*Bulletin of the Iranian Mathematical Society Vol.* 38 *No. 1* (*2012*), *pp 17-26.*

# LIE TRIPLE DERIVATION ALGEBRA OF VIRASORO-LIKE ALGEBRA

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Communicated by Saeid Azam

ABSTRACT. Let $\mathfrak{L}$ be the Virasoro-like algebra and $\mathfrak{g}$ its derived algebra, respectively. We investigate the structure of the Lie triple derivation algebra of $\mathfrak{L}$ and $\mathfrak{g}$. We prove that they are both isomorphic to $\mathfrak{L}$, which provides two examples of invariance under triple derivation.

## 1. **Introduction**

Let $A$ be a Lie algebra over the complex number field $\mathbb{C}$. Recall that a $\mathbb{C}$-linear mapping $\phi : A \to A$ is called a Lie derivation if

$$\phi([a,b]) = [\phi(a),b] + [a,\phi(b)] \qquad (\forall\ a,b \in A).$$

If $\phi$ satisfies

$$\phi([[a,b],c]) = [[\phi(a),b],c] + [[a,\phi(b)],c] + [[a,b],\phi(c)] \tag{1.1}$$

for all $a,b,c \in A$, it is called a Lie triple derivation.

A derivation is a triple derivation. Like derivation algebra, the set of triple derivations is a Lie algebra under the usual bracket. It is denoted by $\mathrm{Der}^{(3)}A$. For some algebra, the derivation algebra $\mathrm{Der}A$ is a proper subalgebra of $\mathrm{Der}^{(3)}A$, see [7]. Lie triple derivation algebra of some kinds of operator algebras have been studied in [1, 4].

MSC(2010): Primary: 17B30; Secondary: 17B68.

Keywords: Lie derivation, Lie triple derivation, Virasoro-like algebra.

Received: 30 January 2010, Accepted: 16 August 2010.

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Recently, Virasoro algebra, as a universal central extension of Witt algebra, has played an important role in mathematics and physics[9]. The $q$-analog Virasoro algebra was introduced by Kirkman et al as a derivation subalgebra of quantum torus (see [6]). On one hand, Virasoro-like algebra is the generalization of Virasoro algebra. On the other hand, it can be regarded as a special case of $q$-analog Virasoro algebra when $q = 1$. Lin and Tan [8] and Wang and Zhao[11] studied the nonzero level Harish-Chandra modules and Verma module of Virasoro-like algebra, respectively. Jiang and Meng[5] gave the structure of its derivation algebra and the automorphism group. Song *et al.* got the quantization of generalized Virsoro-like algebra in [10]. As early as in 1957, Herstein in [3] proved that any Jordan triple derivation is an ordinary derivation on a prime ring of characteristic different from 2. In this paper, we will prove that the Lie triple derivation of Virasoro-like algebra $\mathfrak{L}$ is isomorphic to $\mathfrak{L}$. Meanwhile we obtain that any Lie triple derivation of Virasoro-like algebra is a Lie derivation, i.e.,

$$\mathrm{Der}\mathfrak{L} = \mathrm{Der}^{(3)}\mathfrak{L}. \tag{1.2}$$

The paper is arranged as follows. In Section 2, we give some notations and recall some basic facts on $\mathfrak{L}$. In Section 3, we study the graded version of $\mathrm{Der}^{(3)}\mathfrak{L}$, which is a generalization of [2]. In Section 4, we give the main result of this paper.

## 2. **Preliminaries**

Throughout this paper we use $\mathbb{C}$, $\mathbb{Z}$ to denote the set of complex numbers, and integers, respectively. We assume that $\mathcal{U}$ is a 2-dimensional vector space over $\mathbb{C}$. We choose

$$e_1 = (1, 0), \quad e_2 = (0, 1)$$

as the basis of $\mathcal{U}$ and let $(\cdot, \cdot)$ be the standard bilinear form on $\mathcal{U}$, i.e.,

$$(e_i, e_j) = \delta_{ij}, \quad 1 \le i, j \le 2.$$

Let $A = \mathbb{C}[x_1^{\pm}, x_2^{\pm}]$ be a Laurent polynomial with 2 commuting variables $x_1, x_2$ over $\mathbb{C}$.

The elements of $A$ are denoted by $x^r = x_1^{r_1} x_2^{r_2}$ for $r = (r_1, r_2)$. Set $D_i(r) = x^r x_i(\partial/\partial x_1)$ for $r \in \mathbb{Z}^2$. For all $u = (u_1, u_2) \in \mathcal{U}$, we define

$$D(u, r) = u_1 D_1(r) + u_2 D_2(r).$$

The space Der$A$ is a Lie algebra under the following bracket:

$$[D(u,r), D(u',r')] = D(u'', r+r'),$$

where $u'' = (u,r')u' - (u',r)u$.
A subalgebra $\mathfrak{L}$ of Der$A$ is called (centerless) Virasoro-like algebra if it satisfies

$$\mathfrak{L} = \bigoplus_{r\in\mathbb{Z}^2} \mathfrak{L}_r,$$

where $\mathfrak{L}_r = \{D(u,r) | (u,r) = 0\}$. Let $\mathfrak{g} = [\mathfrak{L}, \mathfrak{L}]$ be the derived algebra of $\mathfrak{L}$, then $\mathfrak{L} = \mathfrak{g} \bigoplus \mathfrak{h}$, where

$$\mathfrak{g}_r = \begin{cases} \mathfrak{L}_r, r \neq 0 \\ 0 \quad\; r = 0 \end{cases}$$

and $\mathfrak{h}$ is the Cartan subalgebra of $\mathfrak{L}$.

In this paper we study the centerless Virasoro-like algebra.

## 3. Gradation of $\mathfrak{L}$ and $\mathbf{Der}^{(3)}\mathfrak{L}$

In this section, we prove that $\mathbf{Der}^{(3)}\mathfrak{L}$ and $\mathbf{Der}^{(3)}\mathfrak{g}$ are both graded algebras.

Let $G$ be an abelian group, $L = \bigoplus_{g\in G} L_g$ a $G$-graded Lie algebra over an algebraically closed field $\mathbb{F}$. Let $V$ be a $G$-graded $L$-module. A linear mapping $\varphi : L \longrightarrow V$ is called a triple derivation if

(3.1) $\varphi([x,[y,z]]) = z.y.\varphi(x) - z.x.\varphi(y) + x.y.\varphi(z) - y.x.\varphi(z)$

for all $x, y, z \in L$. We say that a triple derivation $\varphi$ has degree $g$ if $\varphi(L_h) \subseteq V_{g+h}$.

This definition generalizes (1.1): Take the regular module $V = L$, then (3.1) will be (1.1).

**Theorem 3.1.** *Let $L$ be a finitely generated Lie algebra and $V$ a $G$-graded $L$-module. Then*

$$\mathrm{Der}^{(3)}_{\mathbb{F}}(L,V) = \bigoplus_{g\in G} \mathrm{Der}^{(3)}_{\mathbb{F}}(L,V)_g.$$

This theorem can be proved using a similar argument as in [2].

*Proof.* Let $S$ be a finite generating set of $L$. For each element $g \in G$ we define two canonical projections $p_g : L \longrightarrow L_g$ and $\pi_g : V \longrightarrow V_g$. Suppose $\varphi : L \longrightarrow V$ is a triple derivation and there are finite subsets $Q, R \subset G$ such that

$$S \subset \sum_{g\in Q} L_g \quad \text{and} \quad \varphi(S) \subset \sum_{g\in R} V_g.$$

For $g \in G$ let $\varphi_g := \sum_{h\in G} \pi_{g+h} \circ \varphi \circ p_h$. Now for $x_h \in L_h,\ x_k \in L_k$ and $x_l \in L_l$, we have

$$\begin{aligned}
& \varphi_g([[x_h, x_k], x_l]) \\
= \ & \pi_{g+h+k+l} \circ \varphi([[x_h, x_k], x_l]) \\
= \ & \pi_{g+h+k+l}(x_l.x_k.\varphi(x_h) - x_l.x_h.\varphi(x_h) + x_h.x_k.\varphi(x_l) - x_k.x_h.\varphi(x_l)) \\
= \ & x_l.x_k.\pi_{g+h}(\varphi(x_h)) - x_l.x_h.\pi_{g+h}(\varphi(x_h)) \\
& \qquad\qquad + x_h.x_k.\pi_{g+l}(\varphi(x_l)) - x_k.x_h.\pi_{g+l}(\varphi(x_l)) \\
= \ & x_l.x_k.(\varphi_g(x_h)) - x_l.x_h.(\varphi_g(x_h)) + x_h.x_k.(\varphi_g(x_l)) - x_k.x_h.(\varphi_g(x_l)),
\end{aligned}$$

which follows that $\varphi_g$ is contained in $\mathrm{Der}^{(3)}_{\mathbb{F}}(L, V)_g$.

Let $T = \{g - h | g \in R, h \in Q\}$. For any $y \in S$ we have

$$\begin{aligned}
\varphi(y) &= \sum_{g\in R} \pi_g \circ \varphi(y) = \sum_{g\in R} \sum_{h\in Q} \pi_g \circ \varphi \circ p_h(y) \\
&= \sum_{g\in R} \sum_{h\in Q} \pi_{g-h}(y) \\
&= \sum_{q\in T} \varphi_q(y).
\end{aligned}$$

This shows that $\varphi = \sum_{q\in T} \varphi_q$ on $S$ and subsequently $L$. $\square$

**Lemma 3.2.** *The Lie algebra* $\mathfrak{g}$ *is generated by*

$$D(e_1, \pm e_2), D(e_2, \pm e_1),$$

*where* $e_1, e_2$ *are defined above.*

Note that $\mathfrak{L}$ is generated by $\mathfrak{g}$ and $D(e_1, 0), D(e_2, 0)$, so $\mathfrak{L}$ is also finitely generated with the generators

$$D(e_1, \pm e_2), D(e_2, \pm e_1), D(e_1, 0), D(e_2, 0).$$

Theorem 3.1 and Lemma 3.2 imply that $\mathrm{Der}^{(3)}(\mathfrak{L})$ and $\mathrm{Der}^{(3)}(\mathfrak{g})$ are $\mathbb{Z}^2$-graded Lie algebras.

## 4. Structure of $\mathbf{Der}^{(3)}\mathfrak{g}$ and $\mathbf{Der}^{(3)}\mathfrak{L}$

4.1. **The Structure of $\mathbf{Der}^{(3)}\mathfrak{g}$.** In this section we will characterize the structure $\mathbf{Der}^{(3)}\mathfrak{g}$ and we will show that $\mathbf{Der}^{(3)}\mathfrak{g} \cong \mathfrak{L}$.

**Lemma 4.1.** *For any nonzero $r \in \mathbb{Z}^2$, $(\mathbf{Der}^{(3)}\mathfrak{g})_r = ad\mathfrak{g}_r$.*

*Proof.* We only need to show that there exists an element $y \in \mathfrak{g}_r$ such that $ady = \varphi$ for any $\varphi \in (\mathbf{Der}^{(3)}\mathfrak{g})_r$ and $r \in \mathbb{Z}^2$.

By Lemma 3.2, it is enough to prove $ady = \varphi$ holds for the generators of $\mathfrak{g}$.

Suppose $\varphi(D(e_1, e_2)) = D(w, e_2 + r)$, where $w = (w_1, w_2) \in \mathcal{U}$.

**Case 1:** $r_1 \neq 0$. By choosing

$$u = (-\frac{w_2}{r_1}, -\frac{r_1 w_1 + w_2}{r_1^2}),$$

we get

$$\varphi(D(e_1, e_2)) = adD(u, r)(D(e_1, e_2)).$$

It is convenient to replace $\varphi$ by $\varphi - adD(u, r)$, then $\varphi(D(e_1, e_2)) = 0$.

We assume $\varphi(D(e_1, -e_2)) = D(v, -e_2 + r)$ with $v = (v_1, v_2) \in \mathcal{U}$.

Observe that $[D(e_1, e_2), D(e_1, -e_2)] = 0$, so

$$\varphi([[D(e_1, e_2), D(e_1, -e_2)], D(e_1, e_2)]) = 0.$$

However, $\varphi$ is a triple derivation, so we have

$$\begin{aligned} &\varphi([[D(e_1, e_2), D(e_1, -e_2)], D(e_1, e_2)]) \\ &= [[D(e_1, e_2), D(v, -e_2 + r)], D(e_1, e_2)] \\ &= D(v', e_2 + r) \end{aligned} \tag{4.1}$$

for some $v' \in \mathbb{C}^2$. Subsequently, $\varphi([[D(e_1, e_2), D(e_1, -e_2)], D(e_1, e_2)]) = D(v', e_2 + r) = 0$, which forces $v' = 0$.

On the other hand, direction computation of $v'$ gives that

$$\begin{cases} 2r_1 v_2 - r_1^2 v_1 = 0 \\ r_1^2 v_2 = 0, \end{cases}$$

which implies that $\varphi(D(e_1, -e_2)) = 0$.

We prove $\varphi(D(e_2, e_1)) = \varphi(D(e_2, -e_1)) = 0$ in several steps.

We first assume that $\varphi(D(e_2,e_1)) = D(\overline{v}, e_1+r)$ with $\overline{v} = (\overline{v}_1, \overline{v}_2)$.

If $r \neq (-2, r_2)$, by considering the image of $\varphi$ on the following equality

$$[[D(e_1,e_2), D(e_2,e_1)], D(e_1,-e_2)] = D(e_2,e_1),$$

we know $(r_1+1)^2\overline{v} = \overline{v}$. Hence $\overline{v} = 0$.

Similarly if $r \neq (2, r_2)$, by applying $\varphi$ on

$$[[D(e_1,e_2), D(e_2,-e_1)], D(e_1,-e_2)] = -D(e_2,-e_1),$$

we get $\varphi(D(e_2,-e_1)) = 0$.

So we can conclude that if:
$r = (-2, r_2)$,

$$\varphi(D(e_1,\pm e_2)) = \varphi(D(e_2,-e_1)) = 0;$$

if $r = (2, r_2)$,

$$\varphi(D(e_1,\pm e_2)) = \varphi(D(e_2,e_1)) = 0.$$

Next we consider the case $r = (-2, r_2)$.

If $r_2 \neq 0$,

$$\begin{aligned} & \varphi([[D(e_2,e_1), D(e_2,-e_1)], D(e_1,e_2)]) \\ = \; & [[D(\overline{v}, e_1+r), D(e_2,-e_1)], D(e_1,e_2)] \\ = \; & D(\overline{v''}, e_2+r) \end{aligned}$$

where $\overline{v''} = -((2r_2+1)\overline{v}_1 + r_2\overline{v}_2, 2\overline{v}_1 + 2r_2\overline{v}_2)$, which tells us $\overline{v} = 0$. i.e. $\varphi(D(e_2,e_1)) = 0$.

If $r = (-2, 0)$, then

$$[[D(e_1,e_2), D(e_2,e_1)], D(e_2,-e_1)] = D(e_1,e_2)$$

also implies $\varphi(D(e_2,e_1)) = 0$.

Similar discussion will lead to $\varphi(D(e_2,-e_1)) = 0$ for $r = (-2, r_2)$ with $r_2 \in \mathbb{Z}$.

**Case 2:** $r_1 = 0$. Suppose $\varphi(D(e_2,e_1)) = D(w, e_1+r)$, we choose an element $D(u', r) \in \mathfrak{g}_r$ with

$$u' = \left(-\frac{w_1}{r_2}, -\frac{w_1 + r_2 w_2}{r_2^2}\right).$$

Then we get

$$(\varphi - adD(u', r))(D(e_2,e_1)) = 0.$$

Now we assume $\varphi(D(e_2,-e_1)) = D(\check{w}, -e_1+r)$ with $\check{w} = (\check{w}_1\check{w}_2)$.

Similarly as before (see (4.1)),

$$[[D(e_2,e_1)D(e_2,-e_1)]D(e_2,e_1)]=0$$

implies that we also have

$$\left\{\begin{array}{l} -r_2^2\check{w}_1=0 \\ -r_2^2\check{w}_2+2r_2\check{w}_1=0. \end{array}\right. \tag{4.2}$$

Thus $\varphi(D(e_2,-e_1))=0$.

If $r_2\neq -2$, $\varphi(D(e_1,e_2))=0$ due to the fact that

$$\varphi([[D(e_2,e_1)D(e_1,e_2)]D(e_2,-e_1)])=\varphi(-D(e_1,e_2)).$$

If $r_2=-2$, i.e. $r=(0,-2)$, we also have

$$\varphi([[D(e_2,e_1)D(e_1,e_2)]D(e_1,-e_2)])=\varphi(D(e_2,e_1))=0.$$

Similar discussion on $r_2=2$ or $r_2\neq 2$, gives $\varphi(D(e_1,-e_2))=0$. Therefore $\varphi(D(e_i,e_j))=0,\ 1\leq i\neq j\leq 2$. By Lemma 3.2 we also have $\varphi(\mathfrak{g})=0$ for all nonzero $r\in\mathbb{Z}^2$. This completes the proof. $\square$

**Theorem 4.2.** $\mathbf{Der}^{(3)}\mathfrak{g}\cong\mathfrak{L}$.

*Proof.* We first define the action of $D(u,0)\in\mathfrak{h}$ on $\mathfrak{g}$ as follows:

$$D(u,0).D(v,s)=[D(u,0),D(v,s)]=(u,s)D(v,s),\ \forall s\in\mathbb{Z}^2\setminus\{0\},$$

which is a 0-homogeneous outer derivation of $\mathfrak{g}$.

We note that $\varphi(D(e_i,\pm e_j))=c_{ij}^{\pm}D(e_i,\pm e_j)$ and

$$[[D(e_i,e_j)D(e_j,e_i)]D(e_j,-e_i)])=D(e_i,e_j),\ 1\leq i\neq j\leq 2.$$

By applying $\varphi$ to both sides it follows that $c_{ij}^{+}=-c_{ij}^{-}$.

We then define

$$h=c_{21}^{+}D(e_1,0)+c_{12}^{+}D(e_2,0).$$

Then we can check $(\varphi-h)(\mathfrak{g})=0$ when $r=0$.

Therefore by Section 3 and Lemma 4.1 we have

$$\begin{aligned}\mathbf{Der}^{(3)}\mathfrak{g} &= \oplus_{r\in\mathbb{Z}^2}\mathbf{Der}^{(3)}\mathfrak{g}_r \\ &\cong \oplus_{r\in\mathbb{Z}^2\setminus\{0\}} ad\mathfrak{g}_r \oplus \mathfrak{h} \\ &= ad\mathfrak{g}\oplus\mathfrak{h} \\ &= ad\mathfrak{L} \\ &= \mathfrak{L}.\end{aligned}$$

□

4.2. **The Structure of $\mathbf{Der}^{(3)}\mathfrak{L}$.** In this section we will prove $\mathbf{Der}^{(3)}\mathfrak{L}\cong\mathfrak{L}$ by introducing the following lemma.

**Lemma 4.3.** *Suppose $\varphi\in\mathbf{Der}^{(3)}\mathfrak{L}$. If $\varphi(\mathfrak{g})=0$, then $\varphi(\mathfrak{h})=0$.*

*Proof.* We only consider $D(e_1,0)$.

Assume that $\varphi(D(e_1,0))=D(\tilde{u},r)$, where $\tilde{u}=(\tilde{u_1},\tilde{u_2})$.

**Case 1:** $r\neq 0$.

If $r_1\neq 0$, applying $\varphi$ on both sides of

$$[[D(e_1,e_2),D(e_1,0)],D(e_1,e_2)]=0,$$

we have

$$\left\{\begin{array}{l}2r_1\tilde{u}_2-r_1^2\tilde{u}_1=0\\ r_1^2\tilde{u}_2=0,\end{array}\right. \tag{4.3}$$

which follows that $\tilde{u}=0$ and $\varphi(D(e_1,0))=0$.

If $r_2\neq 0$, it follows from

$$[[D(e_2,e_1),D(e_1,0)],D(e_2,-e_1)]=0.$$

**Case 2:** $r=0$.

Note that

$$[[D(e_1,e_2),D(e_1,0)],D(e_2,e_1)]=0.$$

According to our assumption, the operation of $\varphi$ on the both sides is

$$[[D(e_1,e_2),\tilde{u}_1D(e_1,0)+\tilde{u}_2D(e_2,0)],D(e_j,e_i)]=0.$$

By a simple calculation we can get $\tilde{u}_2=0$.

The same action on

$$[[D(e_2,e_1),D(e_1,0)],D(e_1,0)]=D(e_2,e_1)$$

tells us $\tilde{u}_1=0$. This completes the proof. □

We therefore have:

**Theorem 4.4.** $\mathbf{Der}^{(3)}\mathfrak{L} \cong \mathfrak{L}$.

*Proof.* Let $\phi$ be a triple derivation of $\mathfrak{L}$. By Theorem 3.1 we have

$$\phi = \sum_{r\in\mathbb{Z}^2} \phi_r.$$

Repeat the process of Section 4.1, we deduce that there exists an element $x_r \in \mathfrak{L}_r$ such that $(\phi_r - adx_r)(\mathfrak{g}) = 0$, for all $r \in \mathbb{Z}^2$. By Lemma 4.3, we obtain $(\phi_r - adx_r)(\mathfrak{L}) = 0$. So

$$\begin{aligned} \mathbf{Der}^{(3)}\mathfrak{L} &= \oplus_{r\in\mathbb{Z}^2}\mathbf{Der}^{(3)}\mathfrak{L}_r \\ &= \oplus_{r\in\mathbb{Z}^2} ad\mathfrak{L}_r \\ &= ad\mathfrak{L} \\ &\cong \mathfrak{L}. \end{aligned}$$

□

**Corollary 4.5.** *Suppose* $\mathfrak{g}$ *and* $\mathfrak{L}$ *are defined as above, Then*

$$\mathbf{Der}\mathfrak{g} = \mathbf{Der}^{(3)}\mathfrak{g} = \mathbf{Der}\mathfrak{g} = \mathbf{Der}^{(3)}\mathfrak{L} \cong \mathfrak{L}.$$

**Acknowledgments**

The present work was finished during H. Wang's visit to North Carolina State University. H. Wang would like to thank the support from CSC and the hospitality of the Mathematics Department of NCSU. N. Jing was supported by NSFC Grant (10728102). Q. Li was supported by NSFC Grant (10771056) and 973 program (2009CB326202).

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