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GORENSTEIN GLOBAL DIMENSIONS FOR HOPF ALGEBRA ACTIONS

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ABSTRACT. Let H be a Hopf algebra and A an H-bimodule algebra. In this paper, we investigate Gorenstein global dimensions for Hopf algebras and twisted smash product algebras $A \star H$. Results from the literature are generalized.

Keywords: Hopf algebras, Gorenstein global dimensions, Morita equivalences, twisted smash products.
MSC(2010): Primary: 16T05; Secondary: 18G25.

1. Introduction

Enochs, Jenda and Holm [4–6] introduced Gorenstein projective, injective and flat dimensions for arbitrary (not necessarily finitely generated) module. Using them, Bennis and Mahdou [2] established the (weak) Gorenstein global dimension of a ring, which behaves like the classical (weak) global dimension. For a Hopf algebra over a field k, the left global dimension and the weak global dimension coincide with the projective dimension and the flat dimension of k respectively. A natural question is whether they hold for Gorenstein global dimensions. In Section 2, we shall give an affirmative answer.

Let H be a Hopf algebra and A a bimodule algebra. In 1998, Wang and Li [13] constructed the twisted smash product algebra $A \star H$. The usual smash product, the Drinfel'd double and the Doi-Takeuchi's algebra are all special cases of $A \star H$. Section 3 is devoted to investigating its left Gorenstein global dimensions.

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Throughout this paper, R is a ring, k is a field, A is an algebra and H is a Hopf algebra with antipode S. ${}_R\mathcal{M}$ denotes the category of left R-modules, $\mathcal{P}$ denotes the class of projective left R-modules and $\mathcal{I}$ denotes the class of injective right R-modules. We always work over k and use Sweedler's notations [12].

1.1. A *left integral* in H is an element $l \in H$ such that $hl = \epsilon(h)l$ for every $h \in H$. A *right integral* is defined similarly. The space of left (right) integrals in H is denoted by $\int_l$ $(\int_r)$. If $\int_l = \int_r$, then we say H is *unimodular*.

1.2. Let A be both an algebra and a left H-module by the action $\rightharpoonup: H \otimes A \to A$. If for any $h \in H, a, b \in A$, $h \rightharpoonup (ab) = \Sigma(h_1 \rightharpoonup a)(h_2 \rightharpoonup b)$, $h \rightharpoonup 1_A = \epsilon(h)1_A$ hold, then A is called a *left H-module algebra*.

1.3. Let A be both an algebra and a right H-module by the action $\leftharpoonup: A \otimes H \to A$. If for any $h \in H, a, b \in A$, $(ab) \leftharpoonup h = \Sigma(a \leftharpoonup h_1)(b \leftharpoonup h_2)$, $1_A \leftharpoonup h = \epsilon(h)1_A$ hold, then A is called a *right H-module algebra*.

1.4. Let A be both an algebra and an H-bimodule. If A is both a left H-module algebra and a right H-module algebra, then A is called an *H-bimodule algebra*.

1.5. A left R-module M is *Gorenstein projective* if there exists a $Hom_R(-, \mathcal{P})$-exact exact sequence $\cdots \to P_1 \to P_0 \to P^0 \to P^1 \to \cdots$ with every P^i, P_i projective such that $M = \ker(P^0 \to P^1)$. *Gorenstein injective left R-modules* are defined dually. A left R-module M is *Gorenstein flat* if there is an $\mathcal{I} \otimes_R -$-exact exact sequence $\cdots \to F^1 \to F^0 \to F_0 \to F_1 \to \cdots$ with every F^i, F_i flat such that $M = \ker(F_0 \to F_1)$.

1.6. For a left R-module M, the *Gorenstein projective dimension* $Gpd_R(M)$ is at most n if there is an exact sequence $0 \to G_n \to G_{n-1} \to \cdots \to G_1 \to G_0 \to M \to 0$ with every G_i Gorenstein projective. The *Gorenstein injective dimension* $Gid_R(M)$ is defined dually, the *Gorenstein flat dimension* $Gfd_R(M)$ is defined similarly.

1.7. For any ring R, [2, Theorem 1.1] shows that

$$\sup\{Gpd_R(M) | M \in {}_R\mathcal{M}\} = \sup\{Gid_R(M) | M \in {}_R\mathcal{M}\}.$$

The common value is called the *left Gorenstein global dimension* of R and denote it by $l.Ggldim(R)$. Similarly, we set

$$l.wGgldim(R) = \sup\{Gfd_R(M) | M \in {}_R\mathcal{M}\}$$

and call this quantity the *left weak Gorenstein global dimension* of R.

By [3, Theorem 2.2], $l.Ggldim(R) = 0$ if and only if R is quasi-Frobenius. R is *left Gorenstein hereditary* [8] if every submodule of a projective left R-module is Gorenstein projective, i.e., $l.Ggldim(R) \leq 1$.

1.8. Two rings R and R' are *Morita equivalent* [1] in case ${}_R\mathcal{M} \approx {}_{R'}\mathcal{M}$, that is, there are two covariant functors $F : {}_R\mathcal{M} \to {}_{R'}\mathcal{M}$ and $G : {}_{R'}\mathcal{M} \to {}_R\mathcal{M}$ such that $GF \cong 1_{{}_R\mathcal{M}}$ and $FG \cong 1_{{}_{R'}\mathcal{M}}$.

Lemma 1.1. *Let R and R' be Morita equivalent rings via inverse equivalent functors $F : {}_R\mathcal{M} \to {}_{R'}\mathcal{M}$ and $G : {}_{R'}\mathcal{M} \to {}_R\mathcal{M}$, and let M be a left R-module. Then M is Gorenstein projective if and only if $F(M)$ is Gorenstein projective.*

Proof. Assume that M is a Gorenstein projective left R-module. Then there is a left R-module exact sequence $\mathbb{P} = \cdots \to P_1 \to P_0 \to P^0 \to P^1 \to \cdots$ with every P_i, P^i projective such that $Hom_R(\mathbb{P}, Q)$ is exact for any projective left R-module Q and $M = \ker(P^0 \to P^1)$. By [1, Proposition 21.4 and Proposition 21.6], we get a left R'-module exact sequence

$$F(\mathbb{P}) = \cdots \to F(P_1) \to F(P_0) \to F(P^0) \to F(P^1) \to \cdots$$

with every $F(P_i), F(P^i)$ projective and $F(M) = \ker(F(P^0) \to F(P^1))$. For any projective left R'-module Q', since $G(Q')$ is projective as a left R-module by [1, Proposition 21.6] and $Hom_{R'}(F(\mathbb{P}), Q') cong Hom_R(\mathbb{P}, G(Q'))$ by [1, Lemma 21.3], $Hom_{R'}(F(\mathbb{P}), Q')$ is exact. Hence $F(M)$ is Gorenstein projective.

Similarly, if $F(M)$ is a Gorenstein projective left R'-module, then $GF(M)$ is a Gorenstein projective left R-module, i.e., M is Gorenstein projective. □

Proposition 1.2. *Let R and R' be Morita equivalent rings. Then*

$$l.Ggldim(R) = l.Ggldim(R').$$

Proof. Assume $l.Ggldim(R) = n$, a nonnegative integer. For any left R'-module M, there is a left R-module exact sequence $0 \to G_n \to \cdots \to G_1 \to G_0 \to G(M) \to 0$ with every G_i Gorenstein projective. By [1, Proposition 21.4] and Lemma 1.1, we get a left R'-module exact sequence

$$0 \to F(G_n) \to \cdots \to F(G_1) \to F(G_0) \to M \to 0$$

with every $F(G_i)$ Gorenstein projective. Then $Gpd_{R'}(M) \leq n$. This means that $l.Ggldim(R') \leq l.Ggldim(R)$. Similarly, $l.Ggldim(R) \leq l.Ggldim(R')$. □

2. Gorenstein global dimensions for Hopf algebras

Lemma 2.1. *Let H be a Hopf algebra and X any left H-module.*

(1) *If P is a projective left H-module, then so is $P \otimes_k X$.*
(2) *If F is a flat left H-module, then so is $F \otimes_k X$.*

Proof. (1) If P is a projective left H-module, then there is a left H-module Q such that $P \oplus Q \cong H^{(I)}$ for some set I, thus $P \otimes_k X \oplus Q \otimes_k X \cong (H \otimes_k X)^{(I)}$. Since $H \otimes_k X$ is free as a left H-module, $P \otimes_k X$ is projective.

(2) If F is a flat left H-module, then there is a directed set Λ such that $F \cong \varinjlim F_\lambda$, where $\lambda \in \Lambda$ and F_λ is a free left H-module, thus $F \otimes_k X \cong \varinjlim (F_\lambda \otimes_k X)$. By (1), $F_\lambda \otimes_k X$ is a projective (flat) left H-module, hence $F \otimes_k X$ is flat. □

Proposition 2.2. *Let H be a Hopf algebra and X any left H-module.*
(1) *If M is a Gorenstein projective left H-module, then so is $M \otimes_k X$.*
(2) *If N is a Gorenstein flat left H-module, then so is $N \otimes_k X$.*

Proof. We only give the proof of part (1). If M is a Gorenstein projective left H-module, then there is a $Hom_H(-, \mathcal{P})$-exact exact sequence $\mathbb{P} = \cdots \to P_1 \to P_0 \to P^0 \to P^1 \to \cdots$ with every P^i, P_i projective such that $M = \ker(P^0 \to P^1)$. Thus we get an exact sequence

$$\mathbb{P} \otimes_k X = \cdots \to P_1 \otimes_k X \to P_0 \otimes_k X \to P^0 \otimes_k X \to P^1 \otimes_k X \to \cdots$$

such that $M \otimes_k X = \ker(P^0 \otimes_k X \to P^1 \otimes_k X)$. By Lemma 2.1(1), every $P^i \otimes_k X$ and every $P_i \otimes_k X$ are projective. For any projective left H-module Q,

$$Hom_H(\mathbb{P} \otimes_k X, Q) \cong Hom_k(X, Hom_H(\mathbb{P}, Q)),$$

hence $Hom_H(\mathbb{P} \otimes_k X, Q)$ is exact, as desired. □

It is trivial that k is a left H-module, i.e., $h \cdot k = \epsilon(h)k$.

Theorem 2.3. *$l.Ggldim(H) = Gpd_H(k)$ and $l.wGgldim(H) = Gfd_H(k)$.*

Proof. We only prove the first equality. Clearly, $Gpd_H(k) \leq l.Ggldim(H)$. We shall prove the reverse inequality. Assume that $Gpd_H(k) = n < \infty$. Then there is an exact sequence

$$0 \to G_n \to G_{n-1} \to \cdots \to G_1 \to G_0 \to k \to 0$$

where every G_i is a Gorenstein projective left H-module. For any left H-module X, we get an exact sequence

$$0 \to G_n \otimes_k X \to G_{n-1} \otimes_k X \to \cdots \to G_1 \otimes_k X \to G_0 \otimes_k X \to k \otimes_k X \to 0.$$

Since $k \otimes_k X \cong X$ and every $G_i \otimes_k X$ is Gorenstein projective by Proposition 2.2(1), $Gpd_H(X) \leq n$. This shows $l.Ggldim(H) \leq Gpd_H(k)$. □

3. Gorenstein global dimensions for twisted smash products

Definition 3.1. *Let H be a Hopf algebra and A an H-bimodule algebra. The* twisted smash product $A \star H$ *is defined as follows:* $A \star H = A \otimes H$ *as k-modules and its multiplication is given by*

$$(a \star h)(b \star g) = \Sigma a(h_1 \rightharpoonup b \leftharpoonup S(h_3)) \star h_2 g$$

for any $a, b \in A$ and $h, g \in H$.

By [13], $A \star H$ is an associative algebra with unit $1_A \star 1_H$. Clearly, $i_A : A \to A \star H$, $a \mapsto a \star 1_H$ and $i_H : H \to A \star H$, $h \mapsto 1_A \star h$ are two algebra embedding maps.

It is trivial that $A \star H$ is free as a left A-module, this is also true on the right:

Proposition 3.2. *Let H be a finite dimensional Hopf algebra and A an H-bimodule algebra. Then the map*

$$\phi : A \star H \to H \otimes A, a \star h \mapsto \Sigma h_2 \otimes (S^{-1}(h_1) \rightharpoonup a \leftharpoonup h_3)$$

gives an isomorphism of right A-modules, where the right A-actions on $A \star H$ and $H \otimes A$ are defined respectively as follows

$$(a \star h) \cdot b = \Sigma a(h_1 \rightharpoonup b \leftharpoonup S^{-1}(h_3)) \star h_2, \quad (h \otimes a) \cdot b = h \otimes ab.$$

Proof. We define

$$\psi : H \otimes A \to A \star H, \quad h \otimes a \mapsto \Sigma(h_1 \rightharpoonup a \leftharpoonup S^{-1}(h_3)) \star h_2.$$

It is easy to check that ϕ and ψ are two right A-module maps, and $\psi\phi = id$ and $\phi\psi = id$. □

Let $A^{(-)} :_{A\star H} \mathcal{M} \to {}_A\mathcal{M}$ and ${}^{(-)}A : \mathcal{M}_{A\star H} \to \mathcal{M}_A$ be the left and right restriction functors respectively.

Lemma 3.3. *Let H be a finite dimensional Hopf algebra. Then*

(1) *$(A\star H \otimes_A -, A^{(-)})$ and $(A^{(-)}, A\star H \otimes_A -)$ are double adjunctions.*
(2) *$(- \otimes_A A\star H, {}^{(-)}A)$ and $({}^{(-)}A, - \otimes_A A\star H)$ are double adjunctions.*

Proof. Since H is finite dimensional, the assertions may directly follow from the adjoint isomorphism theorem and the functor isomorphisms $Hom_A(A\star H, -) \simeq A\star H \otimes_A -$ and $Hom_A(A\star H, -) \simeq - \otimes_A A\star H$. □

Corollary 3.4. *Let H be a finite dimensional Hopf algebra.*

(1) *If P is a projective left A-module, then $A\star H \otimes_A P$ is a projective left $A \star H$-module.*

(2) *If P is a projective left $A\star H$-module, then P is a projective left A-module.*

Lemma 3.5. *Let H be a finite dimensional Hopf algebra. If G is a Gorenstein projective left A-module, then $A\star H\otimes_A G$ is a Gorenstein projective left $A\star H$-module.*

Proof. Let $\mathbb{P} = \cdots \to P_{-2} \to P_{-1} \to P_0 \to P_1 \to \cdots$ be an exact sequence with every P_i projective such that $Hom_A(\mathbb{P}, Q)$ is exact for any projective left A-module Q and $G = \ker(P_0 \to P_1)$. By Corollary 3.4(1), every $A\star H\otimes_A P_i$ is a projective left $A\star H$-module. Hence $A\star H\otimes_A \mathbb{P}$ is an exact sequence of projective left $A\star H$-modules such that $A\star H\otimes_A G = \ker(A\star H\otimes_A P_0 \to A\star H\otimes_A P_1)$ since $A\star H$ is a free right A-module. Moreover, for any projective left $A\star H$-module Q',

$$Hom_{A\star H}(A\star H\otimes_A \mathbb{P}, Q') \cong Hom_A(\mathbb{P}, Q'),$$

hence $\mathrm{Hom}_{A\star H}(A\star H\otimes_A \mathbb{P}, Q')$ is exact by Corollary 3.4(2). □

It is easy to see that the equality

$$a\star h = (a\star 1_H)(1_A\star h) \tag{3.1}$$

holds for any $a\in A$ and $h\in H$. If H is a finite dimensional semisimple Hopf algebra, then we can choose a right integral t such that $\epsilon(t) = 1$. Thus we have

$$\Sigma hS(t_1)\otimes t_2 = \Sigma S(t_1)\otimes t_2 h. \tag{3.2}$$

By [10, Formula 15], we have $1\otimes t = \Sigma S^{-1}(t_3)g^{-1}t_1\otimes t_2$, where g is the distinguished group-like element of H. If H^* is unimodular, then $g = 1$, thus we get

$$\Sigma 1\otimes t_1\otimes t_2 = \Sigma S^{-1}(t_4)t_1\otimes t_2\otimes t_3. \tag{3.3}$$

Proposition 3.6. *Let H be a finite dimensional semisimple Hopf algebra such that H^* is unimodular. If M is a left $A\star H$-module, then M is a direct summand of $A\star H\otimes_A M$.*

Proof. We define

$$\phi : A\star H\otimes_A M \to M,\ a\star h\otimes_A m \mapsto (a\star h)\cdot m.$$

Clearly, ϕ is a left $A\star H$-module epimorphism. We also define

$$\psi : M \to A\star H\otimes_A M,\ m \mapsto 1_A\star S(t_1)\otimes_A (1_A\star t_2)\cdot m.$$

For any $a \in A$, $m \in M$ and $h \in H$,

$$\begin{aligned}
\psi((a \star 1_H) \cdot m) &= \Sigma 1_A \star S(t_1) \otimes_A (1_A \star t_2)(a \star 1_H) \cdot m \\
&= \Sigma 1_A \star S(t_1) \otimes_A (t_2 \rightharpoonup a \leftharpoonup S(t_4) \star t_3) \cdot m \\
&\overset{(3.1)}{=} \Sigma(1_A \star S(t_1)) \otimes_A (t_2 \rightharpoonup a \leftharpoonup S(t_4) \star 1_H)(1_A \star t_3) \cdot m \\
&= \Sigma(1_A \star S(t_1))(t_2 \rightharpoonup a \leftharpoonup S(t_4) \star 1_H) \otimes_A (1_A \star t_3) \cdot m \\
&= \Sigma a \leftharpoonup S(S(t_1)t_4) \star S(t_2) \otimes_A (1_A \star t_3) \cdot m \\
&\overset{(3.3)}{=} \Sigma a \star S(t_1) \otimes_A (1_A \star t_2) \cdot m \\
&= \Sigma(a \star 1_H)(1_A \star S(t_1)) \otimes_A (1_A \star t_2) \cdot m \\
&= (a \star 1_H) \cdot \psi(m), \\
\psi((1_A \star h) \cdot m)) &= \Sigma 1_A \star S(t_1) \otimes_A (1_A \star t_2)(1_A \star h) \cdot m \\
&= \Sigma 1_A \star S(t_1) \otimes_A (1_A \star t_2 h) \cdot m \\
&\overset{(3.2)}{=} \Sigma 1_A \star hS(t_1) \otimes_A (1_A \star t_2) \cdot m \\
&= (1_A \star h) \cdot \psi(m).
\end{aligned}$$

Hence, by equality (3.1), ψ is a left $A \star H$-module map. Finally, it is easy to check that $\phi\psi = id_M$ since $\epsilon(t) = 1$, as desired. □

Theorem 3.7. *Let H be a finite dimensional semisimple Hopf algebra such that H^* is unimodular. Then $l.Ggldim(A \star H) \leq l.Ggldim(A)$.*

Proof. Assume $l.Ggldim(A) = n < \infty$. For any $M \in {}_{A\star H}\mathcal{M}$, as a left A-module, there is a Gorenstein projective resolution $0 \to G_n \to \cdots \to G_1 \to G_0 \to M \to 0$ with every G_i Gorenstein projective, which induces an exact sequence

$$0 \to A \star H \otimes_A G_n \to \cdots \to A \star H \otimes_A G_1 \to A \star H \otimes_A G_0 \to A \star H \otimes_A M \to 0$$

since $A \star H \otimes_A -$ is exact. Since every $A \star H \otimes_A G_i$ is a Gorenstein projective left $A \star H$-module by Lemma 3.5, $Gpd_{A\star H}(A \star H \otimes_A M) \leq n$. By [6, Proposition 2.19], $Gpd_{A\star H}(M) \leq n$ since M is a direct summand of $A \star H \otimes_A M$ as left $A \star H$-modules. Hence $l.Ggldim(A \star H) \leq n = l.Ggldim(A)$. □

Next we give a Maschke-type theorem:

Corollary 3.8. *Let H be a finite dimensional semisimple Hopf algebra such that H^* is unimodular.*

(1) *If A is quasi-Frobenius, then so is $A \star H$.*
(2) *If A is left Gorenstein hereditary, then so is $A \star H$.*

In 2006, Jiao and Dong [7] gave the duality theorem for twisted smash product algebras: If H is a finite dimensional cocommutative Hopf algebra, then there is an isomorphism of algebras:

$$(A \star H)\#H^* \cong A \otimes (H\#H^*) \cong A \otimes M_n(k) \cong M_n(A).$$

Thus $(A \star H)\#H^*$ is Morita equivalent to A.

Theorem 3.9. *Let H be a finite dimensional semisimple cosemisimple cocommutative Hopf algebra. Then $l.Ggldim(A \star H) = l.Ggldim(A)$.*

Proof. Since H is cosemisimple, H^* is unimodular by [9, Corollary 2.2.4]. Then,

$$\begin{aligned} l.Ggldim(A) &= l.Ggldim((A \star H)\#H^*) \\ &\leq l.Ggldim(A \star H) \leq l.Ggldim(A). \end{aligned}$$

This means that $l.Ggldim(A \star H) = l.Ggldim(A)$. □

Corollary 3.10. *Let H be a finite dimensional semisimple cosemisimple cocommutative Hopf algebra. Then*

(1) *$A \star H$ is quasi-Frobenius if and only if so is A.*
(2) *$A \star H$ is left Gorenstein hereditary if and only if so is A.*

Corollary 3.11. *Let H be a finite dimensional cocommutative Hopf algebra. If $char(k)$ does not divide $\dim_k(H)$, then*

(1) *$l.Ggldim(A \star H) = l.Ggldim(A)$.*
(2) *$A \star H$ is quasi-Frobenius if and only if so is A.*
(3) *$A \star H$ is left Gorenstein hereditary if and only if so is A.*

Proof. Since H is cocommutative, $S^2 = id$ by [9, Corollary 1.5.12]. Consequently, by [11, Proposition 2], H is semisimple and cosemisimple if and only if $\mathrm{char}(k)$ does not divide $\dim_k(H)$. Hence (1)-(3) follow from Theorem 3.9 and Corollary 3.10. □

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