i \rangle$ is a cyclic group. Hence, $Cyc_G(g) \subseteq Cyc_G(x_i)$. Similarly, $Cyc_G(x_i) \subseteq Cyc_G(g)$. Thus,

$Cyc_G(g) = Cyc_G(x_i)$. This contradiction indicates $Cycl(G) = n$. The converse is clear. \square

Lemma 2.4. *Let $n \geq 2$ be an integer, and $Q_{4n} = \langle x, y | x^{2n} = 1, x^n = y^2, y^{-1}xy = x^{-1} \rangle$. Then, $Cycl(Q_{4n}) = n + 2$.*

Proof. The set of all members of Q_{4n} is $\{1, x^j, x^i y, y | 1 \leq j, i \leq 2n - 1\}$. It is straightforward to check that

(i) for any i , $0 \leq i \leq n - 1$, $Cyc_{Q_{4n}}(x^i y) = \{1, y^2, x^i y, x^{n+i} y\}$;

(ii) $Cyc_{Q_{4n}}(x) = \langle x \rangle$;

and

(iii) for any i , $0 \leq i \leq n - 1$, $Cyc_{Q_{4n}}(x^i y) = Cyc_{Q_{4n}}(x^{n+i} y)$.

Therefore, $Cycl(Q_{4n}) = n + 2$. \square

Corollary 2.5. *Let $n > 1$ be an integer. Then, $Cycl(D_{2n}) = n + 2$.*

Proof. It is well known that $Z(Q_{4n}) = \langle y^2 \rangle$, and we can see that $Z(Q_{4n}) = Cyc(Q_{4n})$ and $Q_{4n}/Z(Q_{4n}) = Q_{4n}/Cyc(Q_{4n}) \cong D_{2n}$, and so the proof follows from Lemma 2.3. \square

Corollary 2.6. *Let $n > 3$ be an integer. Then, there exists a group G with $Cycl(G) = n$.*

Theorem 2.7. *There is no finite n -cyclicizer group for $n = 2, 3$.*

Proof. First, note that there is no cyclic n -cyclicizer group for $n = 2, 3$. Assume G is a finite group such that $Cycl(G) = 2$. Now since the only proper cyclicizer of G is cyclic and G is covered by its all proper cyclicizers, it follows that G is cyclic, which is a contradiction.

Now, suppose for a contradiction that $Cycl(G) = 3$. Assume that $G = Cyc_G(x) \cup Cyc_G(y)$, where $Cyc_G(x)$ and $Cyc_G(y)$ are two distinct cyclicizers of G . By Lemma 2.2, G has at most two maximal cyclic subgroups. If G has exactly two maximal cyclic subgroups, then, without loss of generality, $G = \langle x \rangle \cup \langle y \rangle$, which is a contradiction. Thus, G has only one maximal cyclic subgroup. This means that G is a cyclic group. This contradiction completes the proof. \square

Remark 2.8. Let p be a prime number and $m \in \mathbb{N}$. Then, $\frac{p^m-1}{p-1}$ is the number of subgroups of order p in $(C_p)^m$.

Theorem 2.9. Let p be a prime number and let G be a finite group such that $G/Cycl(G) \cong C_p \times C_p$. Then, $Cycl(G) = p + 2$.

Proof. Let $Cycl(C_p \times C_p) = r$. By Remark 2.8, $C_p \times C_p$ has $p + 1$ maximal cyclic subgroups, and so $r \leq p + 1$. Let $\langle x_1 \rangle, \langle x_2 \rangle, \dots, \langle x_{p+1} \rangle$ be maximal cyclic subgroups of $H = C_p \times C_p$. If $Y = Cycl_H(y) \neq H$ is different from $\langle x_i \rangle$, for any i , $1 \leq i \leq p + 1$, then there exists j , $1 \leq j \leq p + 1$, such that $y \in \langle x_j \rangle$. Therefore, $\langle x_j \rangle = \langle y \rangle \subseteq Y$. Let g be an arbitrary element in Y . Then, for some integer k , $1 \leq k \leq p + 1$, $\langle g, y \rangle = \langle x_k \rangle$. Thus, $y \in \langle x_j \rangle \cap \langle x_k \rangle$. If $j \neq k$, then $y = 1$, and so $Y = H$. This is a contradiction. Therefore, $j = k$. This implies that $Y = \langle x_j \rangle$. Now, Lemma 2.3 completes the proof. \square

Corollary 2.10. Let p be a prime number. Then, $Cycl(C_p \times C_p) = 1$.

Proof. By Lemma 2.9, we have that $C_p \times C_p$ has $p + 1$ proper cyclicizers. Let $Cycl_{C_p \times C_p}(x) = \langle x \rangle$ and $Cycl_{C_p \times C_p}(y) = \langle y \rangle$ be two distinct proper cyclicizers of $C_p \times C_p$. If $\langle x \rangle \cap \langle y \rangle \neq 1$, then $|\langle x \rangle \cap \langle y \rangle| = |\langle x \rangle| = p$. Since $\langle x \rangle \cap \langle y \rangle \leq \langle x \rangle$, then $\langle x \rangle \cap \langle y \rangle = \langle x \rangle$. Therefore, $\langle x \rangle = \langle y \rangle$. This contradiction shows that $\langle x \rangle \cap \langle y \rangle = 1$ and the proof is complete. \square

Lemma 2.11. Let G be a finite p -group, for some prime number p . Then, $Cycl(G) \neq 1$ if and only if G is either a cyclic group or a generalized quaternion group. In this case, $Cycl(G) = Z(G)$.

Proof. It follows from Proposition 2.2 of [2]. \square

Lemma 2.12. Let G and H be finite groups such that $(|G|, |H|) = 1$. Then, $Cycl(G \times H) = Cycl(G) \times Cycl(H)$.

Proof. Let $(a, b) \in Cycl(G \times H)$. Then, for any $(g, h) \in G \times H$, there exists $(x, y) \in G \times H$ such that $\langle (g, h), (a, b) \rangle = \langle (x, y) \rangle$. Therefore, $\langle (g, a) \rangle \leq \langle x \rangle$. So $a \in Cycl(G)$. Similarly, $b \in Cycl(H)$. Thus, $Cycl(G \times H) \subseteq Cycl(G) \times Cycl(H)$.

Now, let $(a, b) \in Cycl(G) \times Cycl(H)$. Then, for any $g \in G$, $\langle g, a \rangle$ is

a cyclic group. Also, for any $h \in H$, $\langle b, h \rangle$ is a cyclic group. Since $\langle (g, h), (a, b) \rangle \leq \langle g, a \rangle \times \langle h, b \rangle$ and $(|H|, |G|) = 1$, then $(a, b) \in Cyc(G \times H)$. Thus, $Cyc(G) \times Cyc(H) \subseteq Cyc(G \times H)$. \square

Lemma 2.13. (i) Let p be a prime number and n be an integer such that $(n, p) = 1$. If $G = C_{pn} \times C_p$, then $G/Cyc(G) \cong C_p \times C_p$.

(ii) Let n be an odd positive integer. If $G = C_n \times Q_8$, then $G/Cyc(G) \cong C_2 \times C_2$.

Proof. (i) Let $H = C_p \times C_p$ and $K = C_n$. Since $(|H|, |K|) = 1$, then $|Cyc(G)| = n$. Thus, $|G/Cyc(G)| = p^2$. If $G/Cyc(G)$ is a cyclic group, then G is also a cyclic group, which is a contradiction. Thus, $G/Cyc(G) \cong C_p \times C_p$.

(ii) Any Sylow subgroup of G is either a cyclic group or a generalized quaternion group, and so by Lemma 2.11, $Cyc(G) = Z(G)$. We have $|G/Cyc(G)| = |G/Z(G)| = 4$ and $G/Cyc(G)$ is not a cyclic group, since G is not a cyclic group. Therefore, $G/Cyc(G) \cong C_2 \times C_2$. \square

Lemma 2.14. Let p be a prime number and let G be a finite group such that $G/Cyc(G) \cong C_p \times C_p$. Then, G is not a cyclic group and

(i) if $p = 2$, then G is isomorphic to either $C_{2n} \times C_2$ or $C_n \times Q_8$, where n is an odd positive integer;

and

(ii) if $p \neq 2$, then $G \cong C_{pn} \times C_p$, where n is an integer such that $(p, n) = 1$.

Proof. If G is a cyclic group, then $|G/Cyc(G)| = 1$, which is a contradiction. If $G/Cyc(G) \cong C_p \times C_p$, then $G/Cyc(G)$ is an abelian group. Since $G/Z(G) \cong \frac{G/Cyc(G)}{Z(G)/Cyc(G)}$, then G is a nilpotent group. Thus, $G = Syl_2 \times Syl_3 \times \cdots \times Syl_p \times \cdots$.

Since $|G/Cyc(G)| = p^2$, then $Cyc(G)$ contains $C = \widehat{Syl}_p$ (\widehat{Syl}_p is the product of all Sylow subgroups of G , except Syl_p). So, C is a cyclic group of size n such that $(p, n) = 1$. Thus, $|Cyc(G)| = p^m \times n$.

If $Cyc(G) \cap Syl_p = \langle 1 \rangle$, then $|Syl_p| = |G/Cyc(G)| = p^2$. If Syl_p is a cyclic group, then G is a cyclic group, which is a contradiction. Thus, $Syl_p \cong C_p \times C_p$. So $G \cong C_{pn} \times C_p$.

If $Cyc(G) \cap Syl_p \neq \langle 1 \rangle$, since $Cyc(G) \cap Syl_p \leq Cyc(Syl_p)$, then Syl_p

is a p -group whose cyclicizer is nontrivial. Thus, Syl_p is a generalized quaternion group.

If $p \neq 2$, then G is not a generalized quaternion group.

If $p = 2$, then $|Cyc(Syl_p)| = 2$. Since $1 \neq |Cyc(G) \cap Syl_2| \leq |Cyc(Syl_2)| = 2$, then $|Syl_2| = 8$. Thus, $G \cong C_n \times Q_8$, and the proof is complete. \square

Lemma 2.15. *Let G be a finite group. Then, $Cycl(G) = 4$ if and only if $G/Cyc(G) \cong C_2 \times C_2$.*

Proof. Suppose that $G/Cyc(G) \cong C_2 \times C_2$. Since $Cycl(C_2 \times C_2) = 4$, then, by Lemma 2.3, $Cycl(G) = 4$.

If $Cycl(G) = 4$, then, by Lemma 2.2, G has at most three maximal cyclic subgroups. Now, Theorem 2.1 completes the proof. \square

Theorem 2.16. *Let n be an odd positive integer, and G be a finite group. Then, $Cycl(G) = 4$ if and only if G is isomorphic to one of the following groups:*

$$C_n \times Q_8, C_{2n} \times C_2.$$

Proof. It follows from Lemmas 2.14 and 2.15. \square

Theorem 2.17. *Let n be an odd positive integer, and G be a finite group. Then, $Cycl(G) = 6$ if and only if G is isomorphic to one of the following groups:*

$$C_n \times D_8, C_{4n} \times C_2, C_n \times Q_{16}.$$

Proof. Let $Cycl(G) = 6$. Then, $Cycl(\bar{G}) = 6$. Since G has at most five maximal cyclic subgroups, then, by Theorem 2.1, $|G/Cyc(G)| \leq 54$. It is easy to see (by the following programs in GAP [11]) that 6-cyclicizer groups whose orders are less than 54 are the followings:

$$C_4 \times C_2, D_8, Q_{16}, C_{12} \times C_2, C_3 \times D_8, C_{20} \times C_2, C_5 \times D_8, C_3 \times Q_{16}.$$

```
a:=function(n)
  local a;
  a:=AllSmallGroups(n);
  return a;
```

```

end;
cycelement:=function(G,x)
  local c, e, i;
  e:=Elements(G);
  c:=[];
  for i in[1..Size(e)] do
    if IsCyclic(Group(x,e[i]))=true then Add(c,e[i]);
    fi;
  od;
  return c;
end;
for n in[4..54] do
  G:=a(n);
  for i in [1..Size(G)] do
    h:=G[i];
    e:=Elements(h);
    l:=List(e,i->[cycelement(h,i)]);
    if Size(Set(l)) = 6 then
      Print(StructureDescription(h),"\n"); fi;
  od;
od;

```

But $|Cyc(G/Cyc(G))| = 1$, therefore, $G/Cyc(G)$ is isomorphic to either $C_4 \times C_2$ or D_8 . We compute $|Cyc(G)|$ by the following program:

```

CycG := function(G)
  local c, e, i;
  c:=G;
  e:=Elements(G);
  for i in[1..Size(G)] do
    c:=Intersection(c,cycelement(G,e[i]));
  od;
  return c;
end;

```

Similar to the proof of Lemma 2.14, we can conclude that $Cycl(G) = 6$ if and only if G is isomorphic to either $C_n \times D_8$ or $C_{4n} \times C_2$ or $C_n \times Q_{16}$. \square

Theorem 2.18. *Let G be a finite group. Then, $Cycl(G) = 5$ if and only if $G/Cyc(G)$ is isomorphic to either S_3 or $C_3 \times C_3$.*

Proof. Let $Cycl(G) = 5$. By Lemma 2.2, G has at most four maximal cyclic subgroups. Since $Cycl(\bar{G}) = 5$, then, by Theorem 2.1, we have $5 \leq |G/Cyc(G)| \leq 9$. On the other hand, $|G/Cyc(G)|$ is not a prime number, and so $|G/Cyc(G)|$ is either 6 or 8 or 9. If $|G/Cyc(G)| = 8$, then (by GAP) $Cycl(\bar{G}) \neq 5$, which is a contradiction. Thus, $|G/Cyc(G)| = 6$ or 9. Therefore, $G/Cyc(G)$ is isomorphic to either S_3 or $C_3 \times C_3$. The converse is clear. \square

A covering for a group G is a collection of subgroups of G whose union is G . An n -cover for a group G is a cover with n members. A cover is irredundant if no proper subcollection is also a cover.

We write $f(n)$ for the largest index $|G : D|$ over all groups G having an irredundant n -cover with intersection D . Bryce et al. obtained $f(5) = 16$ [8]. Also, Abdollahi et al. obtained $f(6) = 36$, and $f(7) = 81$ [3, 4]. We use these results to prove the following theorems.

Theorem 2.19. *Let G be a finite group. Then, $Cycl(G) = 7$ if and only if $G/Cyc(G)$ is isomorphic to one of the following groups:*

$$D_{10}, A = \langle x, y | x^5 = y^4 = 1, x^y = x^3 \rangle, C_5 \times C_5.$$

Proof. Let $Cycl(G) = 7$. By Lemma 2.2, G has at most six maximal cyclic subgroups. Since $f(6) = 36$, then $8 \leq |G/Cyc(G)| \leq 36$. Now, it is easy to see (by GAP) that G is isomorphic to one of the following groups:

$$D_{10}, C_5 \times C_5, A, Q_{20}, C_3 \times D_{10}.$$

On the other hand, $|Cyc(G/Cyc(G))| = 1$, and so G is isomorphic to either D_{10} or $C_5 \times C_5$ or A . The converse is clear. \square

Theorem 2.20. *Let G be a finite group. Then, $Cycl(G) = 8$ if and only if $G/Cyc(G)$ is isomorphic to one of the following groups:*

$$(C_2)^3, A_4, D_{12}, C_8 \times C_2, C_8 : C_2, C_3 \times S_3, C_9 \times C_3, C_9 : C_3.$$

Proof. Let $Cycl(G) = 8$. By Lemma 2.2, G has at most seven maximal cyclic subgroups. As $f(7) = 81$, with an argument similar to the proof of Theorem 2.19, we can prove our claim. The converse is clear. \square

Acknowledgments

This paper has been extracted from my thesis when I was an M.Sc. student at the University of Isfahan. I would like to express my gratitude to my supervisors Professor A. Mohammadi Hassanabadi, and Professor A. Abdollahi for many helpful ideas, suggestions, and their encouragements. This research was financially supported by the Center of Excellence for Mathematics, University of Isfahan.

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L. Mousavi

Department of Mathematics, University of Isfahan, P.O.Box 81746-73441, Isfahan, Iran

Email: lmousavi@ymail.com