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MAXIMAL ELEMENTS OF $\mathscr{F}_{C,\theta}$-MAJORIZED MAPPINGS AND APPLICATIONS TO GENERALIZED GAMES

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Abstract. In the paper, some new existence theorems of maximal elements for $\mathscr{F}_{C,\theta}$-mappings and $\mathscr{F}_{C,\theta}$-majorized mappings are established. As applications, some new existence theorems of equilibrium points for one-person games, qualitative games and generalized games are obtained. Our results unify and generalize most known results in recent literature.
Keywords: Maximal elements, generalized games, $\mathscr{F}_{C,\theta}$-majorized mappings, FC-space.
MSC(2010): Primary: 54A05; Secondary: 47H04.

1. Introduction

The existence of equilibrium is a main problem of investigating various kind of economic models in mathematical economics. In 1976, Borglin and Keiding [3] proved a new existence theorem for a compact generalized games with KF-majorized preference correspondence. Following the ideas, many authors have obtained the equilibrium existence theorem for generalized games, for example, Briec and Horvath [4], Tan and Yuan [27], Yuan and Tarafdar [28], Ansari and Yao [1], Kim et al. [21], Lin and Liu [22], Ding and Xia [9], Du and Deng [13], Deng and Du [7], Deng and Xia [8], Yang and Deng [29], Ding and Yao [10], and Hou [19] etc. In the setting, convexity assumptions play a crucial role. In 2005, Ding [11] introduce the concept of a FC-space, which is a topological space without any convexity structure and linear structure. Moveover,

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FC-space include topological vector spaces, H-space [17, 18], G-convex space [24] and L-convex space [2] as special cases. FC-space will be the framework of this paper.

In the paper, we introduce the notions of $\mathscr{F}_{C,\theta}$-mapping and $\mathscr{F}_{C,\theta}$-majorized mappings with transfer open lower sections in FC-space. We first establish some new existence theorems of maximal elements for $\mathscr{F}_{C,\theta}$-mappings and $\mathscr{F}_{C,\theta}$-majorized mappings in FC-space. Next, New notions for qualitative games and generalized games are introduced. As application of the existence theorems of maximal elements, we establish some existence theorems of equilibrium points for one-person games, qualitative games and generalized games. Our notions and results unify and generalize the corresponding notions and results introduced by many authors, for example, Borglin and Keiding [3], Tan and Yuan [27], Yuan and Tarafdar [28], Ding and Xia [9], Yang and Deng [29], Chowdhury et al. [5], Shen [25], Lin [23], Homidan et al., [16] and etc.

Let X be a nonempty subset of topological space E. We shall denote by 2^X the family of all subsets of X, by $\langle X\rangle$ the family of all nonempty finite subsets of X, by $\text{int}_E(X)$ the interior of X in E, and by $\text{cl}_E(X)$ the closure of X in E. Let $\triangle_n$ denote the standard n-simplex, that is,

$$\triangle_n = \{u \in \mathbb{R}^{n+1} : u = \sum_{i=1}^{n+1} \lambda_i(u)e_i, \lambda_i(u) \geq 0, \sum_{i=1}^{n+1} \lambda_i(u) = 1\},$$

where $e_i \ \ (i = 1, ..., n + 1)$ is the i-th unit vector in $\mathbb{R}^{n+1}$. In the paper, we suppose every topological space is Hausdorff.

If X and Y are topological spaces and $T, S : X \to 2^Y$ are two mappings, for any $D \subset X$ and $y \in Y$, let $S(D) = \cup_{x\in D}S(x)$ and $S^{-}(y) = \{x \in X : y \in S(x)\}$. The notation dom S denotes the domain of S, i.e., dom $S = \{x \in X : S(x) \neq \emptyset\}$, and $T \cap S : X \to 2^Y$ is a mapping defined by $(T \cap S)(x) = T(x) \cap S(x)$ for each $x \in X$. The graph of T is the set $\text{Gr}(T) = \{(x, y) \in X \times Y : y \in T(x)\}$ and the mapping $\bar{T} : X \to 2^Y$ is defined by $\bar{T}(x) = \{y \in Y : (x, y) \in \text{cl}_{X\times Y}(\text{Gr}(T))\}$. The mapping cl $T : X \to 2^Y$ is defined by $(\text{cl } T)(x) = \text{cl}_Y(T(x))$ for each $x \in X$.

Let X be a nonempty set and Y be a topological space. The mapping $F : X \to 2^Y$ is said to be transfer open valued on X if

$$\cup_{x\in X}F(x) = \cup_{x\in X}\text{int}_Y(F(x)).$$

The mapping $F : X \to 2^Y$ is said to be transfer closed valued on X if

$$\cap_{x\in X} F(x) = \cap_{x\in X} \mathrm{cl}_Y(F(x)).$$

It is easy to prove that if F is transfer open valued, then the mapping $G : X \to 2^Y$ with $G(x) = Y \setminus F(x)$ for each $x \in X$ is transfer closed valued.

Definition 1.1. [11] $(E; \varphi_N)$ *is said to be a finitely continuous space (in short, FC-space), if E is a topological space and for each* $N = \{x_0, x_1, \cdots, x_n\} \in \langle E\rangle$, *there exists a continuous mapping* $\varphi_N : \triangle_N \to 2^E$. *If X is a subset of E, X is said to be an FC-subspace of E if for each* $N = \{x_0, x_1, \cdots, x_n\} \in \langle E\rangle$ *and for any* $\{x_{i_0}, x_{i_1} \cdots, x_{i_k}\} \subset N \cap X$, $\varphi_N(\triangle_k) \subset X$, *where* $\triangle_k = co(\{e_{i_0}, \cdots, e_{i_k}\})$.

Definition 1.2. [29] *Let* $(E; \varphi_N)$ *be an FC-space and A be a subset of E, define the FC-hull of A as follows:*

$$FC\text{-}(A) = \cap\{B \subset E : A \subset B \text{ and } B \text{ is an } FC\text{-subspace of } E\}.$$

Definition 1.3. *Let* $(X; \varphi_N)$ *be an FC-space and Y be a topological space. Let* $\theta : X \to Y$ *be a single valued mapping and* $A : X \to 2^Y$ *be a set-valued mapping. Then*

(i) A is said to be a $\mathscr{F}_{C,\theta}$*-mapping if*

(a) $\varphi_N(\triangle_n) \cap \bigcap_{x\in N} A^-(\theta(x)) = \emptyset$ *for each* $N \in \langle X\rangle$; *and*

(b) $A^- : Y \to 2^X$ *is transfer open valued in Y.*

(ii) $(B_x; N_x)$ *is said to be a* $\mathscr{F}_{C,\theta}$*-majorant of A at* $x \in X$ *if* $B_x : X \to 2^Y$ *is a set-valued mapping and* N_x *is an open neighborhood of x in X such that*

(a) $A(z) \subset B_x(z)$ *for each* $z \in N_x$; *and*

(b) B_x *is a* $\mathscr{F}_{C,\theta}$*-mapping.*

(iii) A is said to be a $\mathscr{F}_{C,\theta}$*-majorized mapping if for each* $x \in$*dom A, there exists a* $\mathscr{F}_{C,\theta}$*-majorant* $(B_x; N_x)$ *of A at x.*

Remark 1.4. *Definition 1.3 modifies Definition 1.2 of Shen* [25] *from H-space to FC-space, moreover, Definition 1.2 generalize the according notions in Borglin and Keiding* [3]*, Tan and Yuan* [27]*, Ding and Xia* [9]*, Chowdhury et al.* [5] *and etc.*

The following two propositions show that the intersection or union of finite transfer open valued mappings is still transfer open valued.

Proposition 1.5. *Let X be a nonempty subset of topological space E and Y be a topological space. For each* $i \in I = \{1, 2, \cdots, n\}$, *the mapping*

$S_i : X \to 2^Y$ *is transfer open valued on* X*, then the mapping* $\cap_{i=1}^n S_i$ *is transfer open valued on* X*.*

Proof. It is clear that $\cup_{x\in X}\mathrm{int}_Y(\cap_{i=1}^n S_i(x)) \subset \cup_{x\in X}(\cap_{i=1}^n S_i(x))$, thus we only need to prove that $\cup_{x\in X}(\cap_{i=1}^n S_i(x)) \subset \cup_{x\in X}\mathrm{int}_Y(\cap_{i=1}^n S_i(x))$. If $z \notin \cup_{x\in X}\mathrm{int}_Y(\cap_{i=1}^n S_i(x))$, for each $x \in X$, $z \notin \mathrm{int}_Y(\cap_{i=1}^n S_i(x))$, $z \in \mathrm{cl}_Y(Y\backslash(\cap_{i=1}^n S_i(x))) = \mathrm{cl}_Y(\cup_{i=1}^n(Y\backslash S_i(x)))$, then for each open neighborhood N_z of z, there exists a $i_0 \in I$, such that $N_z \cap (Y\backslash S_{i_0}(x)) \neq \emptyset$. That is $z \in \mathrm{cl}_Y(Y\backslash S_{i_0}(x))$, i.e. $z \notin \mathrm{int}_Y(S_{i_0}(x))$. Since S_{i_0} is transfer open valued, then $z \notin \cup_{x\in X}\mathrm{int}_Y(S_{i_0}(x)) = \cup_{x\in X}(S_{i_0}(x))$. But $\cup_{x\in X}(\cap_{i=1}^n S_i(x)) \subset \cup_{x\in X}S_{i_0}(x)$, thus $z \notin \cup_{x\in X}(\cap_{i=1}^n S_i(x))$. That is $\cup_{x\in X}(\cap_{i=1}^n S_i(x)) \subset \cup_{x\in X}\mathrm{int}_Y(\cap_{i=1}^n S_i(x))$. □

Proposition 1.6. *Let* X *be a nonempty subset of the topological space* E *and* Y *be a topological space. If for each* $i \in I = \{1, 2, \cdots, n\}$*, the mapping* $S_i : X \to 2^Y$ *is transfer open valued on* X*, then the mapping* $\cup_{i=1}^n S_i$ *is transfer open valued on* X*.*

Proof. It is clear that $\cup_{x\in X}\mathrm{int}_Y(\cup_{i=1}^n S_i(x)) \subset \cup_{x\in X}(\cup_{i=1}^n S_i(x))$, thus we only need to prove that $\cup_{x\in X}(\cup_{i=1}^n S_i(x)) \subset \cup_{x\in X}\mathrm{int}_Y(\cup_{i=1}^n S_i(x))$. If $z \notin \cup_{x\in X}\mathrm{int}_Y(\cup_{i=1}^n S_i(x))$, for each $x \in X$, $z \notin \mathrm{int}_Y(\cup_{i=1}^n S_i(x))$, $z \in \mathrm{cl}_Y(Y\backslash(\cup_{i=1}^n S_i(x))) = \mathrm{cl}_Y(\cap_{i=1}^n(Y\backslash S_i(x)))$, for each $i \in I$, that is $z \in \mathrm{cl}_Y(Y\backslash S_i(x))$, $z \notin \mathrm{int}_Y(S_i(x))$. Since S_i is transfer open valued on X, $z \notin \cup_{x\in X}\mathrm{int}_Y(S_i(x)) = \cup_{x\in X}S_i(x)$, that is $z \notin \cup_{x\in X}(\cup_{i=1}^n S_i(x))$. Hence, $\cup_{x\in X}(\cup_{i=1}^n S_i(x)) \subset \cup_{x\in X}\mathrm{int}_Y(\cup_{i=1}^n S_i(x))$. □

The following result is a special case of Lemma 2.3 in Yang [29].

Lemma 1.7. *Let* X *be a nonempty* FC*-subspace of a* FC*-space* $(E; \varphi_N)$*,* K *be a nonempty compact subset of* X *and* $T : X \to 2^E$ *be such that*

(i) T *is transfer closed valued in* X*;*

(ii) for each $N \in \langle X\rangle$*,* $\varphi_N(\triangle_n) \subset \cup_{x\in N}T(x)$*;*

(iii) for each $N \in \langle X\rangle$*, there exists a compact* FC*-subspace* L_N *of* E *containing* N *such that for each* $y \in L_N \setminus K$*, there is an* $x \in L_N \cap X$ *satisfying* $y \notin cl_X(T(x))$*. Then* $K \cap \bigcap_{x\in X} T(x) \neq \emptyset$*.*

2. **Existence theorem of maximal elements**

Let X be a topological space and $T : X \to 2^X$ be a mapping. A point $\hat{x} \in X$ is called a maximal element of T if $T(\hat{x}) = \emptyset$.

In this section, we shall establish some new existence theorems of maximal elements for $\mathscr{F}_{C,\theta}$-mapping and $\mathscr{F}_{C,\theta}$-majorized mapping defined on noncompact FC-space.

Theorem 2.1. *Let X be a nonempty FC-subspace of an FC-space $(E;\varphi_N)$, K be a nonempty compact subset of X and Y be a topological space. Suppose $A : X \to 2^Y$ be a $\mathscr{F}_{C,\theta}$-mapping such that*

(i) $\theta : X \to Y$ is a single valued mapping with $\theta(X) = Y$; and

(ii) for each $N \in \langle X \rangle$, there exists a compact FC-subspace L_N of E containing N such that for each $y \in L_N \setminus K$, there is an $x \in L_N \cap X$ satisfying $y \in int_X(A^-(\theta(x)))$. Then there exists a point $\hat{x} \in K$ such that $A(\hat{x}) = \emptyset$.

Proof. Since A is a $\mathscr{F}_{C,\theta}$-mapping then

(a) $\varphi_N(\triangle_n) \cap \bigcap_{x\in N} A^-(\theta(x)) = \emptyset$ for each $N \in \langle X \rangle$; and

(b) $A^- : Y \to 2^X$ is transfer open valued in Y.

Define a mapping $B : X \to 2^X$ by $B(x) = X \setminus A^-(\theta(x))$, for each $x \in X$. Then we claim that B is transfer closed valued in X. In deed, we only need to prove that $A^- \circ \theta : X \to 2^X$ is transfer open valued in X. Put $x_0 \in X$, $z_0 \in A^-(\theta(x_0)) \subset \cup_{y\in Y} A^-(y)$. Since A^- is transfer open valued in X, i.e., $\cup_{y\in Y} A^-(y) = \cup_{y\in Y} \mathrm{int}_X(A^-(y))$, there exists a point $y' \in Y$ such that $z_0 \in \mathrm{int}_X(A^-(y'))$. By (i), there exists a point $x' \in X$ so that $\theta(x') = y'$. Thus, $z_0 \in \mathrm{int}_X(A^-(\theta(x'))) \subset \cup_{x\in X} \mathrm{int}_X(A^-(\theta(x)))$, and $\cup_{x\in X} A^-(\theta(x)) = \cup_{x\in X} \mathrm{int}_X(A^-(\theta(x)))$, therefore, $A^- \circ \theta$ is transfer open valued in X.

By (a), $\varphi_N(\triangle_n) \subset \cup_{x\in N}(X \setminus A^-(\theta(x))) = \cup_{x\in N} B(x)$ for each $N \in \langle X \rangle$.

By (ii), for each $N \in \langle X \rangle$, there exists a compact FC-subspace L_N of E containing N such that for each $y \in L_N \setminus K$, there is an $x \in L_N \cap X$ satisfying $y \in \mathrm{int}_X(A^-(\theta(x)))$. This follows $y \notin X \setminus \mathrm{int}_X(A^-(\theta(x))) = \mathrm{cl}_X(B(x))$. Therefore, B satisfies all the hypotheses of Lemma 1.7. By lemma 1.7, $K \cap \bigcap_{x\in X} B(x) \neq \emptyset$. Then there exists a point $\hat{x} \in K$ such that $\hat{x} \notin A^-(\theta(x))$ for each $x \in X$. That is $\theta(x) \notin A(\hat{x})$ for each $x \in X$, thus $A(\hat{x}) = \emptyset$ by (i). This completes the proof. □

For a topological space (X, τ), the compactly generated extension of the topology τ is the new topology consisting of all compactly closed [respectively, open] subsets. In this way, we have the following modified form of Theorem 2.1 which is equivalent to Theorem 3.1 of Yang and Deng [29].

Theorem 2.2. *Let X be a nonempty FC-subspace of an FC-space $(E;\varphi_N)$ and K be a nonempty compact subset of X. Suppose the mapping $A : X \to 2^X$ be such that*

(i) $x \notin FC(A(x))$ for each $x \in X$;

(ii) $A^- : X \to 2^X$ *is transfer compactly open valued in* X*;*

(iii) for each $N \in \langle X \rangle$*, there exists a compact* FC*-subspace* L_N *of* E *containing* N *such that for each* $y \in L_N \setminus K$*, there is an* $x \in L_N \cap X$ *satisfying* $y \in int_X(A^-(x))$*. Then there exists a point* $\hat{x} \in K$ *such that* $A(\hat{x}) = \emptyset$*.*

Proof. Replace the topology of E by its compactly generated extension, then $(E; \varphi_N)$ with this new topology is another FC-space. It is easy to prove that $x \notin FC$-$(A(x))$ for all $x \in X$ implies $\varphi_N(\triangle_n) \cap (\bigcap_{x \in N} A^-(x)) = \emptyset$ for each $N \in \langle X \rangle$. Let $\theta = I_X$ be the identity mapping on X, then A becomes a $\mathscr{F}_{C,\theta}$-mapping. All the hypotheses of Theorem 2.1 are satisfied. By Theorem 2.1, there exists a point $\hat{x} \in K$ such that $A(\hat{x}) = \emptyset$. This completes the proof. □

Remark 2.3. *Theorem 2.1 improves Corollary* 2.2 *of Shen* [25] *from* CH*-space to* FC*-space. Moreover, Theorem* 2.1 *generalize Theorem* 3.1 *of Ding and Xia* [9]*, Theorem 3.1 of Chowdhury et al.* [5] *and Theorem 6 of Lin* [23] *with weaker assumptions.*

Moreover, it is easy to prove that Theorem 2.2 is equivalent to the following fixed point theorem.

Theorem 2.4. *Let* X *be a nonempty* FC*-subspace of an* FC*-space* $(E; \varphi_N)$ *and* K *be a nonempty compact subset of* X*. Suppose the mapping* $A : X \to 2^X$ *be such that*

(i) $A(x) \neq \emptyset$ *for each* $x \in K$*;*

(ii) $A^- : X \to 2^X$ *is transfer open valued in* X*;*

(iii) for each $N \in \langle X \rangle$*, there exists a compact* FC*-subspace* L_N *of* E *containing* N *such that for each* $y \in L_N \setminus K$*, there is an* $x \in L_N \cap X$ *satisfying* $y \in int_X(A^-(x))$*.*

Then there exists a point $\hat{x} \in K$ *such that* $\hat{x} \in FC$*-*$(A(\hat{x}))$*.*

Remark 2.5. *Theorem 2.4 generalizes Theorem 3.2 of Yuan and Tarafdar* [28] *and Theorem 3.2 of Ding and Xia* [9] *in several aspects.*

Theorem 2.6. *Let* X *be a nonempty* FC*-subspace of an* FC*-space* $(E; \varphi_N)$*,* K *be a nonempty compact subset of* X *and* Y *be a topological space. Suppose* $A : X \to 2^Y$ *is a* $\mathscr{F}_{C,\theta}$*-majorized mapping such that*

(i) $\theta : X \to Y$ *is a single valued mapping with* $\theta(X) = Y$*;*

(ii) there exists a paracompact subset G *of* X *such that* $\{x \in X : A(x) \neq \emptyset\} \subset G$*; and*

(iii) for each $N \in \langle X \rangle$*, there exists a compact* FC*-subspace* L_N *of* E *containing* N *such that for each* $y \in L_N \setminus K$*, there is an* $x \in L_N \cap X$

satisfying $y \in int_X(A^-(\theta(x)))$.
Then there exists a point $\hat{x} \in K$ *such that* $A(\hat{x}) = \emptyset$.

Proof. Suppose that $A(x) \neq \emptyset$ for each $x \in X$. By (ii), X is paracompact. Since A is a $\mathscr{F}_{C,\theta}$-majorized mapping, for each $x \in X$, there exists an open neighborhood N_x of x in X and a mapping $B_x : X \to 2^Y$ such that

(*a*) $A(z) \subset B_x(z)$ for each $z \in N_x$;
(*b*) $\varphi_N(\triangle_n) \cap \bigcap_{z\in N} B_x^-(\theta(z)) = \emptyset$ for each $N \in \langle X\rangle$; and
(*c*) $B_x^- : Y :\to 2^X$ is transfer open valued in Y.

By Theorem VIII.1.4 of Dugundji [14], the open covering $\{N_x : x \in X\}$ of X has an open precise neighborhood finite refinement N'_x with $\mathrm{cl}_X N'_x \subset N_x$. For each $x \in X$, define $B'_x : X \to 2^Y$ by

$$B'_x(z) = \begin{cases} B_x(z) & \text{if } z \in \mathrm{cl}_X N'_x, \\ Y & \text{if } z \notin \mathrm{cl}_X N'_x. \end{cases}$$

and $B : X \to 2^Y$ by $B(z) = \cap_{x\in X} B'_x(z)$ for each $z \in X$.

For each $N \in \langle X\rangle$, $t \in \cap_{z\in N} B^-(\theta(z))$, then for each $z \in N$, $\theta(z) \in B(t)$. Since $t \in X$, then there exists an $x_0 \in X$ such that $t \in \mathrm{cl}_X N'_{x_0}$, $\theta(z) \in B(t) \subset B_{x_0}(t)$. By (*b*), $t \notin \varphi_N(\triangle_n)$. Thus we have $\varphi_N(\triangle_n) \cap \bigcap_{z\in N} B^-(\theta(z)) = \emptyset$.

Now, we show $B^- : Y \to 2^X$ is transfer open valued in Y. For each $x \in X,\ y \in Y$, we have

$$\begin{aligned} (B'_x)^-(y) &= \{z \in \mathrm{cl}_X N'_x : y \in B'_x(z)\} \cup \{z \in X \setminus \mathrm{cl}_X N'_x : y \in B'_x(z)\} \\ &= \{z \in \mathrm{cl}_X N'_x : y \in B_x(z)\} \cup (X \setminus \mathrm{cl}_X N'_x) \\ &= [B_x^-(y) \cap (\mathrm{cl}_X N'_x)] \cup (X \setminus \mathrm{cl}_X N'_x) \\ &= B_x^-(y) \cup (X \setminus \mathrm{cl}_X N'_x) \qquad (2.1) \end{aligned}$$

For each $y \in Y$ let $t \in B^-(y)$ be arbitrarily fixed. Since $\{N'_x : x \in X\}$ is a neighborhood finite refinement, there exists an open neighborhood V_t of t in X such that $\{x \in X : V_t \cap N'_x \neq \emptyset\} = \{x_1, x_2, \ldots, x_n\}$. If $x \notin \{x_1, x_2, \ldots, x_n\}$, then $V_t \cap N'_x = V_t \cap \mathrm{cl}_X N'_x = \emptyset$. Thus $B'_x(z) = Y$ for all $z \in V_t$, that is $B(z) = \cap_{i=1}^n B'_{x_i}(z)$ for each $z \in V_t$. By formula (2.1), we have

$$\begin{aligned} B^-(y) &= \{z \in X : y \in B(z)\} \supset \left\{ z \in V_t : y \in \bigcap_{i=1}^n B'_{x_i}(z) \right\} \\ &= V_t \cap \bigcap_{i=1}^n (B'_{x_i})^-(y) = \bigcap_{i=1}^n \{V_t \cap [B_{x_i}^-(y) \cup (X \setminus \mathrm{cl}_X N'_{x_i})]\}. \end{aligned}$$

By Proposition 1.5 and Proposition 1.6, $B^- : Y \to 2^X$ is transfer open valued in Y. Thus B is a $\mathscr{F}_{C,\theta}$-mapping.

By (a) and the definition of B, we have $A(z) \subset B(z)$ for each $z \in X$, thus by the assumption (iii), for each $N \in \langle X \rangle$, there exists a compact FC-subspace L_N of E containing N such that for each $y \in L_N \setminus K$, there is an $x \in L_N \cap X$ satisfying $y \in \text{int}_X(A^-(\theta(x))) \subset \text{int}_X(B^-(\theta(x)))$. All conditions of Theorem 2.1 are satisfied. By Theorem 2.1, there exists a point $\hat{x} \in K$ such that $B(\hat{x}) = \emptyset$. Since $A(z) \subset B(z)$ for each $z \in K$, thus $A(\hat{x}) = \emptyset$, which is a contradiction. Hence, there exists a point $\hat{x} \in X$ such that $A(\hat{x}) = \emptyset$. By condition (iii), $\hat{x}$ must be in K. This completes the proof. □

Remark 2.7. *Theorem 2.6 improves Theorem* 1 *of Ding and Tan* [12] *from paracompact topological vector spaces to nonparacompact FC-space. Moreover, Theorem* 2.4 *generalizes Theorem 3.3 of Ding and Xia* [9]*, Theorem 3.3 of Yang and Deng* [29] *and Theorem* 2.3 *of Shen* [25].

3. **Existence of equilibria points**

Let I be a (finite or infinite) set of players. Let its strategy set X be a nonempty FC-subspace of an FC-space $(E; \varphi_N)$, and Y_i be a topological space for each $i \in I$ with $Y = \prod_{i \in I} Y_i$. Let $P_i : X \to 2^{Y_i}$ be the preference correspondence of ith player. The collection $\Lambda = (X; Y_i; P_i)_{i \in I}$ will be called a qualitative game. A point $\hat{x} \in X$ is said to be an equilibrium of the qualitative game, if $P_i(\hat{x}) = \emptyset$ for each $i \in I$.

A generalized game (=abstract economy) is a quintuple family $\Lambda = (X; Y_i; A_i; B_i; P_i; \theta_i)_{i \in I}$ where X is a nonempty FC-subspace of an FC-space $(E; \varphi_N)$, I is a (finite or infinite) set of players such that for each $i \in I$, Y_i is a topological space with $Y = \prod_{i \in I} Y_i$. Let $A_i, B_i : X \to 2^{Y_i}$, $\theta_i : X \to Y_i$ be the constraint correspondences and let $P_i : X \to 2^{Y_i}$ be the preference correspondence. An equilibrium of the generalized game Λ is a point $\hat{x} \in X$ such that for each $i \in I, \theta_i(\hat{x}) \in \bar{B}_i(\hat{x})$ and $A_i(\hat{x}) \cap P_i(\hat{x}) = \emptyset$. If $X = Y = \prod_{i \in I} Y_i$ and $\theta_i = \pi_i : Y \to Y_i$ is the projection of Y onto Y_i, then our definition of an equilibrium point coincides with the standard definition given by Yang and Deng [29], moreover, our definition of an equilibrium point generalizes the standard definition; e.g., Borglin and Keiding [3], Chowdhury et al. [5], Gale and Mas-Colell [15], Kim [20], Yannelis and Prabhakar [30], Cubiotti and Yao [6].

As an application of Theorem 2.6, we firstly prove the following existence theorem of equilibrium points for one person game.

Theorem 3.1. *Let X be a nonempty paracompact FC-subspace of an FC-space $(E;\varphi_N)$, K be a nonempty compact subset of X and Y be a topological space. Suppose the mappings $A, B, P : X \to 2^Y$ are such that*

(i) $\theta : X \to Y$ is a single valued and continuous mapping with $\theta(X) = Y$;

(ii) $domA = X$, A is a $\mathscr{F}_{C,\theta}$-mapping and P is a $\mathscr{F}_{C,\theta}$-majorized mapping;

(iii) for each $N \in \langle X \rangle$, there exists a compact FC-subspace L_N of E containing N such that for each $y \in L_N \setminus K$, there is an $x \in L_N \cap X$ satisfying $y \in int_X((A \cap P)^-(\theta(x)))$.

Then there exists a point $\hat{x} \in K$ such that $\theta(\hat{x}) \in \bar{B}(\hat{x})$ and $A(\hat{x}) \cap P(\hat{x}) = \emptyset$.

Proof. Let $W = \{x \in X : \theta(x) \notin \bar{B}(x)\}$, then W is open in X since θ is continuous. Define $Q : X \to 2^Y$ by

$$Q(z) = \begin{cases} A(z) \cap P(z) & \text{if } z \notin W; \\ A(z) & \text{if } z \in W. \end{cases}$$

By (ii), P is a $\mathscr{F}_{C,\theta}$-majorized mapping, for each $x \in$dom P, there exists an open neighborhood M_x of x in X and $\psi_x : X \to 2^Y$ such that

(a) $\varphi_N(\triangle_n) \cap \bigcap_{z \in N} \psi_x^-(\theta(z)) = \emptyset$ for each $N \in \langle X \rangle$ and $P(z) \subset \psi_x(z)$ for all $z \in M_x$;

(b) $\psi_x^- : Y \to 2^X$ is transfer open valued in Y.

Now, for each $x \in X$ with $Q(x) \neq \emptyset$, let

$$N_x = \begin{cases} M_x & \text{if } x \notin W; \\ W & \text{if } x \in W. \end{cases}$$

and define $\Psi_x : X \to 2^Y$ by

$$\Psi_x(z) = \begin{cases} A(z) \cap \psi_x(z) & \text{if } z \notin W; \\ A(z) & \text{if } z \in W. \end{cases}$$

Then for each $y \in Y$,

$$\begin{aligned} \Psi_x^-(y) &= \{z \in X \setminus W : y \in \Psi_x(z)\} \cup \{z \in W : y \in \Psi_x(z)\} \\ &= \{z \in X \setminus W : y \in A(z) \cap \psi_x(z)\} \cup \{z \in W : y \in A(z)\} \\ &= [(X \setminus W) \cap A^-(y) \cap \psi_x^-(y)] \cup [W \cap A^-(y)] \\ &= [W \cup \psi_x^-(y)] \cap A^-(y) = [W \cap A^-(y)] \cup [\psi_x^-(y) \cap A^-(y)], \end{aligned}$$

thus

$$\Psi_x^-(\theta(z)) = [W \cap A^-(\theta(z))] \cup [\psi_x^-(\theta(z)) \cap A^-(\theta(z))].$$

Hence we have

(a') $\varphi_N(\triangle_n) \cap \bigcap_{z\in N} \Psi_x^-(\theta(z)) = \emptyset$ for each $N \in \langle X\rangle$ by (ii) and (a);

(b') for each $z \in N_x$, $Q(z) \subset \Psi_x(z)$ by (a); and

(c') $\Psi_x^- : Y \to 2^X$ is transfer open valued in Y by Proposition 1.5 and Proposition 1.6. Therefore, $(\Psi_x; N_x)$ is a $\mathscr{F}_{C,\theta}$-majorant of Q at x.

From the definition of Q, it follows that $(A \cap P)(z) \subset Q(z)$ for each $z \in X$. By (iii), for each $N \in \langle X\rangle$, there exists a compact FC-subspace L_N of E containing N such that for each $y \in L_N \setminus K$, there is an $x \in L_N \cap X$ satisfying $y \in \mathrm{int}_X((A\cap P)^-(\theta(x))) \subset \mathrm{int}_X(Q^-(\theta(x)))$. All conditions of Theorem 2.6 are satisfied. By Theorem 2.6, there exists a point $\hat{x} \in K$ such that $Q(\hat{x}) = \emptyset$. By the definition of Q, we have $\theta(\hat{x}) \in \bar{B}(\hat{x})$ and $A(\hat{x}) \cap P(\hat{x}) = \emptyset$. This completes the proof. □

Remark 3.2. *Theorem 3.1 unifies Theorem 4.1 of Chowdhury et al.* [5], *Theorem* 3.1 *of Shen* [25] *and Theorem* 2 *in Ding and Tan* [12] *where* $X = Y$ *and* $\theta = I_X$ *is the identical mapping.*

By using Theorem 2.6, we shall show an equilibrium existence theorem for a qualitative game.

Theorem 3.3. *Let* $\Lambda = (X; Y_i; P_i)_{i\in I}$ *be a qualitative game. Where* X *is a nonempty paracompact* FC*-subspace of an* FC*-space* $(E; \varphi_N)$, K *be a nonempty compact subset of* X *and* $Y_i (i \in I)$ *is a topological space with* $Y = \prod_{i\in I} Y_i$. *For each* $i \in I$, *suppose the mapping* $P_i : X \to 2^{Y_i}$ *be such that*

(i) $\theta_i : X \to Y_i$ *is a single valued mapping with* $\theta_i(X) = Y_i$ *;*

(ii) W_i *=dom* P_i *is open and* $P_i : X \to 2^{Y_i}$ *is a* $\mathscr{F}_{C,\theta_i}$*-majorized mapping; and*

(iii) for each $N \in \langle X\rangle$, *there exists a compact* FC*-subspace* L_N *of* E *containing* N *such that for each* $y \in L_N \setminus K$, *there is an* $x \in L_N \cap X$ *satisfying* $y \in int_X(P_i^-(\theta_i(x)))$.

Then Λ *has an equilibrium point in* $\hat{x} \in K$.

Proof. For each $x \in X$, let $I(x) = \{i \in I : P_i(x) \neq \emptyset\}$. For each $i \in I$, define $P_i' : X \to 2^Y$ by

$$P_i'(x) = \pi_i^-(P_i(x)), \quad \text{for each} \quad x \in X,$$

where $\pi_i : Y \to Y_i$ is the projection of Y onto Y_i. Furthermore, define $P : X \to 2^Y$ by

$$P(x) = \begin{cases} \bigcap_{i\in I(x)} P_i'(x) & \text{if } I(x) \neq \emptyset; \\ \emptyset & \text{if } I(x) = \emptyset. \end{cases}$$

Clearly, for each $x \in \text{dom } P$ if and only if $I(x) \neq \emptyset$. Let $x \in \text{dom } P$, for any fixed $i \in I(x)$, by (ii), there exist an open neighborhood N_x of x in X and a mapping $\psi_{i,\,x} : X \to 2^{Y_i}$ such that

(a) $\varphi_N(\triangle_n) \cap \bigcap_{z\in N} \psi^-_{i,\,x}(\theta_i(z)) = \emptyset$ for each $N \in \langle X\rangle$ and $P_i(z) \subset \psi_{i,\,x}(z)$ for each $z \in N_x$;

(b) $\psi^-_{i,\,x} : Y_i \to 2^X$ is transfer open valued in Y_i.

By (ii), we may assume that $N_x \subset W_i$ so that $P_i(z) \neq \emptyset$ for all $z \in N_x$. Define $\Psi_x : X \to 2^Y$ by

$$\Psi_x(z) = \pi^-_i(\psi_{i,\,x}(z)) \quad \text{for each} \quad z \in X.$$

For each $y \in Y$,

$$\begin{aligned} \Psi^-_x(y) = \{z \in X : y \in \Psi_x(z)\} &= \{z \in X : y_i \in \psi_{i,\,x}(z)\} \\ &= \psi^-_{i,\,x}(y_i). \end{aligned} \tag{3.1}$$

Define $\theta : X \to 2^Y$ by $\theta(x) = \prod_{i\in I} \theta_i(x)$. By (i), $\theta(X) = Y$. Thus we have

(a') by (a) and formula (3.1), for each $N \in \langle X\rangle$, we get

$$\varphi_N(\triangle_n) \cap \bigcap_{z\in N} \Psi^-_x(\theta(z)) = \varphi_N(\triangle_n) \cap \bigcap_{z\in N} \psi^-_{i,\,x}(\theta_i(z)) = \emptyset,$$

and for each $z \in N_x$,

$$P(z) = \bigcap_{i\in I(z)} P'_i(z) \subset P'_i(z) = \pi^-_i(P_i(z)) \subset \pi^-_i(\psi_{i,\,x}(z)) = \Psi_x(z).$$

(b') by (b) and formula (3.1), $\Psi^-_x : Y \to 2^X$ is transfer open valued in Y.

Hence $(N_x; \Psi_x)$ is a $\mathscr{F}_{C,\theta}$-majorant of P at x, P is a $\mathscr{F}_{C,\theta}$-majorized mapping.

By the definitions of P, we have $P^-(y) = P^-_i(\pi_i(y))$ for each $y \in Y$. By condition (iii), for each $N \in \langle X\rangle$, there exists a compact FC-subspace L_N of E containing N such that for each $y \in L_N \setminus K$, there is an $x \in L_N \cap X$ satisfying $y \in \text{int}_X(P^-(\theta(x)))$. All hypotheses of Theorem 2.6 are satisfied. By Theorem 2.6, there exists a point $\hat{x} \in K$ such that $P(\hat{x}) = \emptyset$, which implies $I(\hat{x}) = \emptyset$. Therefore, $P_i(\hat{x}) = \emptyset$ for all $i \in I$. The proof is completed. □

Remark 3.4. *If for each $i \in I$, $X_i = Y_i$, $X = \prod_{i\in I} X_i$ and $\theta_i = \pi_i$ is the projection from X onto X_i, Theorem 3.3 unifies Theorem* 3.2 *of Shen* [25] *from CH-space to FC-space. Moreover, Theorem* 3.2 *improves*

Corollary 3 of Borglin and Keiding [3]*, Theorem* 3.1 *of Tan and Yuan* [27] *and Theorem 4.2 of Chowdhury et al.* [5] *.*

Theorem 3.5. *Let $\Lambda = (X; Y_i; A_i; B_i; P_i; \theta_i)_{i\in I}$ be a generalized qualitative game. Where X is a nonempty paracompact FC-subspace of a FC-space $(E; \varphi_N)$, K be a nonempty compact subset of X and $Y_i (i \in I)$ is a topological space with $Y = \prod_{i\in I} Y_i$. For each $i \in I$, suppose the mapping $A_i, B_i, P_i : X \to 2^{Y_i}$ be such that*

(i) $W_i = \{x \in X : (A_i \cap P_i)(x) \neq \emptyset\}$ is open in X;

(ii) $\theta_i : X \to Y_i$ is a single valued and continuous mapping with $\theta_i(X) = Y_i$;

(iii) dom $A_i = X$, A_i is a $\mathscr{F}_{C,\theta_i}$-mapping and P_i is a $\mathscr{F}_{C,\theta_i}$-majorized mapping; and

(iv) for each $N \in \langle X\rangle$, there exists a compact FC-subspace L_N of E containing N such that for each $y \in L_N \setminus K$, there is an $x \in L_N \cap X$ satisfying $y \in int_X((A_i \cap P_i)^-(\theta_i(x)))$.

Then Λ has an equilibrium point in $\hat{x} \in K$.

Proof. For each $i \in I$, let $U_i = \{x \in X : \theta_i(x) \notin \bar{B}_i(x)\}$, then U_i is open in X by (ii). Define $Q_i : X \to 2^{Y_i}$ by

$$Q_i(x) = \begin{cases} A_i(x) \cap P_i(x) & \text{if } x \notin U_i; \\ A_i(x) & \text{if } x \in U_i. \end{cases}$$

Now, we will show that the qualitative game $\Lambda' = (X; Y_i; Q_i)_{i\in I}$ satisfies all assumptions of Theorem 3.3. For each $i \in I$, the set

$$\begin{aligned} \{x \in X : Q_i(x) \neq \emptyset\} &= \{x \in U_i : A_i(x) \neq \emptyset\} \\ &\cup \{x \in X \setminus U_i : A_i(x) \cap P_i(x) \neq \emptyset\} \\ &= U_i \cup [(X \setminus U_i) \cap W_i] = U_i \cup W_i \end{aligned}$$

is open in X.

Since P_i is a $\mathscr{F}_{C,\theta_i}$-majorized mapping, for each $x \in$ dom P_i, there exist an open neighborhood M_x of x in X and a mapping $\psi_{i,\,x} : X \to 2^{Y_i}$ such that

(a) $\varphi_N(\triangle_n) \cap \bigcap_{z\in N} \psi^-_{i,\,x}(\theta_i(z)) = \emptyset$ for each $N \in \langle X\rangle$ and $P_i(z) \subset \psi_{i,\,x}(z)$ for each $z \in N_x$;

(b) $\psi^-_{i,\,x} : Y_i \to 2^X$ is transfer open valued in Y_i.

Now, for each $x \in X$ with $Q_i(x) \neq \emptyset$, let

$$N_x = \begin{cases} M_x & \text{if } x \notin U_i; \\ U_i & \text{if } x \in U_i. \end{cases}$$

and define $\Psi_{i,\,x} : X \to 2^{Y_i}$ by

$$\Psi_{i,\,x}(z) = \begin{cases} A_i(z) \cap \psi_{i,\,x}(z) & \text{if } z \notin U_i; \\ A_i(z) & \text{if } z \in U_i. \end{cases}$$

For each $y \in Y_i$, we have

$$\begin{aligned} \Psi^-_{i,\,x}(y) &= \{z \in X \setminus U_i : y \in \Psi_{i,\,x}(z)\} \cup \{z \in U_i : y \in \Psi_{i,\,x}(z)\} \\ &= \{z \in X \setminus U_i : y \in A_i(z) \cap \psi_{i,\,x}(z)\} \cup \{z \in U_i : y \in A_i(z)\} \\ &= [(X \setminus U_i) \cap A^-_i(y) \cap \psi^-_{i,\,x}(y)] \cup [U_i \cap A^-_i(y)] \\ &= [U_i \cup \psi^-_{i,\,x}(y)] \cap A^-_i(y) \\ &= [U_i \cap A^-_i(y)] \cup [\psi^-_{i,\,x}(y) \cap A^-_i(y)]. \end{aligned}$$

Thus, we have

(a') $\varphi_N(\triangle_n) \cap \bigcap_{z \in N} \Psi^-_{i,\,x}(\theta_i(z)) = \emptyset$ for each $N \in \langle X \rangle$ and $Q_i(z) \subset \Psi_{i,\,x}(z)$ for each $z \in N_x$ by (a) and (iii);

(b') By Proposition 1.5 and Proposition 1.6, $\Psi^-_{i,\,x} : Y_i \to 2^X$ is transfer open valued in Y_i. Hence $(N_x; \Psi_{i,x})$ is a majorant of Q_i at x, i.e., Q_i is a $\mathscr{F}_{c,\theta_i}$-majorized mapping.

By condition (iv) and the definition of Q_i, for each $N \in \langle X \rangle$, there exists a compact FC-subspace L_N of E containing N such that for each $y \in L_N \setminus K$, there is an $x \in L_N \cap X$ satisfying $y \in \mathrm{int}_X((A_i \cap P_i)^-(\theta_i(x))) \subset \mathrm{int}_X(Q^-_i(\theta_i(x)))$. All assumptions of Theorem 3.3 are satisfied. By Theorem 3.3, there exists a point $\hat{x} \in K$ such that $Q_i(\hat{x}) = \emptyset$ for all $i \in I$. By the definition of Q_i, we must have that for each $i \in I$, $\theta_i(\hat{x}) \in \bar{B}_i(\hat{x})$ and $A_i(\hat{x}) \cap P_i(\hat{x}) = \emptyset$. $\square$

Remark 3.6. *Theorem 3.5 generalized Theorem* 3.3 *of Shen* [25]*, Theorem* 4 *of Ding and Tan* [12] *and Corollary* 3.4 *of Tan and Yuan* [26] *in weaker assumptions.*

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