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Author(s):

J. Liu

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ALMOST SURE EXPONENTIAL STABILITY OF STOCHASTIC REACTION DIFFUSION SYSTEMS WITH MARKOVIAN JUMP

J. LIU

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ABSTRACT. The stochastic reaction diffusion systems may suffer sudden shocks, in order to explain this phenomena, we use Markovian jumps to model stochastic reaction diffusion systems. In this paper, we are interested in almost sure exponential stability of stochastic reaction diffusion systems with Markovian jumps. Under some reasonable conditions, we show that the trivial solution of stochastic reaction diffusion systems with Markovian jumps is almost surely exponentially stable. An example is given to illustrate the theory.

Keywords: Markovian jump, almost sure exponential stability, stochastic reaction diffusion system, Itô differential formula. MSC(2010): Primary: 60H10; Secondary: 60H15.

1. Introduction

Stochastic differential equations with jumps have been widely used in many branches of science and its applications, in particular, in economics, finance and engineering (see, for example, Cont [5], Gukhal [7], Sobczyk [12] and references therein). The investigation of stabilization for stochastic differential equations with jumps has received much more attention in the past few years. Ji and Chizeck [8] studied the stability of linear jump equations and Pakshin [11] studied robust stability

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and stabilization of linear jump systems. Yuan and Lygeros [14] investigated almost sure exponential stability for a class of switching diffusion processes and Bao and Yuan [4] give the exponential stability of switching-diffusion processes with jumps. And for stabilization of partial differential equations one can see [2, 3]. The stability of stochastic reaction diffusion systems has been discussed in many papers, one can see [6, 9, 13, 15] and the systems with the form

(1.1) $du(t,x) = (a(t)\Delta u(t,x) + f(t,x,u(t,x)))dt + g(t,x,u(t,x))dW(t)$

was discussed in [9, 15]. The stochastic reaction diffusion systems may suffer sudden shocks, however, systems described by (1.1) cannot explain this phenomena. In order to explain this phenomena, introducing Markovian jump process into stochastic reaction diffusion systems is one of the important methods.

Motivated by the papers mentioned above, in this paper, we will establish the exponential stability of stochastic reaction diffusion systems with Markovian jumps. In reference to the existing results in the literature, our contributions are as follows:

• We use Markovian jumps to model stochastic reaction diffusion systems when they suffer sudden shocks;

• Under some reasonable conditions, we show that due to the Markovian jumps the overall system can become pathwise exponentially stable although some subsystems are not stable.

• Some new techniques are developed to cope with the difficulty due to the Markovian jumps.

This paper is organized as follows: Section 2 gives some preliminary results, in particular, stochastic reaction diffusion systems with Markovian jump are set up. In section 3, we discussed the almost surely exponentially stable of stochastic reaction diffusion systems under some reasonable conditions. The example is provided in Section 4.

2. Preliminaries

We focus in this paper on stochastic reaction diffusion systems (1.1) with Markovian jumps, that is,

$$(2.1)$$

$$du(t,x) = (a(t)\Delta u(t,x) + f(t,x,u(t,x),r(t))) dt + g(t,x,u(t,x),r(t)) dW(t)$$

for $(t, x) \in \mathbb{R}_+ \times G$, with boundary condition

$$\frac{\partial u(t,x)}{\partial \mathcal{M}} = 0, (t,x) \in \mathbb{R}_+ \times \partial G,$$

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where $u(t,x) \in \mathbb{R}^n$, $\Delta u = \sum_{i=1}^n \frac{\partial^2 u}{\partial x_i}$, $G = \{x, |x| < l < +\infty\} \subset \mathbb{R}^r$, $f \in [\mathbb{R}_+ \times G \times \mathbb{R}^n \times \mathbb{S}, \mathbb{R}^n]$ and $g \in [\mathbb{R}_+ \times G \times \mathbb{R}^n \times \mathbb{S}, \mathbb{R}^{n \times m}]$ are both Borel measurable functions, $|\cdot|$ stands for vector norm, $||A|| := \sqrt{\operatorname{trace}(A^*A)}$ is the Hilbert-Schmidt norm for a matrix A, \mathcal{M} is the normal vector to ∂G ; W(t) an *m*-dimension Wiener process defined in complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ equipped with a filtration $\{\mathcal{F}_{t\geq 0}\}$ satisfying the usual conditions. Let N be some positive integer, $\{r(t), t \in \mathbb{R}_+\}$ a right continuous irreducible Markov chain on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ taking values in a finite state space $\mathbb{S} = \{1, 2, \ldots, N\}$, with generator $\Gamma = (\gamma_{ij})_{N \times N}$ given by

$$\mathbb{P}(r(t+\Delta) = j | r(t) = i) = \begin{cases} \gamma_{ij}\Delta + o(\Delta), & \text{if } i \neq j, \\ 1 + \gamma_{ii}\Delta + o(\Delta), & \text{if } i = j, \end{cases}$$

where $\Delta > 0$ and $\gamma_{ij} > 0$ is the transition rate from *i* to *j*, if $i \neq j$; while $\gamma_{ii} = -\sum_{j\neq i} \gamma_{ij}$, $i = 1, 2, \dots, N$. We further assume that the Wiener process W(t) and Markov chain r(t) are independent. Since we have assumed that the Markov chain r(t) is irreducible, it has a unique stationary probability distribution $\pi := (\pi_1, \pi_2, \dots, \pi_N) \in \mathbb{R}^{1 \times N}$ which

$$\pi\Gamma = 0$$
 s.t. $\sum_{j=1}^{N} \pi_j = 1, \quad \pi_j > 0, \forall j \in \mathbb{S}.$

Throughout the paper we assume:

(**H**): g(t, x, u(t, x), i) satisfies integral linear growth condition and f(t, x, u(t, x), i), g(t, x, u(t, x), i) satisfy Lipschitz condition, i.e., there exists a positive constant L such that for arbitrary $u_1, u_2 \in \mathbb{R}^n, i \in \mathbb{S}$,

$$\begin{split} \|\int_{G}g(t,x,u,i)\mathrm{d}x\| &\leq L(1+|u|_{G}),\\ |\int_{G}[f(t,x,u_{1},i)-f(t,x,u_{2},i)]\mathrm{d}x| &\leq L|u_{1}-u_{2}|_{G},\\ \|\int_{G}[g(t,x,u_{1},i)-g(t,x,u_{2},i)]\mathrm{d}x\| &\leq L|u_{1}-u_{2}|_{G},\\ |u(\cdot,x)|_{G} &:= |\int_{G}u(\cdot,x)\mathrm{d}x|. \end{split}$$

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where

Then by the results of [9], the existence of the solution for system (2.1) can be proved and note that $u(t, x) = u(t, x; t_0, u_0)$.

Definition 2.1. [9] The trivial solution of system (2.1) is said to be almost surely exponentially stable if

$$\limsup_{t \to \infty} t^{-1} ln |u(t, x)|_G < 0.$$

3. Almost sure exponential stability

Similar to ([9], Lemma 1.), we have the following lemma.

Lemma 3.1. For any $u_0 \neq 0$,

$$\mathbb{P}\left\{\int_{G} u(t, x) \mathrm{d}x \neq 0, t \ge 0\right\} = 1.$$

The main result of the paper is the following theorem.

Theorem 3.2. Let $\overline{u} = \int_G u(t, x) dx$, $S_h = \{\xi | |\xi(\cdot)| < h\}$. Assume that there exists $V(t, \overline{u}(t), r(t)) \in C^{1,2}(\mathbb{R}_+ \times S_h \times \mathbb{S}, \mathbb{R}), c_1, c_3(i) > 0, c_2(i), \rho(i) \in \mathbb{R}$, such that for $(t, \overline{u}(t), i) \in \mathbb{R}_+ \times S_h \times \mathbb{S}$

(i) $c_1|u(t)|_G^p \leq V(t, \overline{u}(t), i);$ (ii) $LV(t, \overline{u}, i) \leq c_2(i)V(t, \overline{u}(t), i);$ (iii) $|\frac{\partial V(t, \overline{u}(t), i)}{\partial \overline{u}} \int_G g(s, u(s), i) dx|^2 \geq c_3(i)V^2(t, \overline{u}(t), i);$ (iv) $\sum_{j=1}^N \gamma_{ij}(\ln V(t, \overline{u}(t), j) - \frac{V(t, \overline{u}(t), j)}{V(t, \overline{u}(t), i)}) \leq \rho(i);$ (v) For some $\epsilon \in (0, \frac{1}{2}],$

$$\eta := \limsup_{t \to \infty} \frac{1}{t} \Pi(t) < \infty,$$

where

$$\Pi(t) := \int_0^t \int_{\Theta} \ln(\frac{V(s,\overline{u}(s),i_0 + h(r(s),l))}{V(s,\overline{u}(s),r(s))})^2 - (\frac{V(s,\overline{u}(s),i_0 + h(r(s),l))}{V(s,\overline{u}(s),r(s))})^{\epsilon} m(\mathrm{d}l) \mathrm{d}s$$

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$$\Theta = [0, \sum_{j=1, j \neq i}^{N} \gamma_{ij}].$$

Then the trivial solution of the system satisfies

$$\limsup_{t \to \infty} \frac{1}{t} \ln |u(t,x)|_G \le \frac{1}{p} \sum_{j=1}^N \pi(j) (c_2(j) + \frac{1}{2} c_3(j) + \rho(j)).$$

In particular, the trivial solution of system (2.1) is almost surely exponentially stable if

$$\sum_{j=1}^{N} \pi(j)(c_2(j) + \frac{1}{2}c_3(j) + \rho(j)) < 0.$$

Proof. The argument is motivated by that of ([4], Theorem 3.1). Note that

(3.1)
$$d\int_{G} u(t,x)dx = \int_{G} (a(t)\Delta u(t,x) + f(t,x,u(t,x),r(t))dxdt + \int_{G} g(t,x,u(t,x),r(t))dxdW(t),$$

combining Green formula and boundary condition, we have

$$\int_{G} \Delta u(t, x) \mathrm{d}x = \int_{G} \frac{\partial u}{\partial \mathcal{M}} = 0,$$

then (3.1) can be rewritten as

$$\begin{split} \mathrm{d} \int_{G} u(t,x) \mathrm{d} x &= \int_{G} f(t,x,u(t,x),r(t)) \mathrm{d} x \mathrm{d} t \\ &+ \int_{G} g(t,x,u(t,x),r(t)) \mathrm{d} x \mathrm{d} W(t). \end{split}$$

Define an operator

$$\begin{split} LV(t,\overline{u}(t),i) = & \frac{\partial V(t,\overline{u}(t),i)}{\partial t} + \frac{\partial V(t,\overline{u}(t),i)}{\partial \overline{u}} \int_{G} f(t,x,u(t,x),r(t)) \mathrm{d}x \\ &+ \frac{1}{2} trace \big[\int_{G} g^{T}(t,x,u(t,x),r(t)) \mathrm{d}x \\ &\times \frac{\partial^{2} V(t,\overline{u},i)}{\partial \overline{u} \partial \overline{u}} \int_{G} g(t,x,u(t,x),r(t)) \mathrm{d}x \big] \\ &+ \sum_{j=1}^{N} \gamma_{ij} V(t,\overline{u}(t),j). \end{split}$$

We have the following Itô formula

$$\begin{split} \ln V(t, \overline{u}(t), r(t)) &= \ln V(0, \overline{u}_0, i_0) + \int_0^t L \ln V(s, \overline{u}(s), r(s)) \mathrm{d}s \\ &+ M_1(t) + M_2(t) \\ &= \ln V(0, \overline{u}_0, i_0) + \int_0^t \frac{LV(s, \overline{u}(s), r(s))}{V(s, \overline{u}(s), r(s))} \mathrm{d}s \\ &- \frac{1}{2} \int_0^t \frac{|\frac{\partial V(s, \overline{u}(s), r(s))}{\partial \overline{u}} \int_G g(s, x, u(s, x), r(s)) \mathrm{d}x|^2}{|V(s, \overline{u}(s), r(s))|^2} \mathrm{d}s \\ &+ \int_0^t \sum_{j=1}^N \gamma_{ij} [\ln V(s, \overline{u}(s), j) - \frac{V(s, \overline{u}(s), j)}{V(s, \overline{u}(s), r(s))}] \mathrm{d}s \\ &+ M_1(t) + M_2(t), \end{split}$$

where

$$M_1(t) := \int_0^t \frac{\frac{\partial V(s,\overline{u}(s),r(s))}{\partial \overline{u}} \int_G g(s,x,u(s,x),r(s))dx}{V(s,\overline{u}(s),r(s))} \mathrm{d}W(s),$$
$$M_2(t) := \int_0^t \int_{\Theta} \ln \frac{V(s,\overline{u}(s),i_0 + h(r(s),l))}{V(s,\overline{u}(s),r(s))} \mu(\mathrm{d}s,\mathrm{d}l)$$

and $\mu(ds, dl)$ is a Poisson random measure with intensity $ds \times m(dl)$, in which m is the Lebesgue measure on \mathbb{R} . For more details on the function h and the martingale measure see, e.g., [10].

Note that $M_1(0) = M_2(0) = 0$, by the exponential martingale inequality, for any $n \in \mathbb{N}$, $\delta \in (0, \frac{\epsilon}{2}]$, we have

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$$\begin{split} & \mathbb{P}\{\sup_{0 \le t \le n} [M_1(t) - \frac{\delta}{2} \int_0^t |\frac{\frac{\partial V(s,\overline{u}(s),r(s))}{\partial \overline{u}} \int_G g(s,x,u(s,x),r(s)) \mathrm{d}x}{V(s,\overline{u}(s),r(s))}|^2 \mathrm{d}s] \\ & > \frac{2\ln n}{\delta}\} \le \frac{1}{n^2} \end{split}$$

and denote $\Lambda(t, l) = \frac{V(s, \overline{u}(s), i_0 + h(r(s), l))}{V(s, \overline{u}(s), r(s))}$, then by the exponential martingale inequality with jump ([1], Theorem 5.2.9),

$$\mathbb{P}\{\sup_{0\leq t\leq n} [M_2(t) - \frac{1}{\delta} \int_0^t \int_{\Theta} [\Lambda^{\delta}(s, l) - 1 - \delta \ln \Lambda(s, l)] m(\mathrm{d}l) \mathrm{d}s] > 2\delta^{-1} \ln n\} \leq \frac{1}{n^2}.$$

Applying the Borel-Cantelli Lemma, there exists an $\Omega_0 \subseteq \Omega$ with $\mathbb{P}(\Omega - \Omega_0) = 1$, such that for any $\omega \in \Omega_0$, there exists an integer $n_0 = n_0(\omega) > 0$, if $n \geq n_0$,

$$M_{1}(t) \leq \frac{\delta}{2} \int_{0}^{t} \left| \frac{\frac{\partial V(s,\overline{u}(s),r(s))}{\partial \overline{u}} \int_{G} g(s,x,u(s,x),r(s)) dx}{V(s,\overline{u}(s),r(s))} \right|^{2} ds + \frac{2\ln n}{\delta},$$
$$M_{2}(t) \leq \frac{1}{\delta} \int_{0}^{t} \int_{\Theta} [\Lambda^{\delta}(s,l) - 1 - \delta \ln \Lambda(s,l)] m(dl) ds + \frac{2\ln n}{\delta}.$$

Hence, for $0 \le t \le n$, $\omega \in \Omega_0$ and $n \ge n_0$,

$$\begin{split} \ln V(t,\overline{u}(t),r(t)) &\leq \ln V(0,\overline{u}_{0},i_{0}) + \frac{4\ln n}{\delta} + \int_{0}^{t} \frac{LV(s,\overline{u}(s),r(s))}{V(s,\overline{u}(s),r(s))} \mathrm{d}s \\ &- \frac{1}{2} \int_{0}^{t} \frac{|\frac{\partial V(s,\overline{u}(s),r(s))}{\partial \overline{u}} \int_{G} g(s,x,u(s,x),r(s)) \mathrm{d}x|^{2}}{|V(s,\overline{u}(s),r(s))|^{2}} \mathrm{d}s \\ &+ \int_{0}^{t} \sum_{j=1}^{N} \gamma_{ij} [\ln V(s,\overline{u}(s),j) - \frac{V(s,\overline{u}(s),j)}{V(s,\overline{u}(s),r(s))}] \mathrm{d}s \\ &+ \frac{\delta}{2} \int_{0}^{t} |\frac{\frac{\partial V(s,\overline{u}(s),r(s))}{\partial \overline{u}} \int_{G} g(s,x,u(s,x),r(s)) \mathrm{d}x}{V(s,\overline{u}(s),r(s))}|^{2} \mathrm{d}s \\ &+ \frac{1}{\delta} \int_{0}^{t} \int_{\theta} [\Lambda^{\delta}(s,l) - 1 - \delta \ln \Lambda(s,l)] m(\mathrm{d}l) \mathrm{d}s. \end{split}$$

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Using Taylor expansion, for $\delta \in (0, \frac{\epsilon}{2}]$, we have

$$\Lambda^{\delta}(s,l) = 1 + \delta \ln \Lambda(s,l) + \frac{\delta^2}{2} (\ln \Lambda(s,l))^2 \Lambda^{\xi}(s,l),$$

where ξ lies between 0 and δ . Noting that, for $0 \leq \xi \leq \frac{\epsilon}{2}$, $\Lambda^{\xi} \leq 1$, if $0 < \Lambda < 1$, $\Lambda^{\xi} \leq \Lambda^{\frac{\xi}{2}}$, if $\Lambda \geq 1$ and the inequality

$$\ln x \le \frac{4}{\epsilon} (x^{\frac{\epsilon}{4}} - 1) \quad for \quad x \ge 1,$$

is satisfied we have

$$\begin{split} &\frac{1}{\delta} \int_0^t \int_{\theta} [\Lambda^{\delta}(s,l) - 1 - \delta \ln \Lambda(s,l)] m(\mathrm{d}l) \mathrm{d}s \\ &= \frac{\delta}{2} \int_0^t \int_{\theta} (\ln \Lambda(s,l))^2 \Lambda^{\xi}(s,l)) m(\mathrm{d}l) \mathrm{d}s \\ &= \frac{\delta}{2} \int_0^t \int_{0 < \Lambda < 1} (\ln \Lambda(s,l))^2 \Lambda^{\xi}(s,l)) m(\mathrm{d}l) \mathrm{d}s \\ &+ \frac{\delta}{2} \int_0^t \int_{\Lambda \ge 1} (\ln \Lambda(s,l))^2 \Lambda^{\xi}(s,l)) m(\mathrm{d}l) \mathrm{d}s \\ &\leq \frac{\delta}{2} \int_0^t \int_{\theta} [(\ln \Lambda(s,l))^2 + \frac{16}{\epsilon^2} \Lambda^{\epsilon}(s,l)] m(\mathrm{d}l) \mathrm{d}s. \end{split}$$

Together with (ii)-(iv), implies

$$\ln V(t,\overline{u}(t),r(t)) \leq \ln V(0,\overline{u}_0,i_0) + \frac{4\ln n}{\delta} + \int_0^t [c_2(r(s)) + \frac{\delta-1}{2}c_3(r(s)) + \rho(r(s))] ds + \frac{\delta}{2} \int_0^t \int_{\theta} [(\ln \Lambda(s,l))^2 + \frac{16}{\epsilon^2} \Lambda^{\epsilon}(s,l)] m(dl) ds,$$

which together with (i), for $n-1 \le t \le n, \omega \in \Omega_0$, and $n \ge n_0 + 1$

$$\frac{1}{t}\ln|u(t,x)|_{G} \leq -\frac{\ln c_{1}}{pt} + \frac{1}{pt}[\ln V(0,\overline{u}_{0},i_{0}) + \frac{4\ln n}{\delta} + \int_{0}^{t} (c_{2}(r(s)) + \frac{\delta - 1}{2}c_{3}(r(s)) + \rho(r(s))ds + \frac{8}{\epsilon^{2}}\Pi(t)].$$

By using the ergodic property of Markovian chains, since δ is arbitrary, by the use of (v), we have

$$\limsup_{t \to \infty} \frac{1}{t} \ln |u(t,x)|_G \le \frac{1}{p} \sum_{j=1}^N \pi(j) (c_2(j) + \frac{1}{2} c_3(j) + \rho(j)),$$

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we therefore complete the proof.

Remark 3.3. From Theorem 3.2, we can see due to the Markovian jumps the overall system can become pathwise exponentially stable although some subsystems are not stable.

4. Example

Let W(t) be a Wiener process. Let r(t) be a right-continuous markov chain taking values in $S = \{1, 2\}$ with generator $\Gamma = (\gamma_{ij})_{2\times 2}, \gamma_{11} = \gamma_{21} = 1, \gamma_{12} = \gamma_{22} = -1$. The unique stationary probability distribution of the Markov chain r(t) is

$$\pi = (\pi_1, \pi_2) = (\frac{k}{1+k}, \frac{1}{1+k}), k > 0.$$

Assume that there exist constants $k_i \in \mathbb{R}$, $l_i > 0$, i = 1, 2, such that

$$2\int_{G} u^{T}(t,x) \mathrm{d}x f(t,x,u(t,x),i) + \|g(t,x,u(t,x),i)\|^{2} \le k_{i} |\int_{G} u(t,x) \mathrm{d}x|^{2},$$
$$\int_{G} u^{T}(t,x) \mathrm{d}x g(t,x,u(t,x),i) \ge l_{i} |\int_{G} u(t,x) \mathrm{d}x|^{2}.$$

Consider the following stochastic reaction diffusion system:

(4.1)
$$du(t,x) = (a(t)\Delta u(t,x) + f(t,x,u(t,x),r(t)))dt + g(t,x,u(t,x),r(t))dW(t),$$

for $(t, x) \in \mathbb{R}_+ \times G$ with boundary condition

$$\frac{\partial u(t,x)}{\partial \mathcal{M}} = 0, (t,x) \in \mathbb{R}_+ \times \partial G.$$

Let

$$V(t, \overline{u}, 1) = \left| \int_{G} u(t, x) dx \right|^{2},$$
$$V(t, \overline{u}, 2) = 2 \left| \int_{G} u(t, x) dx \right|^{2}.$$

By the definition of V, we can choose

$$c_1 = \frac{1}{2}, \quad p = 2, \quad c_3(1) = 4l_1^2 |G|, \quad c_3(2) = 16l_2^2 |G|,$$

 $\rho_1 = \ln 2, \quad \rho_2 = -\ln 2 - \frac{3}{2}.$

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Then

$$LV(t, \overline{u}(t), 1) = 2\overline{u}^T \int_G f(t, x, u(t, x), 1) dx + \| \int_G g(t, x, u(t, x), 1) dx \|^2$$

- $|\overline{u}(t)|^2 + 2|\overline{u}(t)|^2$
 $\leq (k_1|G| + 1)V(t, \overline{u}(t), 1)$
:= $c_2(1)V(t, \overline{u}(t), 1)$.

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$$LV(t, \overline{u}(t), 2) \le (k_2|G| - 1)V(t, \overline{u}(t), 2) := c_2(2)V(t, \overline{u}(t), 2).$$

It is obvious that $\exists c_4, c_5 > 0, c_4 \leq \frac{V(t, \overline{u}(t), i)}{V(t, \overline{u}(t), j)} \leq c_5, i, j \in \mathbb{S}$, then condition (v) holds. By Theorem 3.2, we have

$$\limsup_{t \to \infty} \frac{1}{t} \ln |u(t,x)|_G \le \frac{1}{2(1+k)} (k\zeta_1 + \zeta_2),$$

where

$$\zeta_1 = k_1 |G| + 2l_1^2 |G| + 1 + \ln 2,$$

and

$$\zeta_2 = k_2 |G| + 8l_2^2 |G| - \ln 2 - \frac{7}{2}.$$

The stochastic reaction diffusion system (4.1) is almost surely exponentially stable whenever

$$k\zeta_1 + \zeta_2 < 0.$$

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(Jun Liu) Department of Mathematics, Jining University, Qufu, Shandong Province, China

E-mail address: wjzws6@163.com

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