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$\varphi-$AMENABLE AND $\varphi-$BIFLAT BANACH ALGEBRAS

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ABSTRACT. In this paper we study the concept of $\varphi-$ biflatness of a Banach algebra A, where φ is a continuous homomorphism on A. We prove that if φ is a continuous epimorphism on A and A has a bounded approximate identity and A is $\varphi-$ biflat, then A is $\varphi-$ amenable. In the case where φ is an isomorphism on A we show that the $\varphi-$ amenability of A implies its $\varphi-$ biflatness.

1. **Introduction**

A Banach algebra A is called *amenable* if for each Banach A-module X, every bounded derivation from A into the dual A-module X^* is an inner derivation. The notion of a *biflat* Banach algebra was introduced by Helemskii who proved that a Banach algebra is amenable precisely when it is biflat and has a bounded approximate identity [7, 9]. In fact, A is called *biflat* if there exists a bounded A-bimodule map $\theta : (A \hat{\otimes} A)^* \longrightarrow A^*$ such that $\theta \circ \pi^* = id_{A^*}$, where π denotes the product morphism from $A \hat{\otimes} A$ into A given by $\pi(a \otimes b) = ab$ for all $a, b \in A$. Recently, some authors have added a kind of twist to the amenability definition. Given a continuous homomorphism φ from A into A, they defined and studied φ*-derivations* and φ*-amenability* (see [3], [13] and [15]).

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Suppose that A is a Banach algebra and $\varphi \in Hom(A)$, the set of all continuous homomorphisms from A into A. Let X be a Banach A-bimodule. A linear operator $D : A \longrightarrow X$ is a *φ-derivation* if it satisfies $D(ab) = D(a)\varphi(b) + \varphi(a)D(b)$ for all $a, b \in A$. A φ-derivation D is *φ-inner derivation* if there is $x \in X$ such that $D(a) = \varphi(a)x - x\varphi(a)$ for $a \in A$. Let $\mathcal{Z}^1_\varphi(A, X)$ denote the set of all continuous φ-derivations and let $\mathcal{N}^1_\varphi(A, X)$ be the set of all φ-inner derivations from A into X. The first cohomology group $\mathcal{H}^1_\varphi(A, X)$ is defined to be the quotient space $\mathcal{Z}^1_\varphi(A, X)/\mathcal{N}^1_\varphi(A, X)$.

A Banach algebra A is called *φ-amenable* if $\mathcal{H}^1_\varphi(A, X^*) = \{0\}$ for all A-bimodules X. Note that every derivation of a Banach algebra A into an A-bimodule X is an id_A-derivation, where id_A is the identity map on A.

The aim of the present paper is to introduce and investigate $\varphi-$biflat Banach algebras with $\varphi \in Hom(A)$. In particular, we prove that if φ is a continuous epimorphism on A and A has a bounded approximate identity and A is $\varphi-$ biflat, then A is $\varphi-$ amenable. In the case where φ is an isomorphism on A we show that the $\varphi-$ amenability of A implies its $\varphi-$ biflatness.

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2. **The results**

We start this section by introducing the following:

Definition 2.1. *Let A be a Banach algebra and $\varphi \in Hom(A)$. An element M of $(A \hat{\otimes} A)^{**}$ is a $\varphi-$ virtual diagonal for A if*
i) $\varphi(a) \cdot M = M \cdot \varphi(a) \qquad (a \in A)$,
ii) $\pi^{**}(M) \cdot \varphi(a) = \varphi(a) \qquad (a \in A)$.

For an alternative proof of the following result when φ is an idempotent homomorphism on A see [15, Theorem 4.2.]

Theorem 2.2. *Let A be a Banach algebra with a bounded approximate identity and $\varphi \in Hom(A)$. If A is $\varphi-$ amenable then A has a $\varphi-$ virtual diagonal.*

Proof. Let (e_α) be a bounded approximate identity for A and let E be a w^*-cluster point of $(\varphi(e_\alpha) \otimes \varphi(e_\alpha))$ in $(A \hat{\otimes} A)^{**}$. We define a $\varphi-$derivation $D : A \longrightarrow (A \hat{\otimes} A)^{**}$ by $D(a) = \varphi(a) \cdot E - E \cdot \varphi(a)$. Then

$$
\begin{aligned}
\pi^{**}(D(a)) &= w^* - \lim_\alpha \pi[(\varphi(a)(\varphi(e_\alpha)\otimes\varphi(e_\alpha)) - (\varphi(e_\alpha)\otimes\varphi(e_\alpha))\varphi(a)] \\
&= \lim_\alpha \varphi(a)\varphi(e_\alpha^2) - \varphi(e_\alpha^2)\varphi(a) \\
&= \lim_\alpha \varphi(ae_\alpha^2 - e_\alpha^2 a) = 0.
\end{aligned}
$$

Therefore $D(A) \subseteq ker(\pi^{**}) = (ker\pi)^{**}$. Thus there exists $N \in (ker\pi)^{**}$ such that $D = ad_{\varphi;N}$. Put $M = E - N$. Then

$$
\begin{aligned}
\pi^{**}(M)\cdot\varphi(a) &= \pi^{**}(E-N)\cdot\varphi(a) = \pi^{**}(E)\cdot\varphi(a) \\
&= w^* - \lim_\alpha \pi(\varphi(e_\alpha)\otimes\varphi(e_\alpha))\cdot\varphi(a) \\
&= \lim_\alpha \varphi(e_\alpha^2 a) = \varphi(a).
\end{aligned}
$$

Hence M is a $\varphi-$ virtual diagonal for A. □

Definition 2.3. *Let A be a Banach algebra and $\varphi \in Hom(A)$. A bounded $\varphi-$ approximate diagonal for A is a bounded net (m_α) in $(A\hat{\otimes}A)$ such that*
i) $m_\alpha\cdot\varphi(a) - \varphi(a)\cdot m_\alpha \longrightarrow 0 \quad (a\in A)$
ii) $\pi(m_\alpha)\cdot\varphi(a) \longrightarrow \varphi(a) \quad (a \in A)$.

Proposition 2.4. *Let A be a Banach algebra and $\varphi \in Hom(A)$. If A has a $\varphi-$ virtual diagonal. Then A has a bounded $\varphi-$ approximate diagonal.*

Proof. Let M be a $\varphi-$ virtual diagonal for A, and let (m_α) be a net in $(A\hat{\otimes}A)$ such that $M = w^* - \lim_\alpha m_\alpha$. Then, a routine verification shows that for the net (m_α), Definition 2.3 holds in the $weak^*-$ topology. Following the argument given in the proof of [6, Lemma 2.9.64] we can show that there exists a net (m_β) of convex combinations of (m_α)'s satisfying both conditions in Definition 2.3. □

Remark 2.5. *In the case where φ is a continuous idempotent epimorphism on A, M. S. Moslehian [15, Theorem 4.6] has given a generalizations of Johnson's result [11]. It should be emphasized that such an epimorphism φ is nothing but the identity. Indeed, no generalization is given.*

Theorem 2.6. *Let A be a Banach algebra with a bounded φ-approximate diagonal and φ is an epimorphism on A. Then A is φ-amenable.*

Proof. Let (m_α) be a bounded φ-approximate diagonal for A. Then (πm_α) is a bounded approximate identity for A. Let X be a Banach A-bimodule, by [15, Proposition 4.5] there is no loss of generality if we suppose that X is pseudo-unital. Let $D \in \mathcal{Z}^1_\varphi(A,X)$, and $m_\alpha = \Sigma_{n=1}^\infty a_n^{(\alpha)} \otimes b_n^{(\alpha)}$ with $\Sigma_{n=1}^\infty \|a_n^{(\alpha)}\|\|b_n^{(\alpha)}\| < \infty$. Then $(\Sigma_{n=1}^\infty \varphi(a_n^{(\alpha)}) \cdot Db_n^{(\alpha)})$ is a bounded net in X^*, which has a w^*-accumlation point, say ψ; without loss of generality, we may suppose that ψ is the $w^* - \lim$ of $\left(\Sigma_{n=1}^\infty \varphi(a_n^{(\alpha)}) \cdot Db_n^{(\alpha)}\right)$. Then, for $a \in A$ and $x \in X$, we have:

$$\begin{aligned}
\langle x, \varphi(a) \cdot \psi \rangle &= \lim_\alpha \left\langle x, \Sigma_{n=1}^\infty \varphi(a)\varphi(a_n^{(\alpha)}) \cdot Db_n^{(\alpha)} \right\rangle \\
&= \lim_\alpha \left\langle x, \Sigma_{n=1}^\infty \varphi(a_n^{(\alpha)}) \cdot D(b_n^{(\alpha)}\varphi(a)) \right\rangle \\
&= \lim_\alpha \left\langle x, \Sigma_{n=1}^\infty (\varphi(a_n^{(\alpha)} b_n^{(\alpha)}) \cdot D(\varphi(a)) + \varphi(a_n^{(\alpha)}) \cdot \right. \\
&\quad \left. D(b_n^{(\alpha)} \cdot \varphi(a)) \right\rangle \\
&= \lim_\alpha \langle x, (\varphi(\pi(m_\alpha) \cdot D(\varphi(a)) \rangle + \langle x, \psi \cdot \varphi(a) \rangle .
\end{aligned}$$

Since φ is an epimorphism and $\lim_\alpha \varphi(\pi(m_\alpha)a) = \varphi(a)$ for all $a \in A$, we conclude that $D(a) = \varphi(a)\psi - \psi\varphi(a)$. $\square$

Example 2.7. *Let A be a non-amenable Banach algebra with right (left) bounded approximate identity. Then by [16, corollary 2.3.11] $A^\sharp$ is not amenable. Therefore $A^\sharp$(the unitalization of A) is not biflat. We define $\varphi : A^\sharp \longrightarrow A^\sharp$ by $\varphi(a + \lambda e) = \lambda \ \ (a \in A, \ \lambda \in C)$. Therefore, φ is a continuous homomorphism on $A^\sharp$. Let X be a Banach $A^\sharp-$ bimodule. Then for every $\varphi-$ derivation $D : A^\sharp \longrightarrow X$ and every $a, b \in A, \ \lambda, z \in C$ we have,*

$$\begin{aligned}
D((a + \lambda e)(b + ze)) &= (a + \lambda e) \circ D(b + ze) + D(a + \lambda e) \circ (b + ze) \\
&= \varphi(a + \lambda e)D(b + ze) + D(a + \lambda e)\varphi(b + ze) \\
&= \lambda D(b + 0e) + D(a + 0e)z.
\end{aligned}$$

Let (e_α) be a right (left) bounded approximate identity for A and putting $z = \lambda = 0$ and $b = e_\alpha$ in the above equation, we obtain $D(ae_\alpha + 0e) = 0$. Therefore $D(a + 0e) = \lim_\alpha D(ae_\alpha + 0e) = 0$ and $D(a + \lambda e) = D(a + 0e) + D(0 + \lambda e) = 0$ for all $a \in A$ and $\lambda \in C$. Therefore $A^\sharp$ is $\varphi-$ amenable.

Definition 2.8. *Let A be a Banach algebra and let $\varphi \in Hom(A)$. Then A is called n-weakly $\varphi-$amenable if $\mathcal{H}^1_{\varphi}(A, A^{(n)}) = \{0\}$, $(n \in \mathbb{N})$. We say that A is $\varphi-$ weakly amenable if A is 1-weakly $\varphi-$amenable.*

The following two propositions are interesting in their own rights.

Proposition 2.9. *Let A be a Banach algebra and $\varphi \in Hom(A)$ and $n \in \mathbb{N}$. If A is $(n+2)$-weakly $\varphi-$amenable. Then A n-weakly $\varphi-$amenable.*

Proof. Let $D : A \longrightarrow A^{(n)}$ be a φ-derivation. Then $D : A \longrightarrow A^{(n+2)}$ is a φ-derivation. Therefore there exists $\phi \in A^{(n+2)}$ such that $D(a) = \phi \cdot \varphi(a) - \varphi(a) \cdot \phi$ $(a \in A)$. Let $\xi = P(\phi)$ where P is the canonical projection from $A^{(n+2)}$ into $A^{(n)}$. Then $D(a) = P(D(a)) = \xi \cdot \varphi(a) - \varphi(a) \cdot \xi$ $(a \in A)$. □

Proposition 2.10. *Let A be a Banach algebra, $n \in \mathbb{N}$, and let $\varphi \in Hom(A)$ be such that $\varphi(a)b = a\varphi(b)$ for all $a, b \in A$. If A is $(2n-1)$-weakly $\varphi-$amenable. Then $\overline{A^2} = A$ where $\overline{A^2}$ is the closure of A^2 in A.*

Proof. For $n = 1$, the result follows from [3, Proposition 2.1]. Now for $n > 1$, the proof is an immediate consequence of Proposition 2.4. □

We now turn to $\varphi-$ biflat Banach algebras.

Definition 2.11. *Let A be a Banach algebra and $\varphi \in Hom(A)$. Then A is called $\varphi-$ biflat if there exists a bounded A-bimodule map $\theta : A \longrightarrow (A \hat{\otimes} A)^{**}$ such that $\pi^{**} \circ \theta \circ \varphi = \kappa$, where $\kappa : A \longrightarrow A^{**}$ is the natural embedding.*

Proposition 2.12. *Suppose that A is $\varphi-$ biflat ($\varphi \in Hom(A)$) and A has a bounded approximate identity. Then A has a $\varphi-$ virtual diagonal.*

Proof. Assume that $\theta : A \longrightarrow (A \hat{\otimes} A)^{**}$ is a bounded A-bimodule map such that $\pi^{**} \circ \theta \circ \varphi = \kappa$. Let (e_α) be a bounded approximate identity for A and let $M = w^* - \lim_\alpha \theta(\varphi(e_\alpha))$. Then

$$\begin{aligned} \pi^{**}(M) \cdot \varphi(a) &= w^* - \lim_{\alpha} \pi^{**}(\theta(\varphi(e_\alpha))) \cdot \varphi(a) \\ &= w^* - \lim_{\alpha} \pi^{**} \circ \theta \circ \varphi(e_\alpha) \cdot \varphi(a) = \varphi(a). \end{aligned}$$

Hence M is a $\varphi-$ virtual diagonal. □

Remark 2.13. *Let A be a biflat Banach algebra. Then A is id_A- biflat.*

Proposition 2.14. *Let A be a Banach algebra and $\varphi \in Hom(A)$ such that $\varphi^n = 1$ for some $n \in \mathbb{N}$. If A has a virtual diagonal, then A is $\varphi-$ biflat.*

Proof. Let M be a virtual diagonal. Define $\theta : A \longrightarrow (A \hat{\otimes} A)^{**}$ by $a \mapsto \varphi(a)^{n-1} \cdot M$ $(a \in A)$. Then for every $a \in A$

$$\begin{aligned} \pi^{**} \circ \theta \circ \varphi(a) &= \pi^{**}(\varphi^n(a) \cdot M) \\ &= \pi^{**}(a \cdot M) \\ &= a. \end{aligned}$$

□

The following Theorem is the main result of this paper.

Theorem 2.15. *Let A be a Banach algebra with a bounded approximate identity and let φ be a continuous epimorphism on A. If A is $\varphi-$ biflat, then A is $\varphi-$ amenable.*

Proof. By Proposition 2.12, A has a $\varphi-$ virtual diagonal and so by Proposition 2.4, A has a bounded $\varphi-$ approximate diagonal. So by theorem 2.6, A is $\varphi-$ amenable. □

Theorem 2.16. *Let A be a Banach algebra and let φ be a continuous isomorphism on A. If A is $\varphi-$ amenable then A is $\varphi-$ biflat.*

Proof. Suppose that A is $\varphi-$ amenable. By Theorem 2.2, A has a $\varphi-$ virtual diagonal M. Define $\theta : A \longrightarrow (A \hat{\otimes} A)^{**}$ by $a \mapsto \varphi^{-1}(a) \cdot M$ for $a \in A$. Then

$$\begin{aligned} \pi^{**} \circ \theta \circ \varphi(a) &= \pi^{**}(a \cdot M) \\ &= a, \end{aligned}$$

it follow that A is $\varphi-$biflat. □

The following example shows that a biflat Banach algebra is not in general $\varphi-$ biflat.

Example 2.17. *Let D denote the open unit disk and let $A^+(\bar{D})$ be the set of all functions $f = \sum_{n=0}^{\infty} c_n z^n$ in the disk algebra $A(\bar{D})$ which have an absolutely convergent Taylor expansion on $\bar{D}$. Then $A^+(\bar{D})$ is a commutative unital Banach algebra and hence $A^+(\bar{D})$ is biflat. An application of Theorem 2.15, and part two of* [12, *Example* 2.5], *shows that for every $z \in D$ the Banach algebra $A^+(\bar{D})$ is not φ_z-biflat, where φ_z is the continuous idempotent homomorphism on $A^+(\bar{D})$ given by $f \mapsto f(z)$, $(f \in A^+(\bar{D}))$, which obviously is not a surjection.*

In the next result $\varphi : A \longrightarrow A$ is homomorphism and I is a closed ideal of A such that $\varphi(I) \subseteq I$. We define the map $\tilde{\varphi} : A/I \longrightarrow A/I$ by $\tilde{\varphi}(a+I) = \varphi(a) + I$.

Theorem 2.18. *Suppose that A is a $\varphi-$ biflat Banach algebra. If I is a closed deal of A, then A/I is $\tilde{\varphi}-$biflat.*

Proof. Assume that $\theta : A \longrightarrow (A \hat{\otimes} A)^{**}$ is a bounded A-bimodule map such that $\pi^{**} \circ \theta \circ \varphi = \kappa$. Let $q : A \longrightarrow A/I$ be the quotient map. Define the map $\tilde{\theta} : A/I \longrightarrow (A/I \hat{\otimes} A/I)^{**}$ by $a + I \mapsto (q \hat{\otimes} q)^{**} \circ \theta(a)$ $(a \in A)$. We prove that $\tilde{\theta}$ is an A/I- bimodul map. To see this, take $a, b, c \in A$, then we have

$$\begin{aligned} \tilde{\theta}((a+I)(b+I)(c+I)) &= \tilde{\theta}(abc+I) \\ &= (q \hat{\otimes} q)^{**} \circ \theta(abc) \\ &= (q \hat{\otimes} q)^{**}(a \cdot \theta(b) \cdot c) \\ &= a \cdot (q \hat{\otimes} q)^{**}(\theta(b) \cdot c \\ &= (a+I) \cdot \tilde{\theta}(b+I) \cdot (c+I). \end{aligned}$$

We also have

$$\begin{aligned} \pi_{A/I}^{**} \circ \tilde{\theta} \circ \tilde{\varphi}(a+I) &= \pi_{A/I}^{**} \circ \tilde{\theta}(\varphi(a)+I) \\ &= \pi_{A/I}^{**} \circ (q \hat{\otimes} q)^{**} \circ \theta(\varphi(a)) \\ &= q^{**} \circ \pi_{A}^{**} \circ \theta \circ \varphi(a) = q(a) = a + I. \end{aligned}$$

That is, A/I is $\tilde{\varphi}-$biflat. □

We quote the following result from [13].

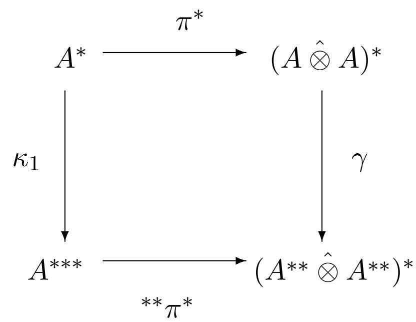
Lemma 2.19. *Let A be a Banach algebra. Then there exists an A-bimodule homomorphism $\gamma : (A \hat{\otimes} A)^* \longrightarrow (A^{**} \hat{\otimes} A^{**})^*$ such that for any functional $f \in (A \hat{\otimes} A)^*$, elements $\varphi, \psi \in A^{**}$ and nets $(a_\alpha), (b_\beta)$ in A with $w^* - lim_\alpha a_\alpha = \varphi$ and $w^* - lim_\beta b_\beta = \psi$ we have*

$$\gamma(f)(\varphi \otimes \psi) = lim_\alpha lim_\beta f(a_\alpha \otimes b_\beta).$$

We close the paper with the following result.

Theorem 2.20. *Suppose that A is a Banach algebra and $\varphi \in Hom(A)$. If A^{**} is $\varphi^{**}-$ biflat. Then A is $\varphi-$biflat.*

Proof. Let $\kappa : A \longrightarrow A^{**}$, $\kappa_1 : A^* \longrightarrow A^{***}$ and $\kappa_* : A^{**} \longrightarrow A^{****}$ denote the natural inclusions, π ($^{**}\pi$, respectively) the product maps on A (A^{**}, respectively) and let γ be defined as in Lemma 2.19. Then the following diagram commutes:



Thus $\gamma \circ \pi^* =^{**} \pi^* \circ \kappa_1$. Hence $\pi^{**} \circ \gamma^* = \kappa_1^* \circ^{**} \pi^{**}$. Since A^{**} is $\varphi^{**}-$biflat, there is an A-bimodule map $\theta_0 : A^{**} \longrightarrow (A^{**} \hat{\otimes} A^{**})^{**}$, such that $\pi^{**} \circ \theta_0 \circ \varphi^{**} = \kappa_*$. Putting $\theta := \gamma^* \circ \theta_0 \circ \kappa$, then for each $a \in A$ we have

$$\begin{aligned} \pi^{**} \circ \theta \circ \varphi(a) &= \pi^{**} \circ \gamma^* \circ \theta_0 \circ \kappa \circ \varphi(a) \\ &= \kappa_1^* \circ \pi^{**} \circ \theta_0 \circ \kappa \circ \varphi(a) \\ &= \kappa_1^* \circ \pi^{**} \circ \theta_0 \circ \varphi^{**}(a) \\ &= \kappa_1^* \circ \kappa_*(a) = a. \end{aligned}$$

That is A is $\varphi-$biflat. □

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