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WEAK BANACH-SAKS PROPERTY IN THE SPACE OF COMPACT OPERATORS

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Abstract. For suitable Banach spaces X and Y with Schauder decompositions and a suitable closed subspace $\mathcal{M}$ of some compact operator space from X to Y, it is shown that the strong Banach-Saks-ness of all evaluation operators on $\mathcal{M}$ is a sufficient condition for the weak Banach-Saks property of $\mathcal{M}$, where for each $x \in X$ and $y^* \in Y^*$, the evaluation operators on $\mathcal{M}$ are defined by $\phi_x(T) = Tx$ and $\psi_{y^*}(T) = T^*y^*$.
Keywords: weak Banach-Saks property, $\mathcal{P}$-property, Schauder decomposition, compact operator, completely continuous operator.
MSC(2010): Primary: 47L05; Secondary: 47L20, 46B28, 46B99.

1. Introduction

A Banach space X has the weak Banach-Saks property if every weakly null sequence (x_n) in X has a subsequence (x_{n_k}) whose arithmetic means sequence is norm convergent to zero, that is,

$$\lim_{k\to\infty} \|\frac{1}{k}(x_{n_1} + ... + x_{n_k})\| = 0.$$

But if every bounded sequence in X has a subsequence whose arithmetic means sequence is norm convergent, we say that X has the Banach-Saks property. It is evident that Banach-Saks property implies the weak Banach-Saks property and in reflexive Banach spaces, they coincide.

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For example, Banach and Saks have shown in [2] that $L^p[0,1]$, for each $1 < p < \infty$, has the Banach-Saks property, while by Szlenk's Theorem, $L^1[0,1]$ has the weak Banach-Saks property [13], but has not the Banach-Saks property, since all Banach spaces with the Banach-Saks property are reflexive [6]. Also the Banach space $C[0,1]$ fails to have the weak Banach-Saks property [12]. There are many Banach spaces with the (weak) Banach-Saks property, and one can see for example, [1, 4, 5, 7] and [8].

Throughout this article, X and Y are arbitrary Banach spaces. The dual of a Banach space X is denoted by X^* and T^* refers to the adjoint of the operator T. We use the standard symbols $L(X,Y)$ and $K(X,Y)$ to denote the Banach spaces of all bounded linear and compact linear operators between Banach spaces X and Y respectively, and $K_{w^*}(X^*,Y)$ is the space of all compact linear operators from X^* to Y that are weak*-weak continuous. The abbreviation $K(X)$ is used for $K(X,X)$. Also, for each closed subspace $\mathcal{M} \subseteq K(X,Y)$, each $x \in X$ and each $y^* \in Y^*$, we denote the evaluation operators at x and y^*, respectively by $\phi_x : \mathcal{M} \to Y$ and $\psi_{y^*} : \mathcal{M} \to X^*$, where $\phi_x(T) = Tx$, $\psi_{y^*}(T) = T^*y^*$ for each $T \in \mathcal{M}$. In the case that $\mathcal{M} \subseteq K_{w^*}(X^*,Y)$, $x^* \in X^*$ and $y^* \in Y^*$, the related evaluation operators $\phi_{x^*} : \mathcal{M} \to Y$ and $\psi_{y^*} : \mathcal{M} \to X$ are well defined, and note that for each weak*-weak continuous operator $T : X^* \to Y$, the adjoint operator T^* maps elements of Y^* into X. We refer the reader for additional notations and terminologies to the standard references [6], [9] and [10].

The main motivations for this article, are the papers [3] and [14] of Brown and A. Ülger. In fact, they proved that if $\mathcal{M}$ is a closed linear subspace of $K(H)$, then the dual $\mathcal{M}^*$ of $\mathcal{M}$ has the Schur property (i.e., weak and norm convergences of sequences in $\mathcal{M}^*$ coincide) if and only if all of the point evaluation sets $\{Tx : T \in \mathcal{M}_1\}$ and $\{T^*x : T \in \mathcal{M}_1\}$ are relatively compact in H, or equivalently all evaluation operators $\phi_x, \psi_x : \mathcal{M} \to H$ are compact operators, where $x \in X$ is arbitrary and $\mathcal{M}_1$ denotes the closed unit ball of $\mathcal{M}$. In 2003, the second author in a joint work with J. Zafarani, extended these results to closed subspaces of $K(X,Y)$ and $K_{w^*}(X^*,Y)$ [11]. Since the Schur property implies the weak Banach-Saks property of any Banach space, it is natural to ask under what conditions, a closed subspace $\mathcal{M}$ of an operator space has the weak Banach-Saks property.

Here, by introducing the concept of strong Banach-Saks for operators between Banach spaces, we will prove that for suitable Banach spaces X and Y, if $\mathcal{M}$ is a closed linear subspace of $K(X,Y)$ or $K_{w^*}(X^*,Y)$ with the $\mathcal{P}$-property (defined bellow), such that all evaluation operators on $\mathcal{M}$ are strong Banach-Saks, then $\mathcal{M}$ has the weak Banach-Saks property.

2. Main results

A bounded operator $T : X \to Y$ between Banach spaces X and Y is said to be completely continuous if T carries weakly convergent sequences to norm convergent ones. T is weakly completely continuous if every weakly null sequence (x_n) in X has a subsequence (x_{n_k}) such that $\|Tx_{n_k}\| \to 0$ as $k \to \infty$. Also T is a (weak) Banach-Saks operator, if every bounded (respectively weakly null) sequence (x_n) in X has a subsequence (x_{n_k}) such that the arithmetic means of the sequence (Tx_{n_k}) is norm convergent.

We say that an operator $T : X \to Y$ is strong Banach-Saks if every weakly null sequence in X has a subsequence (x_n) such that for each subsequence (x_{n_k}) of (x_n),

$$\|\frac{1}{k}\sum_{i=1}^{k} Tx_{n_i}\| \to 0, \ as \ k \to \infty.$$

It is evident that every Banach-Saks operator is weak Banach-Saks and the converse is valid when the domain of the operator contains no copy of l_1, thanks to the Rosenthal's l_1-Theorem [6]. Every completely continuous operator is weakly completely continuous and the class of such operators is strong Banach-Saks. Also, every strong Banach-Saks operator is weak Banach-Saks, but the converse is not true.

At first glance, we have the following connection between the (weak) Banach-Saks property of a Banach space X and the (weak) Banach-Saks-ness of all bounded operators on X:

Theorem 2.1. *A Banach space X has the (weak) Banach-Saks property if and only if for each Banach space Y, every bounded operator $T : X \to Y$ is a (weak) Banach-Saks operator.*

Proof. The proof of this assertion is clear, since the Banach space X has the (weak) Banach-Saks property if and only if the identity operator on X is (weak) Banach-Saks. □

As a corollary, if X and Y are two Banach spaces and the closed subspace $\mathcal{M}$ of $L(X,Y)$ has the (weak) Banach-Saks property, then all evaluation operators on $\mathcal{M}$ are (weak) Banach-Saks operators. We do not know if the converse is true or false. In the following, we shall prove that if $\mathcal{M}$ is a closed subspace of some compact operator spaces, the converse of the last assertion is true under the stronger assumption of strong Banach-Saks-ness of all evaluation operators on $\mathcal{M}$. We recall some notations and a definition from [11].

If V is a complemented subspace of a Banach space X, the projection of X onto V is denoted by P_V and $P_{V'} = I - P_V$ is the projection onto the complementary subspace V' of V. As mentioned in [11], if $(X_n)_{n=1}^{\infty}$ and $(Y_n)_{n=1}^{\infty}$ are Schauder decompositions of X and Y respectively, and $\mathcal{M} \subseteq L(X,Y)$ is a closed subspace, we say that $\mathcal{M}$ has the $\mathcal{P}$-property if for all integers m_0 and n_0 and every operators $T, S \in \mathcal{M}$,

$$\|P_W T P_V + P_{W'} S P_{V'}\| \leq max\{\|P_W T P_V\|, \|P_{W'} S P_{V'}\|\},$$

where $V = X_1 \oplus \cdots \oplus X_{m_0}$ and $W = Y_1 \oplus \cdots \oplus Y_{n_0}$. Finally, if $(X_n)_{n=1}^{\infty}$ is a shrinking Schauder decomposition for X [9], we denote the corresponding Schauder decomposition of X^* by $(X_n^*)_{n=1}^{\infty}$.

Theorem 2.2. *Let X and Y have monotone finite dimensional Schauder decompositions (abb. FDD) such that the decomposition of X is shrinking, and $\mathcal{M} \subseteq K_{w^*}(X^*,Y)$ be a closed subspace with the $\mathcal{P}$-property. If all evaluation operators ϕ_{x^*} and ψ_{y^*} on $\mathcal{M}$ are strong Banach-Saks, then $\mathcal{M}$ has the weak Banach-Saks property.*

Before proving this theorem, we mention Lemma 3.2 of [11] and also we need to prove other lemmas.

Lemma 2.3. [11] *Let X and Y have Schauder decompositions $(X_n)_{n=1}^{\infty}$ and $(Y_n)_{n=1}^{\infty}$, respectively, such that the decomposition of X is shrinking. If $S_1, \ldots, S_n \in K_{w^*}(X^*,Y)$ and $\epsilon > 0$, then there are integers m_0 and n_0 such that*

$$\|S_i P_{V'}\| \leq \epsilon \text{ and } \|P_{W'} S_i\| \leq \epsilon\ ,\ i = 1, 2, \ldots, n,$$

where $V = X_1^* \oplus \cdots \oplus X_{m_0}^*$ and $W = Y_1 \oplus \cdots \oplus Y_{n_0}$, V' and W' are complementary subspaces of V and W in X^* and Y, respectively.

Lemma 2.4. *Let X and Y be Banach spaces and $\mathcal{M} \subseteq L(X,Y)$ be a closed subspace such that all evaluation operators on $\mathcal{M}$ are strong Banach-Saks. If $R \in K(X)$ and $S \in K(Y)$ are two compact operators* *and $\epsilon > 0$ is given, then every weakly null sequence in $\mathcal{M}$ has a subsequence (T_n) such that for a suitable integer N_ϵ and each subsequence (T_{n_k}), the relations*

$$\|\frac{1}{k}\sum_{i=1}^{k} T_{n_i}R\| < \epsilon \text{ and } \|\frac{1}{k}\sum_{i=1}^{k} ST_{n_i}\| < \epsilon,$$

are valid for all integers $k \geq N_\epsilon$.

Proof. Suppose (T_n) is a weakly null sequence in $\mathcal{M}$. We claim that for each relatively compact subset $K \subseteq X$, there corresponds a subsequence of (T_n), denoted again by (T_n), such that the norm of arithmetic means of any of its subsequences is uniformly less than ϵ on K.

If M is a bound for the bounded sequence (T_n) and $\{x_1, \ldots, x_m\}$ is a finite $\frac{\epsilon}{2M}$-net for relatively compact set K, then by hypothesis, there exists a subsequence of (T_n), denoted again by (T_n), such that the arithmetic means of any of its subsequence is norm null in $\{x_1, \ldots, x_m\}$.

Now, if (T_{n_k}) is an arbitrary subsequence of (T_n) and $x \in K$, then there exists $1 \leq j \leq m$ such that $\|x - x_j\| < \frac{\epsilon}{2M}$. So

$$\begin{aligned}\|\frac{1}{k}\sum_{i=1}^{k} T_{n_i}x\| &\leq \|\frac{1}{k}\sum_{i=1}^{k} T_{n_i}x_j\| + \|\frac{1}{k}\sum_{i=1}^{k} T_{n_i}(x - x_j)\| \\ &\leq \epsilon/2 + M\|x - x_j\| < \epsilon,\end{aligned}$$

for a suitable N_ϵ and all $k \geq N_\epsilon$.

Now apply the claim for the relatively compact set $R(X_1)$, where X_1 is the closed unit ball of X.

Since all evaluation operators ψ_{y^*} are strong Banach-Saks, the same argument can be applied to the relatively compact subset $S^*(Y_1^*)$ of Y^*, where Y_1^* is the closed unit ball of Y^*. This completes the proof of the lemma. □

Lemma 2.5. *Let X and Y be Banach spaces and $\mathcal{M} \subseteq L(X, Y)$ be a closed subspace such that all evaluation operators on $\mathcal{M}$ are strong Banach-Saks. Then for every two compact operators $R \in K(X)$ and $S \in K(Y)$, the left and right multiplication operators $T \mapsto ST$ and $T \mapsto TR$ are also strong Banach-Saks on $\mathcal{M}$.*

Proof. If (T_n) is a weakly null sequence in $\mathcal{M}$, then by Lemma 2.4, there exists a subsequence $(T_{1,n})$ of (T_n) and an integer $N_1 > 0$ such that

$$\|\frac{1}{k}\sum_{i=1}^{k} T_{1,n_i}R\| < \epsilon_1 = 1,$$

for each subsequence (T_{1,n_i}) of $(T_{1,n})$ and each $k \geq N_1$. If we apply Lemma 2.4 to the weakly null sequence $(T_{1,n})$, we see that $(T_{1,n})$ has a subsequence $(T_{2,n})$ such that for a suitable integer $N_2 > N_1$,

$$\|\frac{1}{k}\sum_{i=1}^{k} T_{2,n_i}R\| < \epsilon_2 = \frac{1}{2},$$

for each subsequence (T_{2,n_i}) of $(T_{2,n})$ and each $k \geq N_2$. Now apply the induction and the standard argument of the diagonalization process to deduce a subsequence of (T_n), that is also a subsequence of each constructed row sequence, and denote it again by (T_n), such that for each integer $m > 0$ and any subsequence (T_{n_k}) of (T_n),

$$\|\frac{1}{k}\sum_{i=1}^{k} T_{n_i}R\| < \frac{1}{m},$$

for all $k \geq N_m$. This completes the proof. □

Proof of Theorem 2.2. Let $(X_n)_{n=1}^{\infty}$ and $(Y_n)_{n=1}^{\infty}$ be monotone FDDs of X and Y respectively. Since the decompositions of X^* and Y are monotone, $\|P_V\| = \|P_W\| = 1$, $\|P_{V'}\| \leq 2$ and $\|P_{W'}\| \leq 2$, for all $V = X_1^* \oplus \cdots \oplus X_{m_0}^*$ and $W = Y_1 \oplus \cdots \oplus Y_{n_0}$.

Fix a sequence (ϵ_n) of positive numbers such that $\sum n\epsilon_n < \infty$ and suppose that $(T_n) \subseteq \mathcal{M}$ is a weakly null sequence in $\mathcal{M}$. We shall construct by induction a suitable subsequence (T_{n_k}) of (T_n). Set $p_1 = n_1 = 1$. If $p_1 < p_2 < \cdots < p_k$ and $T_{n_1}, \ldots, T_{n_{p_k - 1}}$ have been constructed, for each $1 \leq i \leq k-1$, let $S_i = \sum_{j=p_i}^{p_{i+1}-1} T_{n_j}$. Since $S_1, \ldots, S_{k-1}$ belong to $K_{w^*}(X^*, Y)$, by Lemma 2.3, there exist finite dimensional subspaces V and W of X^* and Y, respectively, such that

$$\|S_i P_{V'}\| < \epsilon_k \text{ and } \|P_{W'} S_i\| < \epsilon_k \text{ for all } i = 1, 2, \ldots, k-1. \qquad (1)$$

On the other hand, P_V and P_W are of finite ranks and so are compact operators and by Lemma 2.5, the left and right multiplication operators

$T \mapsto P_W T$ and $T \mapsto TP_V$ from $\mathcal{M}$ into $K_{w^*}(X^*, Y)$ are strong Banach-Saks. Hence by the hypothesis on (T_n), there exist an integer $p_{k+1} > p_k$ and a subsequence $(T_{n_j})_{j \geq p_k}$ of (T_n) such that

$$\|\tfrac{1}{p_{k+1}-p_k} \textstyle\sum_{j=p_k}^{p_{k+1}-1} P_W T_{n_j}\| < \epsilon_k \text{ and } \|\tfrac{1}{p_{k+1}-p_k} \textstyle\sum_{j=p_k}^{p_{k+1}-1} T_{n_j} P_V\| < \epsilon_k.$$

Let $S_k = \sum_{j=p_k}^{p_{k+1}-1} T_{n_j}$. Then

$$\|P_W S_k\| < (p_{k+1} - p_k)\epsilon_k \quad \text{and} \quad \|S_k P_V\| < (p_{k+1} - p_k)\epsilon_k. \tag{2}$$

This completes the induction process. We claim that the arithmetic means of the constructed subsequence (T_{n_k}) of (T_n) is norm null. If V and W are the constructed subspaces related to S_k, then by (1) and (2),

$$\left\| P_W \sum_{i=1}^{k-1} S_i P_V - \sum_{i=1}^{k-1} S_i \right\| < 4k\epsilon_k$$

and

$$\|P_{W'} S_k P_{V'} - S_k\| < 5(p_{k+1} - p_k)\epsilon_k.$$

Hence by $\mathcal{P}$-property of $\mathcal{M}$ we have:

$$\begin{aligned}
\left\| \sum_{i=1}^{p_{k+1}-1} T_{n_i} \right\| &= \left\| \sum_{i=1}^{k} S_i \right\| \\
&\leq \left\| \sum_{i=1}^{k-1} S_i - P_W \sum_{i=1}^{k-1} S_i P_V \right\| + \|S_k - P_{W'} S_k P_{V'}\| \\
&+ \left\| P_W \sum_{i=1}^{k-1} S_i P_V + P_{W'} S_k P_{V'} \right\| \\
&\leq 4k\epsilon_k + 5(p_{k+1} - p_k)\epsilon_k + max\left\{ \left\| \sum_{i=1}^{k-1} S_i \right\|, 4\|S_k\| \right\} \\
&\leq \\
&\vdots \\
&\leq 4\sum_{i=1}^{k} i\epsilon_i + 5\sum_{i=1}^{k} (p_{i+1} - p_i)\epsilon_i + 4M,
\end{aligned}$$

where M is a bound for the bounded sequence (T_n).

Since $\frac{1}{p_{k+1}}\sum_{i=1}^{k}(p_{i+1}-p_i)\epsilon_i \to 0$, as $k \to \infty$, the sequence $\frac{1}{p_{k+1}}\sum_{i=1}^{p_{k+1}-1} T_{n_i}$ is norm null and so the arithmetic means of the sequence (T_{n_k}) is norm null. □

Under the same assumptions on X and Y, a proof similar to that of Theorem 2.2 can be applied to obtain the following theorem.

Theorem 2.6. *Let X and Y have monotone FDDs, such that the decomposition of X is shrinking, and let $\mathcal{M} \subseteq K(X,Y)$ be a closed subspace with the $\mathcal{P}$-property. If all of the evaluation operators ϕ_x and ψ_{y^*} on $\mathcal{M}$ are strong Banach-Saks, then $\mathcal{M}$ has the weak Banach-Saks property.*

The proof of Theorem 2.2 shows in fact that the arithmetic means of any subsequence of the desired subsequence (T_{n_k}) is norm null. This leads to the following refinement of the above theorems:

Corollary 2.7. *Let X and Y have monotone FDDs such that the decomposition of X is shrinking. If $\mathcal{M}$ is a closed subspace of $K_{w^*}(X^*,Y)$ or $K(X,Y)$ with the $\mathcal{P}$-property and all of the evaluation operators on $\mathcal{M}$ are strong Banach-Saks, then every weakly convergent sequence in $\mathcal{M}$ has a subsequence such that arithmetic means of any of its subsequences is norm convergent.*

The above corollary leads to a familiar property of the Banach space c_0 consisting of all null sequences of scalars with the supremum norm (see for instance [6]):

Corollary 2.8. *Every weakly convergent sequence in c_0 has a subsequences such that arithmetic means of any of its subsequence is norm convergent.*

Proof. Let (e_n) be the standard orthonormal basis of the Hilbert space l_2. Then c_0 is isomorphic to a closed subspace of $K(l_2)$; in fact c_0 is isomorphic to the space of all diagonal elements of $K(l_2)$. Since $c_0^* = l_1$ has the Schur property , then by Theorem 1 of [14], all evaluation operators on c_0 are compact and so are completely continuous. Therefore, the result is an easy consequence of Corollary 2.7, thanks to the $\mathcal{P}$-property of $K(l_2)$. □

If X is an l_p-direct sum and Y is an l_q-direct sum of Banach spaces with $1 < p \leq q < \infty$, or X has a Schauder decomposition and Y is a c_0-direct sum of Banach spaces, then the proofs of Corollaries 3.5 and 3.6 of [11] show that $K(X,Y)$ (resp. $K_{w^*}(X^*,Y)$) and so its closed subspace $\mathcal{M}$ has the $\mathcal{P}$-property. So we have the following two corollaries:

Corollary 2.9. *Let X be an l_p-direct sum and Y be an l_q-direct sum of finite dimensional Banach spaces with $1 < p \leq q < \infty$. If $\mathcal{M}$ is a closed subspace of $K(X,Y)$ such that all evaluation operators on $\mathcal{M}$ are strong Banach-Saks, then $\mathcal{M}$ has the weak Banach-Saks property.*

Corollary 2.10. *Let X have a monotone shrinking FDD and Y be a c_0-direct sum of finite dimensional Banach spaces. If $\mathcal{M}$ is either a closed subspace of $K(X,Y)$ or $K_{w^*}(X^*,Y)$ such that all of the corresponding evaluation operators are strong Banach-Saks, then $\mathcal{M}$ has the weak Banach-Saks property.*

Finally, we state a theorem similar to Theorem 2.2 for operators between two arbitrary Hilbert spaces.

Theorem 2.11. *Let H_1 and H_2 be two Hilbert spaces and $\mathcal{M}$ be a closed subspace of $K(H_1,H_2)$ such that all evaluation operators on $\mathcal{M}$ are strong Banach-Saks. Then $\mathcal{M}$ has the weak Banach-Saks property.*

Proof. By [3], in the Hilbert space setting, a lemma similar to Lemma 2.3 is valid; every closed subspace of a Hilbert space is complemented and an inequality similar to that of the definition of $\mathcal{P}$-property holds for operators between two Hilbert spaces. So the proof is completely similar to Theorem 2.2. $\square$

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