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n-CYCLICIZER GROUPS

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Abstract. The cyclicizer of an element x of a group G is defined as $Cyc_G(x) = \{y \in G | \langle x, y \rangle \text{ is cyclic}\}$. Here, we introduce an n-cyclicizer group and show that there is no finite n-cyclicizer group for $n = 2, 3$. We prove that for any positive integer $n \neq 2, 3$, there exists a finite n-cyclicizer group and determine the structure of finite 4 and 6-cyclicizer groups. Also, we characterize finite $5, 7$ and 8-cyclicizer groups.

1. Introduction

Let G be a group. We know that the centralizer of an element $x \in G$ is defined as follows:

$$C_G(x) = \{y \in G | \langle x, y \rangle \text{ is abelian}\}.$$

If, in this definition, we replace the word abelian by the word cyclic, we get a subset of the centralizer of x. This subset is called the cyclicizer of x in G and it is denoted by $Cyc_G(x)$ [9, 10]. Thus,

$$Cyc_G(x) = \{y \in G | \langle x, y \rangle \text{ is cyclic}\}.$$

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Also, $Cyc(G)$, the cyclicizer of G, is defined as follows:

$$\begin{aligned} Cyc(G) &= \{y \in G | \langle x, y \rangle \text{ is cyclic for all } x \in G\} \\ &= \bigcap_{x \in G} Cyc_G(x). \end{aligned}$$

In general, for an element x of a group G, $Cyc_G(x)$ is not a subgroup of G. For example, in the group $G = \mathbb{Z}_2 \oplus \mathbb{Z}_4$, we have

$$Cyc_G((0,2)) = \{(0,0),(0,1),(0,2),(0,3),(1,1),(1,3)\},$$

which is not a subgroup of G.
In [1] and [2], the cyclicizers of a group are studied via a graph which is called the non-cyclic graph of the group.
For any non-cyclic group G, $Cyc(G)$ is a subgroup, central, cyclic, normal and contained in all maximal cyclic subgroups of G. It is clear that for a nontrivial element x of G, $|Cyc_G(x)| \geq 2$ and $G = \bigcup_{1 \neq x \in G} Cyc_G(x)$. Also, for any group G and $x \in G$, if $\bar{G} = G/Cyc(G)$, then $Cyc_{\bar{G}}(xCyc(G)) = Cyc_G(x)/Cyc(G)$ [2] and it easily follows that $Cyc(\bar{G}) = 1$ (see also [10]).

For a finite group G, let $\mathrm{Cent}(G)$ denote the set of the centralizers of single elements of G. G is called an n-centralizer group if $|\mathrm{Cent}(G)| = n$. We know that there is no n-centralizer group for $n = 2, 3$. Let $Z(G)$ denote the center of a group G. Then, $|\mathrm{Cent}(G)| = 4$ if and only if $G/Z(G) \cong C_2 \times C_2$ and $|\mathrm{Cent}(G)| = 5$ if and only if $G/Z(G) \cong C_3 \times C_3$ or S_3 [7], where C_2 is a cyclic group of size two and S_3 is a symmetric group on three letters .

Moreover, if $|\mathrm{Cent}(G)| = 6$, then $G/Z(G)$ is isomorphic to one of the groups $(C_2)^3$, $(C_2)^4$, A_4 or D_8 [6], where A_4 is an alternating group on four letters and D_8 is a dihedral group of size eight.

Also, $|\mathrm{Cent}\ (G)| = 7$ if and only if $G/Z(G)$ is isomorphic to one of the groups $C_5 \times C_5$, D_{10} or $\langle x, y | x^5 = y^4 = 1, y^{-1}xy = x^3 \rangle$ and if $|\mathrm{Cent}\ (G)| = 8$, then $G/Z(G)$ is isomorphic to one of the groups D_{12}, $(C_2)^3$ or A_4 [5].

Similarly, we can define an n-cyclicizer group, where n is a positive integer.

Definition 1.1. *For a positive integer n, we say that G is an n-cyclicizer group if $|\{Cyc_G(x) | x \in G\}| = n$ and in this case, we write $Cycl(G) = n$.*

It is obvious that G is a 1-cyclicizer group if and only if G is cyclic. Here, we show that there is no finite n-cyclicizer group for $n = 2, 3$ and prove that for any positive integer $n \neq 2, 3$, there exists a finite group

G such that $Cycl(G) = n$. We also study finite n-cyclicizer groups for $n = 4, 5, 6, 7$ and 8.

2. n-Cyclicizer Groups for $n = 4, 5, 6, 7$ and 8

The following theorem is proved in [2].

Theorem 2.1. *Let G be a finite non-cyclic group. Then,* $|G/Cyc(G)| \leq max\{(s-1)^2(s-3)!, (s-2)^3(s-3)!\}$, *where s is the number of maximal cyclic subgroups of G.*

It is clear that if G has n maximal cyclic subgroups, then $Cycl(G) \geq n$.

Lemma 2.2. *Let G be a finite non-cyclic group such that $Cycl(G) = n$. Then, G has at most $n-1$ maximal cyclic subgroups.*

Proof. Assume that $\langle x \rangle$ is a maximal cyclic subgroup of G. Then, $Cyc_G(x) = \langle x \rangle$. Let $Cycl(G) = n$, and $\langle x_1 \rangle, \langle x_2 \rangle, \ldots, \langle x_r \rangle$ be distinct maximal cyclic subgroups of G. Since for any i, $1 \leq i \leq r$, $Cyc_G(x_i) = \langle x_i \rangle$, then $r \leq n$. It is clear that $r \neq n$, since $Cyc_G(1) = G$. This completes the proof. □

Lemma 2.3. *Let G be a finite group. Then, $Cycl(\bar{G}) = n$ if and only if $Cycl(G) = n$.*

Proof. Let $Cycl(\bar{G}) = n$ and $C = Cyc(G)$. The key point of our proof is that $Cyc_G(x) \to Cyc_{\bar{G}}(\bar{x})$ is a one-to-one correspondence between the set of cyclicizers of G and those of $\bar{G}$ (induced by the natural homomorphism $G \to \bar{G} = G/C$). For an element x of G, $\bar{X} = Cyc_G(x)/C$ and $\bar{x} = xC$. We know that $Cyc_{\bar{G}}(\bar{x}) = \bar{X}$. Assume that $\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n$ be distinct cyclicizers of $\bar{x_1} = C, \bar{x_2}, \ldots, \bar{x_n}$, respectively. It is clear that $Cycl(G) \geq n$. Without loss of generality, we can assume that $X_1, X_2, ..., X_n$ are distinct cyclicizers of $x_1, x_2, ..., x_n$, respectively. Suppose that $Y = Cyc_G(g)$ is different from X_i, for any i, $1 \leq i \leq n$. Then, $\bar{Y} = \bar{X_i}$, for some i, $1 \leq i \leq n$. Thus, $Cyc_G(g)C = Cyc_G(x_i)C$. Therefore, for any $h_i \in Cyc_G(g)$, there exist c_i and $z_i \in C$ such that $h_i c_i = k_i z_i$, where $k_i \in Cyc_G(x_i)$, and so $h_i = k_i c_t$, for some $c_t \in C$. Since $k_i \in Cyc_G(x_i)$, it is not hard to see that $\langle h_i, x_i \rangle$ is a cyclic group. Hence, $Cyc_G(g) \subseteq Cyc_G(x_i)$. Similarly, $Cyc_G(x_i) \subseteq Cyc_G(g)$. Thus,

$Cyc_G(g) = Cyc_G(x_i)$. This contradiction indicates $Cycl(G) = n$. The converse is clear. □

Lemma 2.4. *Let $n \geq 2$ be an integer, and*
$Q_{4n} = \langle x, y | x^{2n} = 1, x^n = y^2, y^{-1}xy = x^{-1} \rangle$. *Then,* $Cycl(Q_{4n}) = n + 2$.

Proof. The set of all members of Q_{4n} is $\{1, x^j, x^i y, y | 1 \leq j, i \leq 2n - 1\}$. It is straightforward to check that

(i) for any i, $0 \leq i \leq n - 1$, $Cyc_{Q_{4n}}(x^i y) = \{1, y^2, x^i y, x^{n+i} y\}$;

(ii) $Cyc_{Q_{4n}}(x) = \langle x \rangle$;

and

(iii) for any i, $0 \leq i \leq n - 1$, $Cyc_{Q_{4n}}(x^i y) = Cyc_{Q_{4n}}(x^{n+i} y)$.

Therefore, $Cycl(Q_{4n}) = n + 2$. □

Corollary 2.5. *Let $n > 1$ be an integer. Then,* $Cycl(D_{2n}) = n + 2$.

Proof. It is well known that $Z(Q_{4n}) = \langle y^2 \rangle$, and we can see that $Z(Q_{4n}) = Cyc(Q_{4n})$ and $Q_{4n}/Z(Q_{4n}) = Q_{4n}/Cyc(Q_{4n}) \cong D_{2n}$, and so the proof follows from Lemma 2.3. □

Corollary 2.6. *Let $n > 3$ be an integer. Then, there exists a group G with $Cycl(G) = n$.*

Theorem 2.7. *There is no finite n-cyclicizer group for $n = 2, 3$.*

Proof. First, note that there is no cyclic n-cyclicizer group for $n = 2, 3$. Assume G is a finite group such that $Cycl(G) = 2$. Now since the only proper cyclicizer of G is cyclic and G is covered by its all proper cyclicizers, it follows that G is cyclic, which is a contradiction.

Now, suppose for a contradiction that $Cycl(G) = 3$. Assume that $G = Cyc_G(x) \cup Cyc_G(y)$, where $Cyc_G(x)$ and $Cyc_G(y)$ are two distinct cyclicizers of G. By Lemma 2.2, G has at most two maximal cyclic subgroups. If G has exactly two maximal cyclic subgroups, then, without loss of generality, $G = \langle x \rangle \cup \langle y \rangle$, which is a contradiction. Thus, G has only one maximal cyclic subgroup. This means that G is a cyclic group. This contradiction completes the proof. □

Remark 2.8. *Let p be a prime number and $m \in \mathbb{N}$. Then, $\frac{p^m-1}{p-1}$ is the number of subgroups of order p in $(C_p)^m$.*

Theorem 2.9. *Let p be a prime number and let G be a finite group such that $G/Cyc(G) \cong C_p \times C_p$. Then, $Cycl(G) = p+2$.*

Proof. Let $Cycl(C_p \times C_p) = r$. By Remark 2.8, $C_p \times C_p$ has $p+1$ maximal cyclic subgroups, and so $r \leq p+1$. Let $\langle x_1 \rangle, \langle x_2 \rangle, ..., \langle x_{p+1} \rangle$ be maximal cyclic subgroups of $H = C_p \times C_p$. If $Y = Cyc_H(y) \neq H$ is different from $\langle x_i \rangle$, for any i, $1 \leq i \leq p+1$, then there exists j, $1 \leq j \leq p+1$, such that $y \in \langle x_j \rangle$. Therefore, $\langle x_j \rangle = \langle y \rangle \subseteq Y$. Let g be an arbitrary element in Y. Then, for some integer k, $1 \leq k \leq p+1$, $\langle g, y \rangle = \langle x_k \rangle$. Thus, $y \in \langle x_j \rangle \cap \langle x_k \rangle$. If $j \neq k$, then $y = 1$, and so $Y = H$. This is a contradiction. Therefore, $j = k$. This implies that $Y = \langle x_j \rangle$. Now, Lemma 2.3 completes the proof. □

Corollary 2.10. *Let p be a prime number. Then, $Cyc(C_p \times C_p) = 1$.*

Proof. By Lemma 2.9, we have that $C_p \times C_p$ has $p+1$ proper cyclicizers. Let $Cyc_{C_p \times C_p}(x) = \langle x \rangle$ and $Cyc_{C_p \times C_p}(y) = \langle y \rangle$ be two distinct proper cyclicizers of $C_p \times C_p$. If $\langle x \rangle \cap \langle y \rangle \neq 1$, then $|\langle x \rangle \cap \langle y \rangle| = |\langle x \rangle| = p$. Since $\langle x \rangle \cap \langle y \rangle \leq \langle x \rangle$, then $\langle x \rangle \cap \langle y \rangle = \langle x \rangle$. Therefore, $\langle x \rangle = \langle y \rangle$. This contradiction shows that $\langle x \rangle \cap \langle y \rangle = 1$ and the proof is complete. □

Lemma 2.11. *Let G be a finite p-group, for some prime number p. Then, $Cyc(G) \neq 1$ if and only if G is either a cyclic group or a generalized quaternion group. In this case, $Cyc(G) = Z(G)$.*

Proof. It follows from Proposition 2.2 of [2]. □

Lemma 2.12. *Let G and H be finite groups such that $(|G|, |H|) = 1$. Then, $Cyc(G \times H) = Cyc(G) \times Cyc(H)$.*

Proof. Let $(a, b) \in Cyc(G \times H)$. Then, for any $(g, h) \in G \times H$, there exists $(x, y) \in G \times H$ such that $\langle (g, h), (a, b) \rangle = \langle (x, y) \rangle$. Therefore, $\langle (g, a) \rangle \leq \langle x \rangle$. So $a \in Cyc(G)$. Similarly, $b \in Cyc(H)$. Thus, $Cyc(G \times H) \subseteq Cyc(G) \times Cyc(H)$.

Now, let $(a, b) \in Cyc(G) \times Cyc(H)$. Then, for any $g \in G$, $\langle g, a \rangle$ is a cyclic group. Also, for any $h \in H$, $\langle b, h\rangle$ is a cyclic group. Since $\langle (g,h),(a,b)\rangle \leq \langle g,a\rangle \times \langle h,b\rangle$ and $(|H|,|G|)=1$, then $(a,b) \in Cyc(G \times H)$. Thus, $Cyc(G) \times Cyc(H) \subseteq Cyc(G \times H)$. □

Lemma 2.13. (i) *Let p be a prime number and n be an integer such that $(n,p)=1$. If $G = C_{pn} \times C_p$, then $G/Cyc(G) \cong C_p \times C_p$.*

(ii) *Let n be an odd positive integer. If $G = C_n \times Q_8$, then $G/Cyc(G) \cong C_2 \times C_2$.*

Proof. (i) Let $H = C_p \times C_p$ and $K = C_n$. Since $(|H|,|K|)=1$, then $|Cyc(G)| = n$. Thus, $|G/Cyc(G)| = p^2$. If $G/Cyc(G)$ is a cyclic group, then G is also a cyclic group, which is a contradiction. Thus, $G/Cyc(G) \cong C_p \times C_p$.

(ii) Any Sylow subgroup of G is either a cyclic group or a generalized quaternion group, and so by Lemma 2.11, $Cyc(G) = Z(G)$. We have $|G/Cyc(G)| = |G/Z(G)| = 4$ and $G/Cyc(G)$ is not a cyclic group, since G is not a cyclic group. Therefore, $G/Cyc(G) \cong C_2 \times C_2$. □

Lemma 2.14. *Let p be a prime number and let G be a finite group such that $G/Cyc(G) \cong C_p \times C_p$. Then, G is not a cyclic group and*

(i) *if $p=2$, then G is isomorphic to either $C_{2n} \times C_2$ or $C_n \times Q_8$, where n is an odd positive integer;*
and

(ii) *if $p \neq 2$, then $G \cong C_{pn} \times C_p$, where n is an integer such that $(p,n)=1$.*

Proof. If G is a cyclic group, then $|G/Cyc(G)| = 1$, which is a contradiction. If $G/Cyc(G) \cong C_p \times C_p$, then $G/Cyc(G)$ is an abelian group. Since $G/Z(G) \cong \frac{G/Cyc(G)}{Z(G)/Cyc(G)}$, then G is a nilpotent group. Thus, $G = Syl_2 \times Syl_3 \times \cdots \times Syl_p \times \cdots$.
Since $|G/Cyc(G)| = p^2$, then $Cyc(G)$ contains $C = \widehat{Syl}_p$ ($\widehat{Syl}_p$ is the product of all Sylow subgroups of G, except Syl_p). So, C is a cyclic group of size n such that $(p,n)=1$. Thus, $|Cyc(G)| = p^m \times n$.

If $Cyc(G) \cap Syl_p = \langle 1\rangle$, then $|Syl_p| = |G/Cyc(G)| = p^2$. If Syl_p is a cyclic group, then G is a cyclic group, which is a contradiction. Thus, $Syl_p \cong C_p \times C_p$. So $G \cong C_{pn} \times C_p$.

If $Cyc(G) \cap Syl_p \neq \langle 1\rangle$, since $Cyc(G) \cap Syl_p \leq Cyc(Syl_p)$, then Syl_p

is a p-group whose cyclicizer is nontrivial. Thus, Syl_p is a generalized quaternion group.

If $p \neq 2$, then G is not a generalized quaternion group.

If $p = 2$, then $|Cyc(Syl_p)| = 2$. Since $1 \neq |Cyc(G) \cap Syl_2| \leq |Cyc(Syl_2)| = 2$, then $|Syl_2| = 8$. Thus, $G \cong C_n \times Q_8$, and the proof is complete. □

Lemma 2.15. *Let G be a finite group. Then, $Cycl(G) = 4$ if and only if $G/Cyc(G) \cong C_2 \times C_2$.*

Proof. Suppose that $G/Cyc(G) \cong C_2 \times C_2$. Since $Cycl(C_2 \times C_2) = 4$, then, by Lemma 2.3, $Cycl(G) = 4$.

If $Cycl(G) = 4$, then, by Lemma 2.2, G has at most three maximal cyclic subgroups. Now, Theorem 2.1 completes the proof. □

Theorem 2.16. *Let n be an odd positive integer, and G be a finite group. Then, $Cycl(G) = 4$ if and only if G is isomorphic to one of the following groups:*

$$C_n \times Q_8, C_{2n} \times C_2.$$

Proof. It follows from Lemmas 2.14 and 2.15. □

Theorem 2.17. *Let n be an odd positive integer, and G be a finite group. Then, $Cycl(G) = 6$ if and only if G is isomorphic to one of the following groups:*

$$C_n \times D_8, C_{4n} \times C_2, C_n \times Q_{16}.$$

Proof. Let $Cycl(G) = 6$. Then, $Cycl(\bar{G}) = 6$. Since G has at most five maximal cyclic subgroups, then, by Theorem 2.1, $|G/Cyc(G)| \leq 54$. It is easy to see (by the following programs in `GAP` [11]) that 6-cyclicizer groups whose orders are less than 54 are the followings:

$$C_4 \times C_2, D_8, Q_{16}, C_{12} \times C_2, C_3 \times D_8, C_{20} \times C_2, C_5 \times D_8, C_3 \times Q_{16}.$$

```
a:=function(n)
   local a;
   a:=AllSmallGroups(n);
   return a;
```

```
   end;
cycelement:=function(G,x)
   local c, e, i;
   e:=Elements(G);
   c:=[];
      for i in[1..Size(e)] do
          if IsCyclic(Group(x,e[i]))=true then Add(c,e[i]);
          fi;
      od;
   return c;
   end;
for n in[4..54] do
   G:=a(n);
      for i in [1..Size(G)] do
          h:=G[i];
          e:=Elements(h);
          l:=List(e,i->[cycelement(h,i)]);
             if Size(Set(l)) = 6 then
             Print(StructureDescription(h),"\n"); fi;
      od;
od;
```

But $|Cyc(G/Cyc(G))| = 1$, therefore, $G/Cyc(G)$ is isomorphic to either $C_4 \times C_2$ or D_8. We compute $|Cyc(G)|$ by the following program:

```
CycG := function(G)
   local c, e, i;
   c:=G;
   e:=Elements(G);
      for i in[1..Size(G)] do
      c:=Intersection(c,cycelement(G,e[i]));
      od;
   return c;
   end;
```

Similar to the proof of Lemma 2.14, we can conclude that $Cycl(G) = 6$ if and only if G is isomorphic to either $C_n \times D_8$ or $C_{4n} \times C_2$ or $C_n \times Q_{16}$. □

Theorem 2.18. *Let G be a finite group. Then, $Cycl(G) = 5$ if and only if $G/Cyc(G)$ is isomorphic to either S_3 or $C_3 \times C_3$.*

Proof. Let $Cycl(G) = 5$. By Lemma 2.2, G has at most four maximal cyclic subgroups. Since $Cycl(\bar{G}) = 5$, then, by Theorem 2.1, we have $5 \leq |G/Cyc(G)| \leq 9$. On the other hand, $|G/Cyc(G)|$ is not a prime number, and so $|G/Cyc(G)|$ is either 6 or 8 or 9. If $|G/Cyc(G)| = 8$, then (by GAP) $Cycl(\bar{G}) \neq 5$, which is a contradiction. Thus, $|G/Cyc(G)| = 6$ or 9. Therefore, $G/Cyc(G)$ is isomorphic to either S_3 or $C_3 \times C_3$. The converse is clear. □

A covering for a group G is a collection of subgroups of G whose union is G. An n-cover for a group G is a cover with n members. A cover is irredundant if no proper subcollection is also a cover.

We write $f(n)$ for the largest index $|G : D|$ over all groups G having an irredundant n-cover with intersection D. Bryce et al. obtained $f(5) = 16$ [8]. Also, Abdollahi et al. obtained $f(6) = 36$, and $f(7) = 81$ [3, 4]. We use these results to prove the following theorems.

Theorem 2.19. *Let G be a finite group. Then, $Cycl(G) = 7$ if and only if $G/Cyc(G)$ is isomorphic to one of the following groups:*

$$D_{10}, A = \langle x, y | x^5 = y^4 = 1, x^y = x^3 \rangle, C_5 \times C_5.$$

Proof. Let $Cycl(G) = 7$. By Lemma 2.2, G has at most six maximal cyclic subgroups. Since $f(6) = 36$, then $8 \leq |G/Cyc(G)| \leq 36$. Now, it is easy to see (by GAP) that G is isomorphic to one of the following groups:

$$D_{10}, C_5 \times C_5, A, Q_{20}, C_3 \times D_{10}.$$

On the other hand, $|Cyc(G/Cyc(G))| = 1$, and so G is isomorphic to either D_{10} or $C_5 \times C_5$ or A. The converse is clear. □

Theorem 2.20. *Let G be a finite group. Then, $Cycl(G) = 8$ if and only if $G/Cyc(G)$ is isomorphic to one of the following groups:*

$$(C_2)^3, A_4, D_{12}, C_8 \times C_2, C_8 : C_2, C_3 \times S_3, C_9 \times C_3, C_9 : C_3.$$

Proof. Let $Cycl(G) = 8$. By Lemma 2.2, G has at most seven maximal cyclic subgroups. As $f(7) = 81$, with an argument similar to the proof of Theorem 2.19, we can prove our claim. The converse is clear.

□

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