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ON VERTEX BALANCE INDEX SET OF SOME GRAPHS

CH. ADIGA, C. K. SUBBARAYA, A. S. SHRIKANTH *
M. A. SRIRAJ

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Abstract. Let $Z_2 = \{0,1\}$ and $G = (V, E)$ be a graph. A labeling $f : V \longrightarrow Z_2$ induces an edge labeling $f^* : E \longrightarrow Z_2$ defined by $f^*(uv) = f(u).f(v)$. For $i \in Z_2$, let $v_f(i) = v(i) = card\{v \in V : f(v) = i\}$ and $e_f(i) = e(i) = card\{e \in E : f^*(e) = i\}$. A labeling f is said to be vertex-friendly if $\mid v(0) - v(1) \mid \leq 1$. The vertex balance index set is defined by $\{\mid e_f(0) - e_f(1) \mid : f \text{ is vertex-friendly}\}$. In this paper we completely determine the vertex balance index set of K_n, $K_{m,n}$, $C_n \times P_2$ and Complete binary tree.

1. Introduction

A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. The graph labeling was first introduced in late 1960's. In the intervening years dozens of graph labeling techniques have been studied in over 1000 papers. They have often been motivated by their utility to various applied fields and their intrinsic mathematical interest. There are many graph labeling techniques studied by various authors and one of them is Cordial Graphs [1].

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*Corresponding author

Labeled graphs serve as useful models for a broad range of applications such as; coding theory, X- ray crystallography, astronomy, circuit design, communication network addressing, etc.[2]

Let G be a graph with vertex set V and edge set E, and let A be an abelian group. A labeling $f : V \longrightarrow A$ induces an edge labeling $f^+ : E \longrightarrow A$ defined by $f^+(xy) = f(x) + f(y)$. For $i \in A$, let $v_f(i) = card\{v \in V : f(v) = i\}$ and $e_f(i) = card\{e \in E : f^+(e) = i\}$. A labeling f is said to be A-friendly if $\mid v_f(i) - v_f(j) \mid \leq 1$ for all $(i,j) \in A \times A$, and A-cordial if we also have $\mid e_f(i) - e_f(j) \mid \leq 1$ for all $(i,j) \in A \times A$. When $A = Z_2$, the friendly index set of the graph G is defined as $\{\mid e_f(0) - e_f(1) \mid$:the vertex labeling f is Z_2-friendly$\}$. Recently Harris Kwong, Sin-Min Lee and Ho Kuen Ng [3] have completely determined the friendly index set of 2-regular graphs. In particular, they showed that a 2-regular graph of order n is cordial if and only if $n \neq 2(mod 4)$. Motivated by this, in this paper we introduce vertex balance index set $[VBI(G)]$ of a graph G and determine $VBI(K_n)$, $VBI(K_{m,n})$, $VBI(C_n \times P_2)$ and VBI(Complete binary tree).

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. Each labeling $f : V(G) \longrightarrow Z_2$ induces an edge labeling $f^* : E(G) \longrightarrow Z_2$, defined by $f^*(e) = f^*(uv) = f(u)f(v)$. For $i \in Z_2$, let $e_f(i) = e(i) = card\{e \in E(G) : f^*(e) = i\}$ and $v_f(i) = v(i) = card\{v \in V(G) : f(v) = i\}$.

Definition 1.1. *A labeling of a graph G is said to be vertex-friendly if $\mid v(1) - v(0) \mid \leq 1$.*

Definition 1.2. *The vertex-balance index set, $VBI(G)$, of a graph G is defined as $\{\mid e_f(1) - e_f(0) \mid$: f is vertex-friendly$\}$.*

For convenience, under the vertex labeling f, vertex with label 1 is called 1-vertex, and vertex with label 0 is called 0-vertex. Likewise, an edge is called 0-edge if its induced edge label is 0 and 1-edge if its induced edge label is 1. To prove our main results we need the following observation:

Observation 1.3. If the number of edges in a graph G is even(odd) then the $VBI(G)$ contains only even(odd) numbers.

Remark: The largest number in the $VBI(G)$ is denoted by $maxVBI(G)$.

2. **Vertex Balance Index Set of K_n and Complete Binary Tree**

In this section, we determine vertex balance index set of K_n and Complete binary tree.

Theorem 2.1. *If $n \equiv 0 \ (mod\ 2)$, then $VBI(K_n) = \{\frac{n^2}{4}\}$.*

Proof. Let f be a vertex-friendly labeling of K_n and $n = 2k$. Then $v(0) = v(1) = k$. Therefore, the total number of 1-edges in K_n is $\frac{k(k-1)}{2}$ and the total number of 0-edges is $\frac{k(3k-1)}{2}$. Hence $\mid e(0) - e(1) \mid = k^2 = \frac{n^2}{4}$. □

Theorem 2.2. *If $n \equiv 1 \ (mod\ 2)$, then $VBI(K_n) = \{\frac{(n-1)^2}{4}, \frac{(n-1)(n+3)}{4}\}$.*

Proof. Let f be a vertex-friendly labeling of K_n and $n = 2k+1$. First we choose $v(0) = k$ and $v(1) = k+1$. In this case, the total number of 1-edges in K_n is $\frac{(k+1)k}{2}$ and the total number of 0-edges is $\frac{k(3k+1)}{2}$. Therefore, $\mid e(0) - e(1) \mid = \frac{k(3k+1)}{2} - \frac{(k+1)k}{2} = k^2 = \frac{(n-1)^2}{4}$.

Next we choose $v(0) = k+1$ and $v(1) = k$. In this case, one can easily check that $\mid e(0) - e(1) \mid = \mid \frac{k(3k+3)}{2} - \frac{k(k-1)}{2} \mid = k(k+2) = \frac{(n-1)(n+3)}{4}$. □

Definition 2.3. *A complete binary tree of height h is a binary tree which contains exactly 2^d vertices at depth d, $0 \le d \le h$.*

Theorem 2.4. *If T is a complete binary tree of level n, then $VBI(T) = \{0, 2, 4, \ldots, 2^{n+1} - 2\}$.*

Proof. Let f be a vertex-friendly labeling on T. The complete binary tree T contains $2^{n+1} - 1$ vertices. Since $\mid v(0) - v(1) \mid \le 1$, we have $v(1) = 2^n$ and $v(0) = 2^n - 1$. Now denote the root by $v_{(0,1)}$ and denote the vertices in the k^{th} $(1 \le k \le n)$ level by $v_{(k,1)}, v_{(k,2)}, \ldots, v_{(k,2^k)}$. Now label all the vertices up to $(n-1)^{th}$ level by $'0'$ and all the pendent vertices by $'1'$. Then it is easy to check that $e(0) = 2^{n+1} - 2$ and there is no 1-edge in the graph. Thus $\mid e(0) - e(1) \mid = 2^{n+1} - 2$.

Now we interchange the labels of some vertices to get the remaining VBI numbers. For $0 \le r \le n-1$, by interchanging the labels of the vertices $v_{(n,\ 2^r(2q-1))}$ and $v_{(n-r-1,\ q)}$ $(1 \le q \le 2^{n-r-1})$, we get
$\mid e(0) - e(1) \mid = 2^{n+1-r} - 2(q+1)$, $(1 \le q \le 2^{n-r-1})$. □

3. **Vertex Balance Index Set of** $K_{m,n}$

In this section we find vertex balance index set of $K_{m,n}$ completely.

Theorem 3.1. *If* $m+n \equiv 0 \ (mod\ 4)$ *and* $m, n \geq \frac{m+n}{4}$, *then* $VBI(K_{m,n}) = \{mn - 2i(\frac{m+n}{2} - i) : i = 0, 1, 2, \ldots, \frac{m+n}{4}\}$.

Proof. Suppose that $m \leq n$. Let $u_1, u_2, \ldots, u_m$ and $v_1, v_2, \ldots, v_n$ denote all the vertices of $K_{m,n}$. Let f be a vertex-friendly labeling on $K_{m,n}$. Since $m+n$ is even, we have $v(0) = v(1) = \frac{m+n}{2}$. Now we label the vertices $u_1, u_2, \ldots, u_m$ by $'0'$, $v_1, v_2, \ldots, v_{\frac{m+n}{2}}$ by $'1'$ and $v_{\frac{m+n}{2}+1}, v_{\frac{m+n}{2}+2}, \ldots, v_n$ by $'0'$. Then the number of 0-edges is equal to mn and there is no 1-edge in the graph $K_{m,n}$. Therefore $\mid e(0) - e(1) \mid = mn = maxVBI(K_{m,n})$.

For $1 \leq i \leq \frac{m+n}{4}$, we label the vertices $u_1, u_2, \ldots, u_i$ by $'1'$, $u_{i+1}, u_{i+2}, \ldots, u_m$ by $'0'$, $v_1, v_2, \ldots, v_{\frac{m+n}{2}-i}$ by $'1'$ and $v_{\frac{m+n}{2}-i+1}, v_{\frac{m+n}{2}-i+2}, \ldots, v_n$ by $'0'$. Then it is easy to check that $e(1) = i(\frac{m+n}{2} - i)$ and $e(0) = mn - i(\frac{m+n}{2} - i)$. It follows that $\mid e(0) - e(1) \mid = mn - 2i(\frac{m+n}{2} - i)$.

If $i > \frac{m+n}{4}$, then $i = \frac{m+n}{4} + k$ for some positive integer k. Since $mn - 2(\frac{m+n}{4} + k)[\frac{m+n}{2} - (\frac{m+n}{4} + k)] = mn - 2(\frac{m+n}{4} - k)[\frac{m+n}{2} - (\frac{m+n}{4} - k)]$, we observe that $\mid e(0) - e(1) \mid$ corresponding to $i = \frac{m+n}{4} + k$ is the same as $\mid e(0) - e(1) \mid$ corresponding to $i = \frac{m+n}{4} - k$.

Hence, if $m+n \equiv 0 \ (mod\ 4)$, then $VBI(K_{m,n}) = \{mn - 2i(\frac{m+n}{2} - i) : i = 0, 1, 2, \ldots, \frac{m+n}{4}\}$. □

Theorem 3.2. *If* $m+n \equiv 1 \ (mod\ 4)$ *and* $m, n \geq \frac{m+n-1}{4}$, *then* $VBI(K_{m,n}) = \{mn - 2i(\frac{m+n-1}{2} - i), mn - 2i(\frac{m+n+1}{2} - i) : i = 0, 1, 2, \ldots, \frac{m+n-1}{4}\}$.

Proof. Suppose that $m \leq n$. Let $u_1, u_2, \ldots, u_m$ and $v_1, v_2, \ldots, v_n$ denote all the vertices of $K_{m,n}$. Let f be a vertex-friendly labeling on $K_{m,n}$. Since $m+n$ is odd, first we choose $v(0) = \frac{m+n+1}{2}$ and $v(1) = \frac{m+n-1}{2}$. Now we label the vertices $u_1, u_2, \ldots, u_m$ by $'0'$, $v_1, v_2, \ldots, v_{\frac{m+n-1}{2}}$ by $'1'$ and $v_{\frac{m+n-1}{2}+1}, v_{\frac{m+n-1}{2}+2}, \ldots, v_n$ by $'0'$. Then the number of 0-edges is equal to mn and there is no 1-edge in the graph $K_{m,n}$. Therefore, $\mid e(0) - e(1) \mid = mn = maxVBI(K_{m,n})$.

For $1 \le i \le \frac{m+n-1}{4}$, we label the vertices $u_1, u_2, \ldots, u_i$ by $'1'$, $u_{i+1}, u_{i+2}, \ldots, u_m$ by $'0'$, $v_1, v_2, \ldots, v_{\frac{m+n-1}{2}-i}$ by $'1'$ and $v_{\frac{m+n-1}{2}-i+1}, v_{\frac{m+n-1}{2}-i+2}, \ldots, v_n$ by $'0'$. Then it is easy to check that $e(1) = i(\frac{m+n-1}{2} - i)$ and $e(0) = mn - i(\frac{m+n-1}{2} - i)$. Thus $\mid e(0) - e(1) \mid = mn - 2i(\frac{m+n-1}{2} - i)$.

If $i > \frac{m+n-1}{4}$, then $i = \frac{m+n-1}{4} + k$ for some positive integer k. Since $mn - 2(\frac{m+n-1}{4} + k)[\frac{m+n-1}{2} - (\frac{m+n-1}{4} + k)] = mn - 2(\frac{m+n-1}{4} - k)[\frac{m+n-1}{2} - (\frac{m+n-1}{4} - k)]$, we observe that $\mid e(0) - e(1) \mid$ corresponding to $i = \frac{m+n-1}{4} + k$ is the same as $\mid e(0) - e(1) \mid$ corresponding to $i = \frac{m+n-1}{4} - k$.

Next we choose $v(0) = \frac{m+n-1}{2}$ and $v(1) = \frac{m+n+1}{2}$ and label the vertices $u_1, u_2, \ldots, u_m$ by $'0'$, $v_1, v_2, \ldots, v_{\frac{m+n+1}{2}}$ by $'1'$ and $v_{\frac{m+n+1}{2}+1}, v_{\frac{m+n+1}{2}+2}, \ldots, v_n$ by $'0'$. Then the number of 0-edges is equal to mn and there is no 1-edge in the graph $K_{m,n}$. Therefore, $\mid e(0) - e(1) \mid = mn = maxVBI(K_{m,n})$.

For $1 \le i \le \frac{m+n-1}{4}$, we label the vertices $u_1, u_2, \ldots, u_i$ by $'1'$, $u_{i+1}, u_{i+2}, \ldots, u_m$ by $'0'$, $v_1, v_2, \ldots, v_{\frac{m+n+1}{2}-i}$ by $'1'$ and $v_{\frac{m+n+1}{2}-i+1}, v_{\frac{m+n+1}{2}-i+2}, \ldots, v_n$ by $'0'$. Then it is easy to check that $e(1) = i(\frac{m+n+1}{2} - i)$ and $e(0) = mn - i(\frac{m+n+1}{2} - i)$. Thus $\mid e(0) - e(1) \mid = mn - 2i(\frac{m+n+1}{2} - i)$.

If $i > \frac{m+n-1}{4}$, then $i = \frac{m+n-1}{4} + k$ for some positive integer k. Since $mn - 2(\frac{m+n-1}{4} + k)[\frac{m+n+1}{2} - (\frac{m+n-1}{4} + k)] = mn - 2(\frac{m+n-1}{4} - k + 1)[\frac{m+n+1}{2} - (\frac{m+n-1}{4} - k + 1)]$, we observe that $\mid e(0) - e(1) \mid$ corresponding to $i = \frac{m+n-1}{4} + k$ is the same as $\mid e(0) - e(1) \mid$ corresponding to $i = \frac{m+n-1}{4} - k + 1$.

Hence if $m + n \equiv 1 \ (mod\ 4)$, then $VBI(K_{m,n}) = \{mn - 2i(\frac{m+n-1}{2} - i), mn - 2i(\frac{m+n+1}{2} - i) : i = 0, 1, 2, \ldots, \frac{m+n-1}{4}\}$. □

Theorem 3.3. *If* $m+n \equiv 2 \ (mod\ 4)$ *and* $m, n \ge \frac{m+n-2}{4}$, *then* $VBI(K_{m,n}) = \{mn - 2i(\frac{m+n}{2} - i) : i = 0, 1, 2, \ldots, \frac{m+n-2}{4}\}$.

Theorem 3.4. *If* $m+n \equiv 3 \ (mod\ 4)$ *and* $m, n \ge \frac{m+n+1}{4}$, *then* $VBI(K_{m,n}) = \{mn - 2i(\frac{m+n-1}{2} - i) : i = 0, 1, 2, \ldots, \frac{m+n-3}{4}\} \bigcup \{mn - 2i(\frac{m+n+1}{2} - i) : i = 0, 1, 2, \ldots, \frac{m+n+1}{4}\}$.

As the proofs of Theorems 3.3 and 3.4 are same as that of Theorems 3.1 and 3.2, we omit the proofs.

4. Vertex Balance Index Set of $C_n \times P_2$

Recently Ying Wang, Yuge Zheng and Sin-Min Lee [4] obtained the edge balance index set of $C_n \times P_2$. In this section we determine the vertex balance index set of $C_n \times P_2$.

Theorem 4.1. *If $n \equiv 0$ (mod 2), then the* $VBI(C_n \times P_2) = \{4, 6, 8, \ldots, 3n-6, 3n-4, 3n\}$.

Proof. Let f be a vertex-friendly labeling on $C_n \times P_2$. In $C_n \times P_2$, there are $2n$ vertices and $3n$ edges. Since the number of vertices is even and $\mid v(0) - v(1) \mid \leq 1$, we have $v(0) = v(1) = n$. First we label the vertices of $C_n \times P_2$ in order to obtain the $maxVBI(C_n \times P_2)$.

We denote the vertices of outer cycle of $C_n \times P_2$ by $v_1, v_2, \ldots, v_n$ and vertices of inner cycle by $u_1, u_2, \ldots, u_n$. We label the vertices $v_{2q-1}, u_{2q} (1 \leq q \leq \frac{n}{2})$ by $'1'$, and $v_{2q}, u_{2q-1} (1 \leq q \leq \frac{n}{2})$ by $'0'$. Then the number of 0-edges is equal to $3n$ and there is no 1-edge in the construction. Thus $\mid e(0) - e(1) \mid = 3n = maxVBI(C_n \times P_2)$.

Now we interchange the labels of some vertices to get the remaining VBI numbers. For example, by interchanging the labels of v_1 and v_n ($v_1 \longleftrightarrow v_n$) the number of 0-edges will decrease where as the number of 1-edges will increase and hence the difference $\mid e(0) - e(1) \mid$ will reduce.

i.e, $u_1 \longleftrightarrow u_n$ $\quad \mid e(0) - e(1) \mid = 3n - 4$

$u_1 \longleftrightarrow u_n, v_2 \longleftrightarrow u_2$ $\quad \mid e(0) - e(1) \mid = 3n - 6$

$v_2 \longleftrightarrow u_2, v_4 \longleftrightarrow u_4$ $\quad \mid e(0) - e(1) \mid = 3n - 8$

$u_1 \longleftrightarrow u_n, v_2 \longleftrightarrow u_2, v_4 \longleftrightarrow u_4$ $\quad \mid e(0) - e(1) \mid = 3n - 10$

$\vdots$

$v_2 \longleftrightarrow u_2, v_4 \longleftrightarrow u_4, \ldots, v_{n-2} \longleftrightarrow u_{n-2}$ $\quad \mid e(0) - e(1) \mid = n + 4$

$u_1 \longleftrightarrow u_n, v_2 \longleftrightarrow u_2, v_4 \longleftrightarrow u_4,$
$\ldots, v_{n-2} \longleftrightarrow u_{n-2}$ $\quad \mid e(0) - e(1) \mid = n + 2$

$u_1 \longleftrightarrow u_n, v_2 \longleftrightarrow u_2, v_4 \longleftrightarrow u_4, \ldots, v_{n-2} \longleftrightarrow u_{n-2},$
$v_{n-1} \longleftrightarrow u_2$ $\quad \mid e(0) - e(1) \mid = n$

$u_1 \longleftrightarrow u_n, v_2 \longleftrightarrow u_2, v_4 \longleftrightarrow u_4 \ldots v_{n-2} \longleftrightarrow u_{n-2},$
$v_{n-1} \longleftrightarrow u_2, v_{n-2} \longleftrightarrow u_3$ $\quad \mid e(0) - e(1) \mid = n - 2$

$\vdots$

$u_1 \longleftrightarrow u_n, v_2 \longleftrightarrow u_2, v_4 \longleftrightarrow u_4, \ldots, v_{n-2} \longleftrightarrow u_{n-2},$
$v_{n-1} \longleftrightarrow u_2, v_{n-2} \longleftrightarrow u_3, \ldots, v_{\frac{n}{2}+2} \longleftrightarrow u_{\frac{n}{2}-1}$ $\quad | e(0) - e(1) |= 6$
$u_1 \longleftrightarrow u_n, v_2 \longleftrightarrow u_2, v_4 \longleftrightarrow u_4, \ldots, v_{n-2} \longleftrightarrow u_{n-2},$
$v_{n-1} \longleftrightarrow u_2, v_{n-2} \longleftrightarrow u_3, \ldots, v_{\frac{n}{2}+1} \longleftrightarrow u_{\frac{n}{2}}$ $\quad | e(0) - e(1) |= 4.$

And by the similar construction it is easy to show that the numbers 0, 2 and $3n-2$ do not belong to $VBI(C_n \times P_2)$. □

Theorem 4.2. *If* $n \equiv 1(mod\ 2)$*, then the* $VBI(C_n \times P_2) = \{5, 7, 9, \ldots, 3n-4, 3n-2\}$.

Proof. Let f be a vertex-friendly labeling of $C_n \times P_2$. There are $2n$ vertices and $3n$ edges in the graph $C_n \times P_2$. Since the number of vertices is even and $| v(0) - v(1) |\leq 1$, we have $v(0) = v(1) = n$. First we label the vertices of $C_n \times P_2$ in order to obtain the $maxVBI(C_n \times P_2)$.

Denote the vertices of $C_n \times P_2$ as in Theorem 4.1. Now we label the vertices $v_{2q-1}(1 \leq q \leq \frac{n-1}{2}+1)$ and $u_{2q}(1 \leq q \leq \frac{n-1}{2})$ by $'1'$, $v_{2q}(1 \leq q \leq \frac{n-1}{2})$ and $u_{2q-1}(1 \leq q \leq \frac{n-1}{2})$ by $'0'$. Then the number of 0-edges is equal to $3n-1$ and 1-edges is equal to one. Thus, $| e(0) - e(1) |=| 3n-1-1 |= 3n-2 = maxVBI(C_n \times P_2)$.

Now we interchange the labels of some vertices to get the remaining VBI numbers. For example, by interchanging the labels of u_n and u_{n-1}, the number of 0-edges will decrease, where as the number of 1-edges will increase and hence the difference $| e(0) - e(1) |$ will reduce.

i.e, $u_n \longleftrightarrow u_{n-1}$ $\quad | e(0) - e(1) |= 3n-4.$
$v_2 \longleftrightarrow u_2$ $\quad | e(0) - e(1) |= 3n-6.$
$u_n \longleftrightarrow u_{n-1}, v_2 \longleftrightarrow u_2$ $\quad | e(0) - e(1) |= 3n-8$
$v_2 \longleftrightarrow u_2, v_4 \longleftrightarrow u_4$ $\quad | e(0) - e(1) |= 3n-10$

$\vdots$

$v_2 \longleftrightarrow u_2, v_4 \longleftrightarrow u_4, \ldots, v_{n-3} \longleftrightarrow u_{n-3}$ $\quad | e(0) - e(1) |= n+4$
$u_n \longleftrightarrow u_{n-1}, v_2 \longleftrightarrow u_2, v_4 \longleftrightarrow u_4, \ldots,$
$v_{n-3} \longleftrightarrow u_{n-3}$ $\quad | e(0) - e(1) |= n+2$
$u_n \longleftrightarrow u_{n-1}, v_2 \longleftrightarrow u_2, v_4 \longleftrightarrow u_4, \ldots,$
$v_{n-3} \longleftrightarrow u_{n-3}, v_{n-2} \longleftrightarrow u_1$ $\quad | e(0) - e(1) |= n$
$u_n \longleftrightarrow u_{n-1}, v_2 \longleftrightarrow u_2, v_4 \longleftrightarrow u_4, \ldots,$
$v_{n-3} \longleftrightarrow u_{n-3}, v_{n-2} \longleftrightarrow u_1, v_{n-3} \longleftrightarrow u_2$ $\quad | e(0) - e(1) |= n-2$

$\vdots$

$u_n \longleftrightarrow u_{n-1}, v_2 \longleftrightarrow u_2, v_4 \longleftrightarrow u_4, \ldots, v_{n-3} \longleftrightarrow u_{n-3},$
$v_{n-2} \longleftrightarrow u_1, v_{n-3} \longleftrightarrow u_2, \ldots v_{\frac{n+3}{2}} \longleftrightarrow u_{\frac{n-5}{2}} \qquad \mid e(0) - e(1) \mid = 7$
$u_n \longleftrightarrow u_{n-1}, v_2 \longleftrightarrow u_2, v_4 \longleftrightarrow u_4, \ldots, v_{n-3} \longleftrightarrow u_{n-3},$
$v_{n-2} \longleftrightarrow u_1, v_{n-3} \longleftrightarrow u_2, \ldots v_{\frac{n+1}{2}} \longleftrightarrow u_{\frac{n-3}{2}} \qquad \mid e(0) - e(1) \mid = 5.$

And by the similar construction it is easy to show that the numbers 1 and 3 do not belong to $VBI(C_n \times P_2)$.

□

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Ch. Adiga, A. S. Shrikanth and M. A. Sriraj
Department of Studies in Mathematics, University of Mysore, Manasagangothri, Mysore-577006, India
Email: `c_adiga@hotmail.com`
Email: `shrikanth.ait@gmail.com`
Email: `srinivasa_sriraj@yahoo.co.in`

C. K. Subbaraya
Department of Mathematics, Adichunchanagiri Institute of Technology, Chikamagalur - 577102, India
Email: `subrayack@gmail.com`