ISSN: 1017-060X (Print)

ISSN: 1735-8515 (Online)

Bulletin of the

Iranian Mathematical Society

Vol. 40 (2014), No. 5, pp. 1195–1212

Title:

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Published by Iranian Mathematical Society
http://bims.ims.ir

Bull. Iranian Math. Soc.
Vol. 40 (2014), No. 5, pp. 1195–1212
Online ISSN: 1735-8515

LOWER SEMICONTINUITY FOR PARAMETRIC SET-VALUED VECTOR EQUILIBRIUM-LIKE PROBLEMS

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(Communicated by Behzad Djafari-Rouhani)

Abstract. A concept of weak f-property for a set-valued mapping is introduced, and then under some suitable assumptions, which do not involve any information about the solution set, the lower semicontinuity of the solution mapping to the parametric set-valued vector equilibrium-like problems are derived by using a density result and scalarization method, where the constraint set K and a set-valued mapping H are perturbed by different parameters.
Keywords: Lower semicontinuity, parametric set-valued vector equilibrium-like problem, weak f-property, positive proper efficient solution.
MSC(2010): Primary: 47H09; Secondary: 90C33, 49J40.

1. Introduction

It is well known that the vector equilibrium problem provides a unified model of several problems such as the vector optimization, the vector variational inequalities and the vector complementarities. In the past decades, various types of vector equilibrium problems have intensively been considered (see, e.g., [2,6,8,12–15,18,26], etc.). Among many properties of vector equilibrium problems, the stability analysis of solution mappings is of considerable interest. Recently, there have been many

Article electronically published on October 27, 2014.
Received: 22 May 2013, Accepted: 30 August 2013.

results on the stability such as continuity, semicontinuity, lower semicontinuity of the solution mapping for parametric optimization, parametric vector variational inequalities and parametric vector equilibrium problems in the literature (see, e.g., [4, 9, 20, 21, 27–29], etc.).

In [16], Cheng and Zhu studied the lower semicontinuity of the solution mapping to weak vector variational inequalities in finite dimension spaces by using the scalarization method. By applying a density result and scalarization approach, Gong and Yao [19] also explored the lower semicontinuity of the set of efficient solutions to parametric vector equilibrium problems under some conditions which involve some information about the solution set. Khanh and Luu [23] studied parametric multi-valued quasi-variational inequalities and obtained the semicontinuity of the solution sets and approximate solution sets. Zhong and Huang [32] studied the lower semicontinuity of the solution mapping for the parametric weak vector variational inequalities and also obtained the lower semicontinuity of the solution mapping by degree-theoretic method. Zhao [31] derived a sufficient and necessary condition $(H1)$ for the Hausdorff lower semicontinuity of the solution mapping to a parametric optimization problems. Under mild assumptions, Kien [24] also proved the sufficient and necessary condition $(H1)$ for the Hausdorff lower semicontinuity of the solution mapping to a parametric optimization problems. Later on, in order to obtain the sufficient or necessary conditions for the lower semi-continuity of solution sets, many authors introduced a key assumption (Hg) similar to what is given in [31] for parametric variational inequalities, parametric vector equilibrium problems and parametric quasi-equilibrium problems (see, e.g., Agarwal et al. [1], Chen et al. [10], Li and Chen [25], Zhong and Huang [32,33], etc.). Very recently, by using a scalarization technique, Chen and Huang [7] investigated sufficient conditions for the continuity of the solution mappings to the two kinds of parametric generalized vector equilibrium problems under suitable conditions. Xu and Li [30] discussed the lower semicontinuity of solution mappings to a parametric generalized strong vector equilibrium problem without any information about its solution set. It is worth noting that, many authors studied the stability of solution sets for optimization problems, vector variational inequalities and vector equilibrium problems involving the information about the solution set of the considered problems such as the key assumption (Hg) or (H1)(see, e.g., [1, 7, 10, 16, 19, 23–25, 31–33], etc.). However, we do not know ahead of the information of the solutions including the existence in many practical problems.

Motivated and inspired by the results mentioned above, the aim of this paper is devoted to study the lower semicontinuity of the solution mapping to the parametric set-valued vector equilibrium-like problems (for short, (PSVEP)), where the constraint set K and a set-valued mapping H are perturbed by different parameters. To this end, we introduce a concept of weak f-property for set-valued mapping, and then the existence results of f-efficient solutions and the behavior of f-efficient solution set for (PSVEP) are established under some suitable conditions. The density of the positive proper efficient solution set relative to the solution set of a set-valued vector equilibrium-like problem are proved without involving any information about the solution set. Finally, the lower semicontinuity of the solution mapping to (PSVEP) are derived by using a density result and scalarization method. The results presented in this paper generalize and improve some main results of Xu and Li (2013) [30].

2. **Preliminaries**

Let X, Y, Z and W be locally convex Hausdorff topological vector spaces, and Y^* be the topological dual space of Y. Let K be a nonempty subset of X, C be a pointed closed convex cone in Y with nonempty interior $\text{int}C \neq \emptyset$, Λ and Ξ be nonempty subsets of Z and W, respectively. Let $\eta : X \times X \to X$ be a vector-valued mapping, and $H : X \times X \to 2^X$ be a set-valued mapping, where 2^X means the family of all nonempty subsets of X. The zero vector of X (Y, Z or W) is denoted by 0. The dual cone (positive polar cone) of C is defined as

$$C^* = \{f \in Y^* : f(y) \geq 0, \quad \forall y \in C\}.$$

The quasi-interior of C^* is defined as

$$C^\sharp = \{f \in Y^* : f(y) > 0, \quad \forall y \in C \setminus \{0\}\}.$$

It is well-known that $\text{int}C^* \subseteq C^\sharp$ and equality holds whenever $\text{int}C^* \neq \emptyset$ (see, e.g., [22]).

We consider the following *set-valued vector equilibrium-like problem* (SVEP): find $x \in K$ such that

$$H(x, \eta(y, x)) \cap (-C \setminus \{0\}) = \emptyset, \quad \forall y \in K. \tag{2.1}$$

Denote the solution set of this (SVEP) by S.

If the mappings K and H are perturbed by parameters $\lambda \in \Lambda$ and $\mu \in \Xi$, respectively, then for any given $(\lambda, \mu) \in \Lambda \times \Xi$, we define the *parametric set-valued vector equilibrium-like problem* (PSVEP): find $x \in K(\lambda)$ such that

$$H(\mu, x, \eta(y, x)) \cap (-C \setminus \{0\}) = \emptyset, \quad \forall y \in K(\lambda), \tag{2.2}$$

where $H : \Xi \times X \times X \to 2^Y$ and $K : \Lambda \to 2^X$ are two set-valued mappings such that for some $(\bar{\lambda}, \bar{\mu}) \in \Lambda \times \Xi, K(\bar{\lambda}) = K$ and $H(\bar{\mu}, x, \eta(y, x)) = H(x, \eta(y, x))$ for any $x, y \in K$. For each $(\lambda, \mu) \in \Lambda \times \Xi$, denote the solution set of (PSVEP) by $S(\lambda, \mu)$, i.e.,

$$S(\lambda, \mu) = \{x \in K(\lambda) : H(\mu, x, \eta(y, x)) \cap (-C \setminus \{0\}) = \emptyset, \quad \forall y \in K(\lambda)\}.$$

So, $S : \Lambda \times \Xi \to 2^X$ is a set-valued mapping, which is called the solution mapping of (2.2).

We also consider the following *weak set-valued vector equilibrium-like problem* (WSVEP): find $x \in K$ such that

$$H(x, \eta(y, x)) \cap (-\text{int}C) = \emptyset, \quad \forall y \in K.$$

For each $(\lambda, \mu) \in \Lambda \times \Xi$, the corresponding *parametric weak set-valued vector equilibrium-like problem* (PWSVEP) defined as follows: find $x \in K(\lambda)$ such that

$$H(\mu, x, \eta(y, x)) \cap (-\text{int}C) = \emptyset, \quad \forall y \in K(\lambda).$$

Denote the solution sets of these (WSVEP) and (PWSVEP) by S_w and $S_w(\lambda, \mu)$, respectively.

Similar to [19, 30], we define the f-efficient solution of (SVEP) and (PSVEP), respectively, i.e.,

for each $f \in C^* \setminus \{0\}$, the f-efficient solution set of (SVEP) is defined by

$$S_f = \{x \in K : \inf_{v \in H(x, \eta(y, x))} f(v) \geq 0, \quad \forall y \in K\}$$

and for each $(\lambda, \mu) \in \Lambda \times \Xi$, the f-efficient solutions set of (PSVEP) is defined by

$$S_f(\lambda, \mu) = \{x \in K(\lambda) : \inf_{v \in H(\mu, x, \eta(y, x))} f(v) \geq 0, \quad \forall y \in K(\lambda)\}.$$

It is worth noting that if $f = 0$, then $S_f = K$ and $S_f(\lambda, \mu) = K(\lambda)$ for each $(\lambda, \mu) \in \Lambda \times \Xi$.

Special cases of the problem (2.2) are as follows:

(I) If $\Lambda = \Xi$,$\lambda = \mu, \eta(y, x) = y$ and $H(\mu, x, \eta(y, x)) = F(x, y, \mu)$ for all $x, y \in X$ and $\mu \in \Xi$, then the problem (2.2) is reduced to the following *generalized strong vector equilibrium problem*: find $x \in K(\mu)$ such that

$$F(x, y, \mu) \cap (-C \setminus \{0\}) = \emptyset, \quad \forall y \in K(\mu),$$

which has been studied by Gong and Yao [19], Xu and Li [30] and the references therein.

(II) If $\Lambda = \Xi$,$\lambda = \mu$ and $H : \Xi \times X \times X \to Y$ is vector-valued, $F(x, y, \mu) = H(\mu, x, \eta(y, x))$ for all $x, y \in X$ and $\mu \in \Xi$, then the problem (2.2) is reduced to the following *generalized strong vector equilibrium problem*: find $x \in K(\mu)$ such that

$$F(x, y, \mu) \notin -C \setminus \{0\}, \quad \forall y \in K(\mu),$$

which has been studied by Ansari, Oettli and Schlaer [2], Bianchi, Hadjisavvas and Schaible [6] and the references therein.

We firstly recall some basic concepts and well-known results.

Definition 2.1. *[19] A point $x \in K$ is called a positive proper efficient solution to (SVEP) if there exists $f \in C^\sharp$ such that*

$$\inf_{v \in H(x, \eta(y,x))} f(v) \geq 0, \forall y \in K.$$

Definition 2.2. *[3,5] Let Γ be a Hausdorff topological space and X be a locally convex Hausdorff topological vector space. A set-valued mapping $F : \Gamma \to 2^X$ is said to be:*

(1) *upper semicontinuous in the sense of Berge at $\gamma_0 \in \Gamma$ if, for each open set V with $F(\gamma_0) \subset V$, there exists $\delta > 0$ such that*

$$F(\gamma) \subset V, \quad \forall \gamma \in B(\gamma_0, \delta);$$

(2) *lower semicontinuous in the sense of Berge at $\gamma_0 \in \Gamma$ if, for each open set V with $F(\gamma_0) \cap V \neq \emptyset$, there exists $\delta > 0$ such that*

$$F(\gamma) \cap V \neq \emptyset, \quad \forall \gamma \in B(\gamma_0, \delta).$$

We say that F is upper semicontinuous (resp., lower semicontinuous) on Γ if it is upper semicontinuous (resp., lower semicontinuous) at each $\gamma \in \Gamma$. F is called continuous on Γ if it is both upper semicontinuous and lower semi-continuous on Γ.

Definition 2.3. *Let E be a nonempty convex subset of X. A set-valued mapping $F : E \to 2^Y$ is said to be C-convex on E if for any $x_1, x_2 \in E$*

and $l \in [0,1]$,

$$lF(x_1) + (1-l)F(x_2) \subseteq F(lx_1 + (1-l)x_2) + C.$$

Definition 2.4. *[17] Let* K *be a nonempty subset of a Hausdorff topological vector space* X. *A set-valued mapping* $F : K \to 2^X$ *is called a KKM mapping if, for each finite subset* $\{x_1, x_2, \cdots, x_m\}$ *of* K, $co\{x_1, \cdots, x_m\} \subseteq \bigcup_{i=1}^m F(x_i)$, *where co denotes the convex hull.*

Definition 2.5. *Let* $f \in C^* \setminus \{0\}$ *be given and* D *be a nonempty subset of* X. *A set-valued mapping* $\Pi : D \to 2^Y$ *is said to have the weak* f*-property at* $\bar{x} \in D$ *if, for all* $x_\alpha \xrightarrow{D} \bar{x}$ *with* $x_\alpha \neq \bar{x}$ *and* $\inf_{v \in \Pi(\bar{x})} f(v) \geq 0$ *implies that there exists an index* α_0 *such that*

$$\inf_{v \in \Pi(x_\alpha)} f(v) \geq 0, \quad \forall \alpha \geq \alpha_0, \tag{2.3}$$

where the notation $x_\alpha \xrightarrow{D} \bar{x}$ *means that* $x_\alpha \to \bar{x}, x_\alpha \in D$.

We say that $\Pi : D \to 2^Y$ *is said to have the weak* f*-property on* D *if it has the weak* f*-property at every point* $x \in D$.

Remark 2.6. *If* $D = X$, *and the inequality (2.3) is replaced by*

$$\inf_{v \in \Pi(x_\alpha)} f(v) > 0, \quad \forall \alpha \geq \alpha_0,$$

then the weak f*-property is reduced to the* f*-property of Xu and Li [30]. That is, the* f*-property implies the weak* f*-property. But the converse is not true.*

Example 2.7. *[30] Let* $X = R, Y = R^2, C = R^2_+, \bar{x} = 0, f = (1,1) \in C^* \setminus \{0\}$ *and let*

$$\Pi(x) = \begin{cases} [-1,1] \times [1,2], & \text{if} \quad x = 0, \\ (-1,1) \times [1,3], & \text{otherwise.} \end{cases}$$

Then Π *has the* f*-property at* $\bar{x} = 0$, *moreover,* Π *also has the weak* f*-property at* $\bar{x} = 0$.

Example 2.8. *Let* X, Y, C *be the same as Example 2.7,* $\bar{x} = 0$ *and* $f = (1,1) \in C^* \setminus \{0\}$, *and let*

$$\Pi(x) = \begin{cases} [-1,1] \times [1,2], & \text{if} \quad x = 0, \\ [-1,1) \times [1,3], & \text{otherwise.} \end{cases}$$

Then, for any net $\{x_\alpha\} \subseteq R$, $x_\alpha \to \bar{x}$ *with* $x_\alpha \neq 0$ *for all index* α, $\inf_{v \in \Pi(0)} f(v) = 0 \geq 0$. *Simple computation shows that, for all index* α, $\inf_{v \in \Pi(x_\alpha)} f(v) = 0 \geq 0$. *Therefore,* Π *has the weak* f*-property at* $\bar{x} = 0$.

Particularly, taking up $x_n = \frac{1}{n}$. *Then* $x_n \to 0$ *and* $x_n \neq 0$. *However, there exists a positive integer number* n_0 *such that*

$$0 = \inf_{v \in \Pi(x_n)} f(v) \not> 0, \quad \forall n \geq n_0.$$

So, Π *dose not have the* f*-property at* $\bar{x} = 0$.

Lemma 2.9. *[17] Let* K *be a nonempty subset of a Hausdorff topological vector space* X *and* $F : K \to 2^X$ *be a KKM mapping such that, for all* $y \in K$, $F(y)$ *is closed and* $F(y^*)$ *is compact for some* $y^* \in K$. *Then* $\bigcap_{y \in K} F(y) \neq \emptyset$.

Lemma 2.10. *[3] Let* Γ *be a Hausdorff topological space,* X *be a locally convex Hausdorff topological vector space and* $F : \Gamma \to 2^X$ *be a set-valued mapping. Then the following hold:*

(1) F *is lower semicontinuous at* $\gamma_0 \in \Gamma$ *if and only if, for any net* $\{\gamma_\alpha\} \subseteq \Gamma$ *with* $\gamma_\alpha \to \gamma_0$ *and* $x_0 \in F(\gamma_0)$, *there exists a net* $\{x_\alpha\} \subseteq X$ *with* $x_\alpha \in F(\gamma_\alpha)$ *for all* α *such that* $x_\alpha \to x_0$;

(2) *If* F *is compact-valued, then* F *is upper semicontinuous at* $\gamma_0 \in \Gamma$ *if and only if, for any net* $\{\gamma_\alpha\} \subseteq \Gamma$ *with* $\gamma_\alpha \to \gamma_0$ *and* $\{x_\alpha\} \subseteq X$ *with* $x_\alpha \in F(\gamma_\alpha)$ *for all* α, *there exists* $x_0 \in F(\gamma_0)$ *and a subnet* $\{x_\beta\}$ *of* $\{x_\alpha\}$ *such that* $x_\beta \to x_0$;

(3) *If* F *is upper semicontinuous and closed-valued, then* F *is closed. Conversely, if* F *is closed and* X *is compact, then* F *is upper semicontinuous.*

Lemma 2.11. *[3] The union* $\Upsilon = \bigcup_{j \in J} \Upsilon_j$ *of a family of lower semicontinuous set-valued mappings* Υ_j *from a topological space* X *into a topological space* Y *is also an lower semicontinuous set-valued mapping from* X *into* Y, *where* J *is an index set.*

Lemma 2.12. *[3, 26] Let* X *and* Y *be two topological spaces. Let* $F : X \times Y \to R = (-\infty, +\infty)$ *be a bifunction and* $G : X \to 2^Y$ *be a set-valued mapping with nonempty values and let* $g(x) = \sup_{y \in G(x)} F(x, y)$. *The following statements hold:*

(1) *if* F *and* G *are both lower semicontinuous, then* g *is also lower semicontinuous;*

(2) *if* F *is upper semicontinuous and* G *is upper semicontinuous with compact values, then* g *is also upper semicontinuous.*

3. **Main results**

In this section, we shall investigate the lower semi-continuity of the solution mapping $S(\lambda, \mu)$ for (PSVEP) corresponding to a pair (λ, μ) of parameters under some suitable assumptions which do not involve the information of solution to (PSVEP).

Lemma 3.1. *Let $\eta : X \times X \to X$ be continuous. For each $(\lambda, \mu) \in \Lambda \times \Xi$, $K(\lambda)$ is a nonempty compact convex subset of X, and assume that the following conditions hold:*

(i) for each $y \in K(\lambda), H(\mu, \cdot, \eta(y, \cdot))$ is lower semicontinuous on $K(\lambda)$, and for each $x \in K(\lambda)$, $H(\mu, x, \eta(\cdot, x))$ is nonempty compact-valued and C-convex on $K(\lambda)$;

(ii) $H(\mu, x, \eta(x, x)) \subseteq C$ for all $x \in K(\lambda)$.

Then, for each $f \in C^* \setminus \{0\}, S_f(\lambda, \mu)$ is a nonempty compact set.

Proof. Let $f \in C^* \setminus \{0\}$. Define a set-valued mapping $\Theta : K(\lambda) \to 2^{K(\lambda)}$ by

$$\Theta(y) = \{x \in K(\lambda) : \inf_{v \in H(\mu, x, \eta(y,x))} f(v) \geq 0\}, \quad \forall y \in K(\lambda).$$

Clearly, $S_f(\lambda, \mu) = \bigcap_{y \in K(\lambda)} \Theta(y)$. To prove that $S_f(\lambda, \mu) \neq \emptyset$, we only need to prove that $\bigcap_{y \in K(\lambda)} \Theta(y) \neq \emptyset$. By condition (ii), one has

$$\inf_{v \in H(\mu, y, \eta(y,y))} f(v) \geq 0,$$

which shows that $y \in \Theta(y)$ for each $y \in K(\lambda)$.

Let us firstly show that, for each $y \in K(\lambda), \Theta(y)$ is a closed set. Taking any sequence $\{x_n\} \subseteq \Theta(y)$ such that $x_n \to \bar{x} \in K(\lambda)$ as $n \to \infty$. Then

$$\inf_{v \in H(\mu, x_n, \eta(y, x_n))} f(v) \geq 0. \tag{3.1}$$

Set $g_y(x) = \inf_{v \in H(\mu, x, \eta(y,x))} f(v) = -\sup_{v \in H(\mu, x, \eta(y,x))} -f(v)$. According to (i) and Proposition 19 of [3, CH.3,SEC.1,P.118], it follows that g_y is upper semicontinuous on $K(\lambda)$. By (3.1), we have $g_y(x_n) \geq 0$ and so,

$$\inf_{v \in H(\mu, \bar{x}, \eta(y, \bar{x}))} f(v) = g_y(\bar{x}) \geq \limsup_{n \to \infty} g_y(x_n) \geq 0.$$

Namely, $\bar{x} \in \Theta(y)$. Hence, for each $y \in K(\lambda), \Theta(y)$ is a closed set. Taking into account the compactness of $K(\lambda)$ and $\Theta(y) \subseteq K(\lambda)$, for each $y \in K(\lambda), \Theta(y)$ is a compact set.

Secondly, let us prove that Θ is a KKM mapping. Suppose to the contrary that there exists a finite subset $\{y_1, y_2, \cdots, y_m\} \subseteq K(\lambda)$ such that $\mathrm{co}\{y_1, y_2, \cdots, y_m\} \not\subseteq \bigcup_{j=1}^{m} \Theta(y_j)$. That is, there exist $t_j \in [0,1], j = 1, 2, \cdots, m$ with $\sum_{j=1}^{m} t_j = 1$ such that $\tilde{y} = \sum_{j=1}^{m} t_j y_j \notin \Theta(y_j)$, for $j = 1, 2, \cdots, m$. Then, for each $j \in \{1, 2, \cdots, m\}$, $\inf_{v \in H(\mu, \tilde{y}, \eta(y_j, \tilde{y}))} f(v) < 0$. By condition (i), $H(\mu, \tilde{y}, \eta(y_j, \tilde{y}))$ is a compact set and so, there exists $v_j \in H(\mu, \tilde{y}, \eta(y_j, \tilde{y}))$ such that $f(v_j) = \min_{v \in H(\mu, \tilde{y}, \eta(y_j, \tilde{y}))} f(v) < 0$. Since for each $y \in K(\lambda)$, $H(\mu, y, \eta(\cdot, y))$ is C-convex on $K(\lambda)$, we have

$$\sum_{j=1}^{m} t_j v_j \in \sum_{j=1}^{m} t_j H(\mu, \tilde{y}, \eta(y_j, \tilde{y})) \subseteq H(\mu, \tilde{y}, \eta(\tilde{y}, \tilde{y})) + C \subseteq C + C \subseteq C.$$

Noting that to f is linear continuous and $f \in C^* \setminus \{0\}$, by the above inclusion, we get

$$0 \leq f(\sum_{j=1}^{m} t_j v_j) = \sum_{j=1}^{m} t_j f(v_j) < 0,$$

which is a contradiction. Therefore, Θ is a KKM mapping. This follows from Lemma 2.9 that $\bigcap_{y \in K(\lambda)} \Theta(y) \neq \emptyset$. So, $S_f(\lambda, \mu) = \bigcap_{y \in K(\lambda)} \Theta(y) \neq \emptyset$. By the compactness of $Theta(y)$ for $yinK(lambda)$, we have that $S_f(lambda, mu)$ is compact. □

The following result is a direct consequence of Lemma 3.1.

Corollary 3.2. *Let K be a nonempty compact convex subset of X and $\eta : X \times X \to X$ be continuous. Assume that the following conditions hold:*

(i) for each $y \in K, H(\cdot, \eta(y, \cdot))$ is lower semicontinuous on K, and for each $x \in K$, $H(x, \eta(\cdot, x))$ is nonempty compact-valued and C-convex on K;

(ii) $H(x, \eta(x, x)) \subseteq C$ for all $x \in K$.

Then, for each $f \in C^* \setminus \{0\}, S_f$ is nonempty and compact.

Theorem 3.3. *Let $f \in C^* \setminus \{0\}$ be given. Assume that all assumptions of Lemma 3.1 are satisfied, and the following conditions hold:*

(i) $K(\cdot)$ is continuous on Λ;

(ii) $H(\cdot, \cdot, \cdot)$ has the weak f-property on $\Xi \times X \times X$.

Then, $S_f(\cdot, \cdot)$ is lower semicontinuous on $\Lambda \times \Xi$.

Proof. By Lemma 3.1, we have $S_f(\lambda, \mu) \neq \emptyset$ for $(\lambda, \mu) \in \Lambda \times \Xi$. Suppose that there exists $(\hat{\lambda}, \hat{\mu}) \in \Lambda \times \Xi$ such that $S_f(\cdot, \cdot)$ is not lower semicontinuous at $(\hat{\lambda}, \hat{\mu})$. Therefore there exist $\hat{x} \in S_f(\hat{\lambda}, \hat{\mu})$ and a net $\{(\lambda_\alpha, \mu_\alpha)\} \subseteq \Lambda \times \Xi$ with $(\lambda_\alpha, \mu_\alpha) \to (\hat{\lambda}, \hat{\mu})$ satisfying $x_\alpha \not\to \hat{x}$ for all $x_\alpha \in S_f(\lambda_\alpha, \mu_\alpha)$, $\hat{x} \in K(\hat{\lambda})$ and so

$$\inf_{v \in H(\hat{\mu}, \hat{x}, \eta(y, \hat{x}))} f(v) \geq 0, \quad \forall y \in K(\hat{\lambda}). \tag{3.2}$$

By (i), there exists a net $\{\tilde{x}_\alpha\} \subseteq K(\lambda_\alpha)$ such that $\tilde{x}_\alpha \to \hat{x}$. Moreover, one can conclude that there exists a subnet $\{\tilde{x}_\iota\} \subseteq \{\tilde{x}_\alpha\}$ such that $\tilde{x}_\iota \notin S_f(\lambda_\iota, \mu_\iota)$. This follows that there exists some $y_\iota \in K(\lambda_\iota)$ such that

$$\inf_{v_\iota \in H(\mu_\iota, \tilde{x}_\iota, \eta(y_\iota, \tilde{x}_\iota))} f(v_\iota) < 0. \tag{3.3}$$

It follows from (i) that there exists $\hat{y} \in K(\hat{\lambda})$ such that $y_\iota \to \hat{y}$ (taking a subnet if necessary). Consequently, one has $(\tilde{x}_\iota, y_\iota, \lambda_\iota, \mu_\iota) \to (\hat{x}, \hat{y}, \hat{\lambda}, \hat{\mu})$. Since $\eta : X \times X \to X$ is continuous, we obtain $\eta(y_\iota, \tilde{x}_\iota) \to \eta(\hat{y}, \hat{x})$. This, together with (3.2) and (ii), yields that there exists an index ι_0 such that

$$\inf_{v_\iota \in H(\mu_\iota, \tilde{x}_\iota, \eta(y_\iota, \tilde{x}_\iota))} f(v_\iota) \geq 0, \quad \iota \geq \iota_0,$$

which contradicts (3.3). $\square$

Next, we give the following two examples to illustrate Theorem 3.3.

Example 3.4. *Let* $X = Z = W = R, Y = R^2, C = R^2_+, \Lambda = [0,1], \Xi = [-1,1]$, *and for each* $x, y \in X, \lambda \in \Lambda, \mu \in \Xi$, *let* $K(\lambda) = [\lambda, 1], \eta(y,x) = y - 2x$ *and*

$$H(\mu, x, y) = \begin{cases} [0, 2+\mu] \times [1,2], & \text{if} \quad \mu \in [-1,1] \setminus \{0\}, \\ (y + x + 1, 2), & \text{if} \quad \mu = 0. \end{cases}$$

Then

$$H(\mu, x, \eta(y,x)) = \begin{cases} [0, 2+\mu] \times [1,2], & \text{if} \quad \mu \in [-1,1] \setminus \{0\}, \\ (y - x + 1, 2), & \text{if} \quad \mu = 0. \end{cases}$$

It is easy to verify that all conditions of Lemma 3.1 and conditions (i),(ii) are satisfied. For each $f = (f_1, f_2) \in C^* \setminus \{0\}$, *simple computation shows that, for each* $(\lambda, \mu) \in \Lambda \times \Xi$, $S_f(\lambda, \mu) = [\lambda, 1]$. *Therefore,* $S_f(\cdot, \cdot)$ *is lower semicontinuous on* $\Lambda \times \Xi$.

Example 3.5. *Let* $X = Z = W = R, Y = R^2, C = R^2_+, \Lambda = [0, \frac{1}{2}], \Xi = [-1, 1]$, *and for each* $x, y \in X, \lambda \in \Lambda, \mu \in \Xi$, *let* $K(\lambda) = [\lambda, -\lambda + 2], \eta(y, x) = y + 2x$ *and*

$$H(\mu, x, y) = \begin{cases} [0, 2+\mu] \times [1, 2], & \textit{if} \quad \mu = 0, \\ (-y + x + 1, 2), & \textit{if} \quad \mu \in [-1, 1] \setminus \{0\}. \end{cases}$$

Then

$$H(\mu, x, \eta(y, x)) = \begin{cases} [0, 2+\mu] \times [1, 2], & \textit{if} \quad \mu = 0, \\ (-y - x + 1, 2), & \textit{if} \quad \mu \in [-1, 1] \setminus \{0\}. \end{cases}$$

It is easy to verify that all conditions of Lemma 3.1 and condition (i) are satisfied. For $f = (1, 0) \in C^* \setminus \{0\}$, *simple computation shows that, for* $(\lambda, \mu) \in \Lambda \times \{0\}$, $S_f(\lambda, \mu) = [\lambda, -\lambda+2]$ *and for each* $(\lambda, \mu) \in \Lambda \times (\Xi \setminus \{0\})$, $S_f(\lambda, \mu) = \emptyset$. *Therefore,* $S_f(\cdot, \cdot)$ *is not lower semicontinuous at* $(\lambda, \mu) \in \Lambda \times \{0\}$. *Indeed, let* $\lambda = \mu = x = 0, y = 1$, $H(0, 0, 1) = [0, 2+\mu] \times [1, 2]$, *one has*

$$\inf_{v \in H(0,0,1)} f(v) = 0.$$

But, there exist $\lambda_n = \mu_n = x_n = \frac{1}{n}, y_n = 1 + \frac{3}{n}$ *with* $(\lambda_n, \mu_n, x_n, y_n) \to (0, 0, 0, 1)$ *such that* $H(\frac{1}{n}, \frac{1}{n}, 1 + \frac{3}{n}) = -\frac{2}{n}$ *and so, for any positive integer numbers* $n_0, n \geq n_0$,

$$\inf_{v \in H(\frac{1}{n}, \frac{1}{n}, 1+\frac{3}{n})} f(v) = -\frac{2}{n} < 0.$$

Remark 3.6. *Examples 3.4 and 3.5 show that the* f*-efficient solution set* $S_f(\cdot, \cdot)$ *of (PSVEP) is not a singleton, but a general set. Moreover, when* $S_f(\cdot, \cdot)$ *is lower semicontinuous on* $\Lambda \times \Xi$, *there is no need to impose the continuity on the set-valued mapping* $H(\cdot, \cdot, \cdot)$. *Example 3.5 also illustrates that the weak* f*-property of* H *in Theorem 3.3 is essential. The advantage of the weak* f*-property does not include the information of solution to (PSVEP).*

Motivated by the works of [19, 30], we shall prove that, under some appropriate assumptions, the set of positive proper efficient solutions to (SVEP) is dense in relative to the solutions set of (SVEP). For this purpose, we define a set-valued mapping $\Omega : C^* \to 2^K$ as follows:

$$\Omega(f) = \begin{cases} S_f, & \text{if} \quad f \in C^* \setminus \{0\}, \\ K, & \text{if} \quad f = 0. \end{cases}$$

Lemma 3.7. *Let K be a nonempty compact subset of X and $\eta : X \times X \to X$ be continuous. Assume that the following conditions hold:*

(i) $H(\cdot, \eta(\cdot, \cdot))(K)$ is a compact subset of Y, where $H(\cdot, \eta(\cdot, \cdot))(K) = \{H(x, \eta(y, x)) : x, y \in K\}$;

(ii) for each $f \in C^* \setminus \{0\}, H(\cdot, \cdot)$ has the weak f-property on $X \times X$. Then, $\Omega(f)$ is lower semicontinuous on C^* with respect to the strong topology $B(Y^*, Y)$ in Y.

Proof. The proof is similar to the one of [30, Lemma 3.2] and so it is omitted here. □

Lemma 3.8. *Assume that for each $x \in K, H(x, \eta(K, x))$ is a convex set. Then*

$$S_w = \bigcup_{f \in C^* \setminus \{0\}} S_f.$$

Proof. The proof is similar to the one of [11, Lemma 3.1, p.313] and so it is omitted here. □

Theorem 3.9. *Assume that all assumptions of Lemma 3.1 are satisfied, and the following conditions hold:*

(i) $K(\cdot)$ is continuous on Λ;

(ii) for each $f \in C^* \setminus \{0\}, H(\cdot, \cdot, \cdot)$ has the weak f-property on $\Xi \times X \times X$;

(iii) for each $(\lambda, \mu) \in \Lambda \times \Xi$, $x \in K(\lambda), H(\mu, x, \eta(K(\lambda), x))$ is a convex set.

Then, for each $(\lambda, \mu) \in \Lambda \times \Xi, S_w(\lambda, \mu)$ is nonempty. Moreover, $S_w(\cdot, \cdot)$ is lower semicontinuous on $\Lambda \times \Xi$.

Proof. By Lemma 3.8, for each $(\lambda, \mu) \in \Lambda \times \Xi$, we have

$$S_w(\lambda, \mu) = \bigcup_{f \in C^* \setminus \{0\}} S_f(\lambda, \mu).$$

In view of Lemma 3.1, one has $S_w(\lambda, \mu) \neq \emptyset$. It follows from Theorem 3.3 that for each $f \in C^* \setminus \{0\}$, S_f is lower semicontinuous on $\Lambda \times \Xi$. Therefore, by Lemma 2.11, we derive that $S_w(\cdot, \cdot)$ is lower semicontinuous on $\Lambda \times \Xi$. □

Example 3.10. *Let $X, Z, W, Y, C, \Lambda, \Xi, K(\cdot), \eta(\cdot, \cdot)$ and $H(\cdot, \cdot, \cdot)$ be the same as Example 3.4. Then*

$$H(\mu, x, \eta(y, x)) = \begin{cases} [0, 2 + \mu] \times [1, 2], & \text{if } \mu \in [-1, 1] \setminus \{0\}, \\ (y - x + 1, 2), & \text{if } \mu = 0. \end{cases}$$

It is easy to verify that all conditions of Lemma 3.1 and conditions (i),(ii) and (iii) of Theorem 3.9 are satisfied. Simple computation allows that, for each $(\lambda, \mu) \in \Lambda \times \Xi$, $S_w(\lambda, \mu) = \bigcup_{f \in C^* \setminus \{0\}} S_f(\lambda, \mu) = [\lambda, 1]$. *Therefore,* $S_w(\cdot, \cdot)$ *is lower semicontinuous on* $\Lambda \times \Xi$.

Theorem 3.11. *Let* $C^\sharp \neq \emptyset$. *Assume that all assumptions of Corollary 3.2 are satisfied, and the following conditions hold:*

(i) $H(\cdot, \eta(\cdot, \cdot))(K)$ *is a compact subset of* Y, *where* $H(\cdot, \eta(\cdot, \cdot))(K) = \{H(x, \eta(y, x)) : x, y \in K\}$;

(ii) for each $f \in C^* \setminus \{0\}, H(\cdot, \cdot)$ *has the weak* f*-property on* $X \times X$;

(iii) for each $x \in K, H(x, \eta(K, x))$ *is a convex set.*

Then,

$$\bigcup_{f \in C^\sharp} S_f \subseteq S \subseteq cl(\bigcup_{f \in C^\sharp} S_f).$$

Proof. By Corollary 3.2, for each $f \in C^* \setminus \{0\}, S_f \neq \emptyset$. By definition, one has

$$\bigcup_{f \in C^\sharp} S_f \subseteq S \subseteq S_w.$$

This, together with Lemma 3.8, implies that

$$\bigcup_{f \in C^\sharp} S_f \subseteq S \subseteq \bigcup_{f \in C^* \setminus \{0\}} S_f.$$

Let us show that

$$\bigcup_{f \in C^* \setminus \{0\}} S_f \subseteq \mathrm{cl}(\bigcup_{f \in C^\sharp} S_f).$$

Let $x \in \bigcup_{f \in C^* \setminus \{0\}} S_f$. Then there exists $\bar{f} \in C^* \setminus \{0\}$ such that $x \in S_{\bar{f}}$. By the definition of $\Omega(\bar{f})$, we get $x \in S_{\bar{f}} = \Omega(\bar{f})$. In view of $C^\sharp \neq \emptyset$, let $g \in C^\sharp$ and set $f_m = \bar{f} + \frac{g}{m}$. Then $f_m \in C^\sharp$. By the similar method of proof in [19, Theorem 2.1], we show that f_m converges to $\bar{f}$ with respect to the topology $\beta(Y^*, Y)$.

For any neighborhood V of 0 with respect to $\beta(Y^*, Y)$, there exist bounded subsets $B_j \subseteq Y (j = 1, 2, \cdots, n)$ and $\epsilon > 0$ such that

$$\bigcap_{j=1}^{n} \{f \in Y^* : \sup_{y \in B_j} |f(y)| < \epsilon\} \subseteq V.$$

By the boundedness of B_j and $g \in Y^*, |g(B_j)|$ is bounded for $j = 1, 2, \cdots, n$. Therefore, there exists N such that

$$\sup_{y \in B_j} |\frac{g(y)}{m}| < \epsilon, \quad j = 1, 2, \cdots, n, m \geq N.$$

This implies that $\frac{g}{m} \in V$, i.e., $f_m - \bar{f} \in V$. So, f_m converges to $\bar{f}$ with respect to the topology $\beta(Y^*, Y)$. From Lemma 3.7, it follows that $\Omega(\cdot)$ is lower semicontinuous at $\bar{f}$. Then, for $f_m \to \bar{f}$ and any $x \in \Omega(\bar{f}) = S_{\bar{f}}$, there exists $x_m \in \Omega(f_m) = S_{f_m} \subseteq \bigcup_{f \in C^\sharp} S_f$ such that $x_m \to x$. It is easy to see that

$$x \in \mathrm{cl}(\bigcup_{f \in C^\sharp} S_f).$$

Again, from $x \in \bigcup_{f \in C^* \setminus \{0\}} S_f$, one has

$$\bigcup_{f \in C^* \setminus \{0\}} S_f \subseteq \mathrm{cl}(\bigcup_{f \in C^\sharp} S_f).$$

Therefore

$$\bigcup_{f \in C^\sharp} S_f \subseteq S \subseteq \mathrm{cl}(\bigcup_{f \in C^\sharp} S_f).$$

$\square$

Theorem 3.12. *Let $C^\sharp \neq \emptyset$. Assume that all assumptions of Lemma 3.1 are satisfied, and the following conditions hold:*

(i) $K(\cdot)$ is continuous on Λ;

(ii) for each $f \in C^* \setminus \{0\}, H(\cdot, \cdot, \cdot)$ has the weak f-property on $\Xi \times X \times X$;

(iii) for each $(\lambda, \mu) \in \Lambda \times \Xi$, $x \in K(\lambda), H(\mu, x, \eta(K(\lambda), x))$ is a convex set;

(iv) for each $(\lambda, \mu) \in \Lambda \times \Xi$, $H(\mu, \cdot, \eta(\cdot, \cdot))(K(\lambda))$ is a compact subset of Y, where

$$H(\mu, \cdot, \eta(\cdot, \cdot))(K(\lambda)) = \{H(\mu, x, \eta(y, x)) : x, y \in K(\lambda)\}.$$

Then, $S(\cdot, \cdot)$ is lower semicontinuous on $\Lambda \times \Xi$.

Proof. Let $(\bar{\lambda}, \bar{\mu}) \in \Lambda \times \Xi$ be any given. For each $f \in C^* \setminus \{0\}$, it follows from Theorem 3.3 that S_f is lower semicontinuous at $(\bar{\lambda}, \bar{\mu})$. In the light of Lemma 3.1 and Theorem 3.11, for each $(\lambda, \mu) \in \Lambda \times \Xi$, one has $S_f(\lambda,\mu)\neq\emptyset$ and

$$\bigcup_{f\in C^\sharp} S_f(\lambda,\mu)\subseteq S(\lambda,\mu)\subseteq \mathrm{cl}(\bigcup_{f\in C^\sharp} S_f(\lambda,\mu)). \tag{3.4}$$

For any $x\in S(\bar\lambda,\bar\mu)$ and any neighborhood $V(x)$ of x, by (3.4), $x\in \mathrm{cl}(\bigcup_{f\in C^\sharp} S_f(\bar\lambda,\bar\mu))$. Moreover, one has

$$V(x)\bigcap(\bigcup_{f\in C^\sharp} S_f(\bar\lambda,\bar\mu))\neq\emptyset.$$

Hence, there exists $f\in C^\sharp$ such that

$$V(x)\bigcap S_f(\bar\lambda,\bar\mu)\neq\emptyset.$$

By the lower semicontinuity of S_f at $(\bar\lambda,\bar\mu)$, there exists a neighborhood $V(\bar\lambda,\bar\mu)$ of $(\bar\lambda,\bar\mu)$ such that

$$S_f(\lambda,\mu)\bigcap V(x)\neq\emptyset,\quad \forall(\lambda,\mu)\in V(\bar\lambda,\bar\mu). \tag{3.5}$$

Again from (3.4), one has $S_f(\lambda,\mu)\subseteq S(\lambda,\mu)$. This, together with (3.5), shows that

$$S(\lambda,\mu)\bigcap V(x)\neq\emptyset,\quad \forall(\lambda,\mu)\in V(\bar\lambda,\bar\mu).$$

By Definition 2.2, $S(\cdot,\cdot)$ is lower semicontinuous at $(\bar\lambda,\bar\mu)$. By the randomicity of $(\bar\lambda,\bar\mu)$, we can obtain that $S(\cdot,\cdot)$ is lower semicontinuous on $\Lambda\times\Xi$. □

Example 3.13. *Let X,Z,W,Y,C,Λ,Ξ,$K(\cdot),\eta(\cdot,\cdot)$ and $H(\cdot,\cdot,\cdot)$ be the same as Example 3.4. It is easy to verify that all conditions of Lemma 3.1 and conditions (i)-(iv) of Theorem 3.12 are satisfied. Simple computation allows that, for each $(\lambda,\mu)\in\Lambda\times\Xi$, $S(\lambda,\mu)=[\lambda,1]$. Therefore, $S(\cdot,\cdot)$ is lower semicontinuous on $\Lambda\times\Xi$.*

Example 3.14. *Let us consider Example 3.5. Easily verify that all conditions of Lemma 3.1 and conditions (i),(iii) and (iv) of Theorem 3.12 are satisfied. From Example 3.5, the condition (ii) does not hold. Simple computation allows that, for $(\lambda,\mu)\in\Lambda\times\{0\}$, $S(\lambda,\mu)=[\lambda,-\lambda+2]$ and for each $(\lambda,\mu)\in\Lambda\times(\Xi\setminus\{0\})$, $S(\lambda,\mu)=\emptyset$. Therefore, $S(\cdot,\cdot)$ is not lower semicontinuous at $(\lambda,\mu)\in\Lambda\times\{0\}$. That is, the weak f-property of H is necessary to the lower semicontinuity for the solution mapping $S(\cdot,\cdot)$ in Theorem 3.12.*

Remark 3.15. *Lemma 3.7 and Theorems 3.3, 3.11 and 3.12 improve and generalize Lemma 3.2, Lemma 3.1, Lemma 3.3 and Theorem 3.1 of Xu and Li [30], respectively. Particularly, we may assume that* Λ *is the same as* Ξ, $\lambda = \mu, \eta(y, x) = y$ *for all* $x, y \in X$. *Then, if we replace the weak* f*-property by the* f*-property, one can easily obtain the results presented by Xu and Li [30].*

Acknowledgments

The author wish to thank the anonymous referees and the associate editor for their very careful and valuable comments which led to an improved presentation of this manuscript. The work was partially supported by the Natural Science Foundation of China (11401487, 11401058), the Fundamental Research Fund for the Central Universities (SWU113037, XDJK2014C073) and the Technology Project of Chongqing Education Committee (KJ130732).

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