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FIRST ORDER COHOMOLOGY OF ℓ^1-MUNN ALGEBRAS AND CERTAIN SEMIGROUP ALGEBRAS

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Abstract. We characterize cyclic and weakly amenable ℓ^1-Munn algebras. In the special case of Rees matrix semigroups, we obtain a new proof of the following result due to Blackmore: The semigroup algebra of every Rees matrix semigroup is weakly amenable. Characterizations of Connes-amenable ℓ^1-Munn algebras with square sandwich matrix and semigroup algebras of Rees matrix semigroups are also provided.

1. **Introduction**

Cohomology of Banach algebras has received extensive study since the major exposition of Hochschild cohomology theory for Banach algebras by B. E. Johnson[15], with emphasis on the first order cohomology groups. Let A be a Banach algebra. Vanishing of the first cohomology groups $H^1(A, X)$ for certain classes of Banach A-modules X has been given different names, depending on the class of Banach A-modules under investigation among which are cyclic, weak and Connes-amenability. The reader may see [1,2,4,14,15,16,18] for more information. Here, we consider these notions for ℓ^1-Munn algebras and semigroup algebras of Rees matrix semigroups.

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Blackmore showed that the semigroup algebra of every Rees matrix semigroup is weakly amenable [3]. Bowling and Duncan [4] considered weak and cyclic amenability of the convolution algebras of Rees matrix semigroups and gave another proof of Blackmore's result. As shown in [9], these convolution algebras are certain types of ℓ^1- Munn algebras, whose introduction in [9], was motivated by two open problems in [8]. Some of the properties and the structure of these algebras have been studied in [9-11]. In Sections 2 and 3 we study weak and cyclic amenable ℓ^1-Munn algebras and semigroup algebras of Rees matrix semigroups. Our results involve Bowling and Duncan's result. Connes-amenability has been proved an appropriate version of amenability for dual Banach algebras. For more information on this subject, see [18]. In the last section we consider Connes amenability of ℓ^1-Munn algebras and semigroup algebras of Rees matrix semigroups.

Before proceeding further we set up our notations.

Let G be a group , I and J be arbitrary nonempty sets, and $G^0 = G\bigcup\{0\}$. An $I \times J$ matrix A over G^0 that has at most one nonzero entry $a = A(i,j)$ is called a Rees $I \times J$ matrix over G^0 and is denoted by $(a)_{ij}$. Let P be a $J \times I$ matrix over G. The set $S = G \times I \times J$ with the composition $(a,i,j) \circ (b,l,k) = (aP_{jl}b, i, k), \quad (a,i,j), (b,l,k) \in S$, is a semigroup that we denote by $M(G,p)$. Similarly, if P is a $J \times I$ matrix over G^0, then $S = G \times I \times J \cup \{0\}$ is a semigroup under the following composition operation which is denoted by $M^0(G,P)$:

$$(a,i,j) \circ (b,l,k) = \begin{cases} (aP_{jl}b, i, k), & \text{if } p_{jl} \neq 0 \\ 0 & \text{if } p_{jl} = 0, \end{cases}$$

$$(a,i,j) \circ 0 = 0 \circ (a,i,j) = 0 \circ 0 = 0.$$

$M^0(G,P)$ is isomorphic to the semigroup of all Rees $I \times J$ matrices over G^0 with binary operation $A \circ B = APB$. $M^0(G,P)$ [resp. $M(G,P)$] is called the Rees $I \times J$ matrix semigroup over G^0 [resp. G] with sandwich matrix P.

Throughout, A is a Banach algebra and A module means Banach A-bimodule. An A-bimodule X is called dual if there is a closed submodule X_* of X^* such that $X = (X_*)^*$. A is called dual if it is dual as a Banach A-bimodule. Let A be a dual Banach algebra and X be a dual A-bimodule. If for each $x \in X$ the maps $a \longmapsto a.x$ and $a \longmapsto x.a$ from A into X are $\omega^* - \omega^*$ continuous, then X is called a normal A-bimodule.

Let X be an A module. We will denote the set of all bounded [resp. inner] derivations from A into X by $Z^1(A,X)$ [resp. $B^1(A,X)$]. Also, set $H^1(A,X)=Z^1(A,X)/B^1(A,X)$. A bounded derivation $D:A\longrightarrow A^*$ is called cyclic if $\langle Da,b\rangle+\langle Db,a\rangle=0$, for all $a,b\in A$. The set of all cyclic derivations is denoted by $Z^1_\lambda(A,A^*)$ and $Z^1_\lambda(A,A^*)/B^1(A,A^*)$ by $H^1_\lambda(A,A^*)$. A is called weakly amenable [resp. cyclic amenable] if $H^1(A,A^*)=0$ [resp. $H^1_\lambda(A,A^*)$]. A dual Banach algebra A is called Connes-amenable if for every normal dual Banach A-bimodule X, every bounded $\omega^*-\omega^*$ continuous derivation from A into X is inner.

Suppose A is unital, I and J are arbitrary index sets and P is a $J\times I$ matrix over A such that all of its nonzero entries are invertible and $\| P\|_\infty\leq 1$. The space $\ell^1(I\times J,A)$ with product $X\circ Y=XPY$ is a Banach algebra which we call the ℓ^1-Munn algebra over A with sandwich matrix P and we denote it with $LM(A,P)$. As shown in [9], Proposition 5.6, semigroup algebras of Rees matrix semigroups are concrete examples of ℓ^1-Munn algebras.

2. Weak and cyclic amenability of ℓ^1-Munn algebras

Throughout this section, we assume A is unital.

Theorem 2.1. *If A is cyclic amenable , then so is $LM(A,P)$.*

Proof. Suppose $\alpha\in J$, $\beta\in I$ are such that $P_{\alpha\beta}\neq 0$ and $q=P^{-1}_{\alpha\beta}$. Let $D:LM(A,P)\longrightarrow LM(A,P)^*$ be a bounded cyclic derivation and define $\widehat{D}$ by:

$$\widehat{D}:A\longrightarrow A^*,\quad \langle\widehat{D}a,b\rangle=\langle D(qa\varepsilon_{\beta\alpha}),qb\varepsilon_{\beta\alpha}\rangle,\quad a,b\in A.$$

Clearly $\widehat{D}$ is a bounded linear map. Let $a,b,c\in A$. Then,

$$\begin{aligned}\langle\widehat{D}(ab),c\rangle&=\langle D(qa\varepsilon_{\beta\alpha}\circ qb\varepsilon_{\beta\alpha}),qc\varepsilon_{\beta\alpha}\rangle\\&=\langle D(qa\varepsilon_{\beta\alpha}),qb\varepsilon_{\beta\alpha}\circ qc\varepsilon_{\beta\alpha}\rangle+\langle D(qb\varepsilon_{\beta\alpha}),qc\varepsilon_{\beta\alpha}\circ qa\varepsilon_{\beta\alpha}\rangle\\&=\langle D(qa\varepsilon_{\beta\alpha}),qbc\varepsilon_{\beta\alpha}\rangle+\langle D(qb\varepsilon_{\beta\alpha}),qca\varepsilon_{\beta\alpha}\rangle\\&=\langle\widehat{D}a,bc\rangle+\langle\widehat{D}b,ca\rangle\\&=\langle\widehat{D}a.b+a.\widehat{D}b,c\rangle.\end{aligned}$$

On the other hand, by assumption we have,

$$\langle\widehat{D}a,b\rangle+\langle\widehat{D}b,a\rangle=\langle D(qa\varepsilon_{\beta\alpha}),qb\varepsilon_{\beta\alpha}\rangle+\langle D(qb_{\beta\alpha}),qa\varepsilon_{\beta\alpha}\rangle=0.$$

So, $\widehat{D}$ is a bounded cyclic derivation. Let $\widehat{\psi} \in A^*$ be such that $\widehat{D}(a) = a.\widehat{\psi} - \widehat{\psi}.a, (a \in A)$. Define,

$$\psi(a\varepsilon_{ij}) = \widehat{\psi}(p_{ji}a) + \langle D(q\varepsilon_{\beta j}), a\varepsilon_{i\alpha}\rangle, \quad i \in I, j \in J, a \in A.$$

It is easy to see that $\psi \in LM(A,P)^*$. Let $S = a\varepsilon_{ij}$ $T = b\varepsilon_{kl}$ be nonzero elements in $LM(A,P)$, $U = q\varepsilon_{\beta j}$, $V = q\varepsilon_{\beta l}$, $X = a\varepsilon_{i\alpha}$ and $Y = qp_{jk}b\varepsilon_{\beta\alpha}$. Then, $S = X \circ U$ and $U \circ T = Y \circ V$. So,

$$\langle D(X), U \circ T\rangle = -\langle D(Y \circ V), X\rangle$$

$$= -\langle D(Y), V \circ X\rangle - \langle D(V), X \circ Y\rangle$$

$$= \langle D(V \circ X), Y\rangle - \langle D(V), X \circ Y\rangle. \tag{2.1}$$

But,

$$\langle D(V \circ X), Y\rangle = \langle D(qp_{li}a\varepsilon_{\beta\alpha}), qp_{jk}b\varepsilon_{\beta\alpha}\rangle = \langle \widehat{D}(p_{li}a), p_{jk}b\rangle$$

$$= \langle (p_{li}a).\widehat{\psi} - \widehat{\psi}.(p_{li}a), p_{jk}b\rangle$$

$$= \langle \widehat{\psi}, p_{jk}bp_{li}a\rangle - \langle \widehat{\psi}, p_{li}ap_{jk}b\rangle. \tag{2.2}$$

So by (2.1) and (2.2), we have,

$$\begin{aligned}
\langle DS, T\rangle &= \langle DU, T \circ X\rangle + \langle DX, U \circ T\rangle \\
&= \langle DU, T \circ X\rangle + \langle D(V \circ X), Y\rangle - \langle DV, X \circ Y\rangle \\
&= \langle D(q\varepsilon_{\beta j}), bp_{li}a\varepsilon_{k\alpha}\rangle + \langle \widehat{\psi}, p_{jk}bp_{li}a\rangle \\
&\quad -\langle \widehat{\psi}, p_{li}ap_{jk}b\rangle - \langle D(q\varepsilon_{\beta l}), ap_{jk}b\varepsilon_{i\alpha}\rangle \\
&= \langle \psi, bp_{li}a\varepsilon_{kj}\rangle - \langle \psi, ap_{jk}b\varepsilon_{il}\rangle = \langle \delta_{-\psi}(S), T\rangle.
\end{aligned}$$

Therefore, D is inner. □

Lemma 2.2. *If A is weakly amenable, then every bounded derivation $D : LM(A,P) \longrightarrow LM(A,P)^*$ is cyclic.*

Proof. It is enough to show the cyclic identity for Rees matrices. Let $S = a\varepsilon_{ij}$ and $T = b\varepsilon_{kl}$, where a,b$\neq 0$.

Step I: $p_{jk} = 0$. Then, $0 = D0 = D(S \circ T) = DS.T + S.DT$. So, for every $X \in LM(M,P)$,

$$0 = \langle DS.T, X\rangle + \langle S.DT, X\rangle$$

$$= \langle DS, T \circ X\rangle + \langle DT, X \circ S\rangle. \tag{2.3}$$

If $p_{li} \neq 0$, then take $X = E = p_{li}^{-1}\varepsilon_{il}$. Clearly, E is an idempotent, $T \circ E = T$, $E \circ S = S$ and hence if we substitute X with E in (2.3), then we get,

$$\langle DS, T\rangle + \langle DT, S\rangle = 0.$$

So, the cyclic condition holds. If $p_{li} = 0$, then choose $\alpha \in J$ and $\beta \in I$ such that $p_{\alpha\beta} \neq 0$. Let $Y = a\varepsilon_{i\alpha}$ and $Z = p_{\alpha\beta}^{-1}\varepsilon_{\beta j}$. Then, $Y \circ Z = S$ and $Z \circ T = T \circ Y = 0$. So,

$$\begin{aligned}\langle DS, T\rangle &= \langle D(Y \circ Z), T\rangle = \langle DY.Z, T\rangle + \langle Y.DZ, T\rangle \\ &= \langle DY, Z \circ T\rangle + \langle DZ, T \circ Y\rangle = 0.\end{aligned}$$

Similarly, $\langle DT, S\rangle = 0$ and hence the cyclic condition holds.

Step II: $p_{jk} \neq 0$. By the symmetry of the cyclic condition, we may assume $p_{li} \neq 0$ as well. As above, let $E = p_{li}^{-1}\varepsilon_{il}$. Then, we have,

$$\begin{aligned}\langle D(S \circ T), E\rangle &= \langle DS.T, E\rangle + \langle S.DT, E\rangle \\ &= \langle DS, T \circ E\rangle + \langle DT, E \circ S\rangle \\ &= \langle DS, T\rangle + \langle DT, S\rangle. \end{aligned} \tag{2.4}$$

Now, define the map $\phi : A \longrightarrow LM(A, P)$ by $\phi(c) = p_{li}^{-1}c\varepsilon_{il}$. Clearly, ϕ is an injective bounded linear map. Let $c, d \in A$. Then,

$$\phi(cd) = p_{li}^{-1}cd\varepsilon_{il} = (p_{li}^{-1}c\varepsilon_{il}) \circ (p_{li}^{-1}d\varepsilon_{il}) = \phi(c) \circ \phi(d).$$

Therefore, ϕ is a bounded algebra monomorphism. Now, define the map,

$$\widehat{D} : A \longrightarrow A^*, \qquad \langle \widehat{D}a, b\rangle = \langle (D\phi)(a), p_{li}^{-1}b\varepsilon_{il}\rangle \qquad a, b \in A.$$

Then, $\widehat{D}$ is a bounded linear map. If $a, b, c \in A$, then,

$$\begin{aligned}\langle \widehat{D}(ab), c\rangle &= \langle D(\phi(ab)), p_{li}^{-1}c\varepsilon_{il}\rangle \\ &= \langle (D\phi(a)).\phi(b), p_{li}^{-1}c\varepsilon_{il}\rangle + \langle \phi(a).(D\phi(b)), p_{li}^{-1}c\varepsilon_{il}\rangle \\ &= \langle D(\phi(a)), \phi(b) \circ p_{li}^{-1}c\varepsilon_{il}\rangle + \langle D(\phi(b)), p_{li}^{-1}c\varepsilon_{il} \circ \phi(a)\rangle \\ &= \langle D(\phi(a)), p_{li}^{-1}b\varepsilon_{il} \circ p_{li}^{-1}c\varepsilon_{il}\rangle + \langle D(\phi(b)), p_{li}^{-1}c\varepsilon_{il} \circ p_{li}^{-1}a\varepsilon_{il}\rangle \\ &= \langle D(\phi(a)), p_{li}^{-1}bc\varepsilon_{il}\rangle + \langle D(\phi(b)), p_{li}^{-1}ca\varepsilon_{il}\rangle \\ &= \langle \widehat{D}a, bc\rangle + \langle \widehat{D}b, ca\rangle \\ &= \langle \widehat{D}a.b + a.\widehat{D}b, c\rangle.\end{aligned}$$

Therefore, $\widehat{D}$ is a bonded derivation and, by assumption, it is inner. So, for every $a \in A$, $\langle \widehat{D}a, 1\rangle = 0$, and hence,

$$\langle D(S \circ T), E\rangle = \langle D(ap_{jk}b\varepsilon_{il}), p_{li}^{-1}\varepsilon_{il}\rangle$$

$$= \langle D\phi(p_{li}ap_{jk}b), p_{li}^{-1}\varepsilon_{il}\rangle$$

$$= \langle \widehat{D}(p_{li}ap_{jk}b), 1\rangle = 0. \tag{2.5}$$

Now, by (2.4) and (2.5),

$$\langle DS, T\rangle + \langle DT, S\rangle = \langle D(S \circ T), E\rangle = 0.$$

Therefore, in either case, the cyclic condition holds. □

The following theorem is an immediate consequence of Theorem 2.1 and Lemma 2.2.

Theorem 2.3. *If A is weakly amenable, then so is $LM(A, P)$.*

Remark 2.4. In the proof of the following theorem we use Lemma 3.7 in [9] which is true only for the case that the sandwich matrix P is square; i.e., the index sets I and J are equal. If P is a regular square matrix and $LM(A, P)$ has a bounded approximate identity, then the converse of theorems 2.1 and 2.3 are also true.

Theorem 2.5. *Suppose P is a regular square matrix and $LM(A, P)$ has a bounded approximate identity. Then, $LM(A, P)$ is cyclic [resp. weakly] amenable if and only if A is cyclic [resp. weakly] amenable.*

Proof. we need only to prove the converse. By Lemma 3.7 in [9], the index set I is finite and $LM(A, P)$ is topologically isomorphic to $A\widehat{\otimes}M_n$, for some $n \in N$. If $D : A \longrightarrow A^*$ is a bounded derivation, then $D \otimes 1$ is a bounded derivation from $LM(A, P)$ to $LM(A, P)^*$. Moreover, if D is cyclic, then so is $D \otimes 1$, since the action of M_n on itself as its dual, is componentwise. Now, suppose $LM(A, P)$ is weakly amenable and $D : A \longrightarrow A^*$ is a bounded derivation. There is a $\phi = \sum_{i=1}^{n} f^i \otimes B^i \in A^* \otimes M_n = LM(A, P)^*$ such that $D \otimes 1 = \delta_\phi$. It is easy to see that $D = \delta_f$, where $f = \sum_{i=1}^{m} B_{11}^i f^i$. The argument for cyclic derivations is the same. □

Now, we apply the preceding results to the semigroup algebras of Rees matrix semigroups. The following theorem is (Corollary 5.3 in [3]) comes with a different proof. Another proof can also be found (see Theorem 2.4 in [4]).

Theorem 2.6. *If S is a Rees matrix semigroup, then $\ell^1(S)$ is weakly amenable.*

Proof. Suppose $S = M^\circ(G,P)$. Then, by Proposition 5.6 in [9], $\ell^1(S)/\ell^1(0)$ is isomorphic to $LM(\ell^1(G),P)$. By Johnson' Theorem, $\ell^1(G)$ is weakly amenable. So, by Theorem 2.3, $LM(\ell^1(G),P)$ is weakly amenable, and hence so is $\ell^1(S)$. □

3. Connes-amenability of ℓ^1-Munn algebras and Rees matrix semigroup algebras

Throughout this section, we assume A ia a dual Banach algebra, $I = J$, P and $LM(A,P)$ are as in Section 1. It is well known that $c_\circ(I \times J, A_*)^* = \ell^1(I \times J, A) = LM(A,P)$. Moreover, $c_\circ(I \times J, A_*)$ is an $LM(A,P)$ submodule of $LM(A,P)^* = \ell^\infty(I \times J, A^*)$. Therefore, $LM(A,P)$ is a dual Banach algebra. In the proof of the following theorem, we use Lemma 3.7 in [9], which is true only for the case that the index sets I and J are equal.

Theorem 3.1. *Suppose A is a unital dual Banach algebra and the index sets I and J are equal. Then, $LM(A,P)$ is Connes-amenable if and only if it has a bounded approximate identity and A is Connes-amenable.*

Proof. Suppose $LM(A,P)$ has a bounded approximate identity and A is Connes-amenable. By Lemma 3.7 in [9], $LM(A,P) \simeq A\widehat{\otimes}M_n$. Since $A\widehat{\otimes}M_n = (A_*\check{\otimes}M_n)^*$ and both of A and M_n are Connes-amenable, then by using the argument of Theorem 5.4 in [15] with appropriate modifications, we can see that $LM(A,P)$ is Connes-amenable.

Conversely, suppose $LM(A,P)$ is Connes-amenable and X is a normal A-bimodule. Then, $LM(A,P)$ is unital and hence by Lemma 3.7 in [9], $LM(A,P) \simeq A\widehat{\otimes}M_n$. Since $X\widehat{\otimes}M_n = (X_*\check{\otimes}M_n)^*$, then $X\widehat{\otimes}M_n$ is a dual $A\widehat{\otimes}M_n-$ bimodule. Moreover, by the above remark, elements of $A\widehat{\otimes}M_n$ and $X\widehat{\otimes}M_n$ have finite representations in terms of elementary tensors and action of $X\widehat{\otimes}M_n$ on $X_*\check{\otimes}M_n$ is componentwise. Thus, normality of $X\widehat{\otimes}M_n$ follows from normality of X. Now, suppose $D : A \longrightarrow X$ is a bounded $\omega^* - \omega^*$ continuous derivation. Then, $D \otimes 1 : A\widehat{\otimes}M_n \longrightarrow X\widehat{\otimes}M_n$ is a bounded derivation, and $\omega^* - \omega^*$ continuity of $D \otimes 1$ follows from the fact that elements of $A\widehat{\otimes}M_n$ and $X\widehat{\otimes}M_n$ are finite sums of elementary tensors and action of $X\widehat{\otimes}M_n$ on $X_*\check{\otimes}M_n$ is componentwise. Therefore, by assumption, $D \otimes 1 = \delta_\phi$, for some $\phi \in X\widehat{\otimes}M_n$. By the above remark, ϕ has a unique representation of the form $\phi = \sum_{i,j=1}^n x_{ij}\otimes$

ε_{ij}. Let $a \in A$ and $r \leq n$ be a natural number. Then,

$$\begin{aligned} Da \otimes \varepsilon_{rr} &= (D \otimes 1)(a \otimes \varepsilon_{rr}) = (a \otimes \varepsilon_{rr}).\phi - \phi.(a \otimes \varepsilon_{rr}) \\ &= \sum_{j=1}^{n} a x_{rj} \otimes \varepsilon_{rj} - \sum_{i=1}^{n} x_{ir} a \otimes \varepsilon_{ir}. \end{aligned}$$

Letting $a = 1$, we conclude that for all $i, j \neq r$, $x_{rj} = 0 = x_{ir}$. Thus, $\phi = \sum_{i=1}^{n} x_{ii} \otimes \varepsilon_{ii}$. Now, the identities,

$$Da \otimes \varepsilon_{11} = (a \otimes \varepsilon_{11}).\phi - \phi.(a \otimes \varepsilon_{11}) = (a x_{11} - x_{11} a) \otimes \varepsilon_{11}$$

imply that $D = \delta_{x_{11}}$. □

Theorem 3.2. *Suppose P is a square matrix over G° and $S = M^\circ(G, P)$. Then, the following conditions are equivalent.*

i) *G is amenable and $\ell^1(S)$ has a bounded approximate identity.*

ii) *$\ell^1(S)$ is Connes-amenable.*

Proof. Proposition 5.6 and Lemma 5.1(ii) of [9] imply that existence of a bounded approximate identity in $\ell^1(S)$ is equivalent to the existence of a bounded approximate identity in $LM(\ell^1(G), P)$. Also, Theorem 4.4.13 in [18], $\ell^1(G)$ is Connes-amenable if and only if G is amenable. On the other hand, by Lemma 5.1(ii) and Proposition 5.6 in [9], Connes-amenability of $\ell^1(S)$ is equivalent to Connes-amenability of $LM(\ell^1(G), P)$. Therefore, equivalence of (i) and (ii) follows from Theorem 3.1 . □

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