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LINEAR WEINGARTEN HYPERSURFACES IN A UNIT SPHERE

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Abstract. In this paper, by modifying Cheng-Yau′s technique to complete hypersurfaces in $S^{n+1}(1)$, we prove a rigidity theorem under the hypothesis of the mean curvature and the normalized scalar curvature being linearly related which improve the result of [H. Li, Hypersurfaces with constant scalar curvature in space forms, *Math. Ann.* 305 (1996), 665–672].

Keywords: Linear Weingarten hypersurface, generalized maximum principle, rigidity theorem.
MSC(2010): Primary: 53C20; Secondary: 53C42.

1. Introduction

Let M be an n-dimensional hypersurfaces in the unit sphere $S^{n+1}(1)$. The investigation of curvature structures of compact hypersurfaces of $S^{n+1}(1)$ is important and interesting, with so much attention on it. Cheng and Yau [5] studied compact hypersurfaces with constant scalar curvature in the unit sphere $S^{n+1}(1)$. They proved that if M is an n-dimensional hypersurface with constant scalar curvature $n(n-1)r$, $r \geq 1$ and the sectional curvatures of M are nonnegative, then M is isometric to a totally umbilical hypersurface $S^n(c)$ or to the Riemannian product $S^k(c) \times S^{n-k}(\sqrt{1-c^2})$, $1 \leq k \leq n-1$, where $S^k(c)$ denote the sphere of radius c. Furthermore, by using of the same method which was used in [4] and the differential operator in [5], Li [6] prove that if M is an n-dimensional compact hypersurface with constant scalar curvature $n(n-1)r$, $r \geq 1$ and $S \leq (n-1)\frac{n(r-1)+2}{n-2} + \frac{n-2}{n(r-1)+2}$, then M is isometric to either totally umbilical hypersurface or the Riemannian product $S^1(\sqrt{1-c^2}) \times S^{n-1}(c)$ with $c^2 = \frac{n-2}{nr} \leq \frac{n-2}{n}$, where S is the squared norm of the second fundamental form of M. In the proof of these results, the fact that the differential operator $\Box$ defined by $\Box f = \sum_{ij}^{n}(nH\delta_{ij} - h_{ij})f_{ij}$ is self-adjoint and degenerate elliptic

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is indispensable. So the condition of $r \geq 1$ and the assumption of constant scalar curvature is essential. Q.M.Cheng [3] generalized the results to the case $r \geq \frac{n-2}{n-1}$ under a topological condition. More precisely, he proved that if M is an n-dimensional compact hypersurface with infinite fundamental group in $S^{n+1}(1)$, If $r \geq \frac{n-2}{n-1}$ and $S \leq (n-1)\frac{n(r-1)+2}{n-2} + \frac{n-2}{n(r-1)+2}$, then M is isometric to the Riemannian product $S^1(\sqrt{1-c^2}) \times S^{n-1}(c)$, where $n(n-1)r$ is the scalar curvature of M and $c^2 = \frac{n-2}{nr}$.

On the other hand, Li [7] studied some hypersurfaces in the unit sphere with scalar curvature proportional to the mean curvature and proved the following theorem.

Theorem 1.1 ([7]). *Let M be an n-dimensional compact hypersurface in the unit sphere $S^{n+1}(1)$. If*

(1) M has nonnegative sectional curvature,

(2) the normalized scalar curvature r and the mean curvature H of M satisfy the following conditions:

$$r = aH, a^2 \geq \frac{4n}{n-1},$$

where a is a constant, then M is either totally umbilical, or $M = S^{n-k}(c) \times S^k(\sqrt{1-c^2}), 1 \leq k \leq n-1$.

Recently, Li, Suh and Wei [9] considered linear Weingarten hypersurfaces in an sphere and obtained the following rigidity theorems:

Theorem 1.2 ([9]). *Let M be an n-dimensional compact hypersurface in the unit sphere $S^{n+1}(1)$. If*

(1) M has nonnegative sectional curvature,

(2) the normalized scalar curvature r and the mean curvature H of M satisfy the following conditions:

$$r = aH + b, (n-1)a^2 - 4n + 4nb \geq 0,$$

then M is either totally umbilical, or $M = S^{n-k}(c) \times S^k(\sqrt{1-c^2}), 1 \leq k \leq n-1$.

In these theorems, M is compact and M has nonnegative sectional curvature. Without the assumption of nonnegative sectional curvature, Li, Suh and Wei [9] obtained the following result, as well.

Theorem 1.3 ([9]). *Let M be an n-dimensional compact hypersurface in the unit sphere $S^{n+1}(1)$. If*

(1) $r = aH + b, (n-1)a^2 - 4n + 4nb \geq 0$,

(2) $|B|^2 \leq 2\sqrt{n-1}$,

then either $|B|^2 = 0$ and M is a totally umbilical hypersurface or $|B|^2 = 2\sqrt{n-1}$ and $M = S^1(c) \times S^{n-1}(\sqrt{1-c^2})$.

In this paper, we consider complete linear Weingarten hypersurface in $S^{n+1}(1)$ with no sectional curvature restriction. By modifying Cheng-Yau's technique for complete hypersurfaces in $S^{n+1}(1)$, we will prove a rigidity theorem under the hypothesis of the mean curvature and the normalized scalar curvature being linearly related. More precisely, we prove the following theorem.

Theorem 1.4. *Let M^n be a complete hypersurface of $S^{n+1}(1)$ with bounded mean curvature. If $r = aH+b, a \leq 0, b \geq 1$, then M^n is either totally umbilical, or $M = S^1(c) \times S^{n-1}(\sqrt{1-c^2})$.*

If we choose $a = 0$ and $b \geq 1$ in Theorem 1.4, we get

Corollary 1.5. *Let M^n be a complete hypersurface of $S^{n+1}(1)$ with constant normalized scalar curvature r satisfying $r \geq 1$. If M^n has bounded mean curvature, then M^n is either totally umbilical, or $M = S^1(c) \times S^{n-1}(\sqrt{1-c^2})$.*

Remark 1.6. *Corollary 1.5 is a real improvement of the result in [6] because it has no compactness restriction on M and no restriction on $|B|^2$.*

2. Preliminaries

Let M be an n-dimensional complete hypersurface in the $(n+1)$-dimensiomal unit sphere $S^{n+1}(1)$. For any $p \in M$, we choose a local orthonormal frame $e_1, \cdots, e_n, e_{n+1}$ on $S^{n+1}(1)$ around p such that $e_1, \cdots, e_n$ are tangent to M. Take the corresponding dual coframe $\omega_1, \cdots, \omega_n, \omega_{n+1}$, We shall make use of the following standard convention on the range of indices:

$$1 \leq A, B, C, D \cdots \leq n+1, \quad 1 \leq i, j, k, l \cdots \leq n.$$

Then the structure equations of $S^{n+1}(1)$ are given by

$$d\omega_A = \sum_B \omega_{AB} \wedge \omega_B, \quad \omega_{AB} + \omega_{BA} = 0, \tag{2.1}$$

$$d\omega_{AB} = \sum_C \omega_{AC} \wedge \omega_{CB} - \omega_A \wedge \omega_B. \tag{2.2}$$

Restricting those forms to M, we have $\omega_{n+1} = 0$ and

$$0 = d\omega_{n+1} = \sum_i^n \omega_{n+1i} \wedge \omega_i. \tag{2.3}$$

By Cartan's lemma, there exist functions h_{ij} such that

$$\omega_{n+1i} = \sum_j^n h_{ij}\omega_i, \quad h_{ij} = h_{ji}. \tag{2.4}$$

The structure equations of M are given by

$$d\omega_i = \sum_{j=1}^{n} \omega_{ij} \wedge \omega_j, \quad \omega_{ij} + \omega_{ji} = 0, \tag{2.5}$$

$$d\omega_{ij} = \sum_{k=1}^{n} \omega_{ik} \wedge \omega_{kj} - \frac{1}{2} \sum_{k,l=1}^{n} R_{ijkl} \omega_k \wedge \omega_l. \tag{2.6}$$

The Gauss equations are

$$R_{ijkl} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk} + h_{ik}h_{jl} - h_{il}h_{jk}, \tag{2.7}$$

$$n(n-1)r = n(n-1) + n^2H^2 - |B|^2, \tag{2.8}$$

where R_{ijkl} denotes the components of the Riemannian curvature tensor of M, r is the normalized scalar curvature of M and $|B|^2 = \sum_{i,j} h_{ij}^2$ is the norm square of the second fundamental form of M.

By taking the exterior differentiation of (2.4), we obtain the Codazzi equations

$$h_{ijk} = h_{ikj} = h_{jik}, \tag{2.9}$$

where the covariant derivative of h_{ij} is defined by

$$\sum_k h_{ijk}\omega_k = dh_{ij} + \sum_k h_{kj}\omega_{ki} + \sum_k h_{ik}\omega_{kj}. \tag{2.10}$$

Similarly, the components h_{ijkl} of the second derivative $\nabla^2 h$ are given by

$$\sum_l h_{ijkl}\omega_l = dh_{ijk} + \sum_l h_{ljk}\omega_{li} + \sum_l h_{ilk}\omega_{lj} + \sum_l h_{ijl}\omega_{lk}. \tag{2.11}$$

By exterior differentiation of (2.10), we can get the following *Ricci identities*

$$h_{ijkl} - h_{ijlk} = \sum_m h_{im}R_{mjkl} + \sum_m h_{jm}R_{mikl}. \tag{2.12}$$

The Laplacian $\triangle h_{ij}$ of h_{ij} is defined by $\triangle h_{ij} = \sum_k h_{ijkk}$, from the Codazzi equation (2.9) and Ricci identities (2.12), we have

$$\triangle h_{ij} = \sum_k h_{kkij} + \sum_{m,k} h_{km}R_{mijk} + \sum_{m,k} h_{im}R_{mkjk}. \tag{2.13}$$

Set $\phi_{ij} = h_{ij} - H\delta_{ij}$, it is easy to check that ϕ is traceless and $|\phi|^2 = \sum_{i,j} \phi_{ij}^2 = |B|^2 - nH^2$. Following Cheng-Yau [5], as in [1] and [2], for any $a \geq 0$, we introduce a modified operator $\Box$ acting on any C^2- function f by

$$\Box(f) = \sum_{i,j} \big((nH - \frac{n-1}{2}a)\delta_{ij} - h_{ij}\big) f_{ij}, \tag{2.14}$$

where f_{ij} is given by the following

$$\sum_j f_{ij}\omega_j = df_i + f_j\omega_{ij}.$$

Lemma 2.1 ([9])**.** *Let M^n be a hypersurface of $S^{n+1}(1)$ with $r = aH+b, a, b \in \mathbb{R}$ and $(n-1)a^2 - 4n + 4nb \geq 0$. Then we have*

$$|\nabla B|^2 \geq n^2|\nabla H|^2. \tag{2.15}$$

Lemma 2.2. *Let M^n be a complete hypersurface of $S^{n+1}(1)$ with $r = aH + b, a, b \in \mathbb{R}$. Then we have*

$$\begin{aligned}\Box(nH) \geq{}& |\nabla B|^2 - n^2|\nabla H|^2 \\ &+ |\phi|^2\Big(- |\phi|^2 - \frac{n(n-2)}{\sqrt{n(n-1)}}|H||\phi| + n(1+H^2)\Big).\end{aligned} \tag{2.16}$$

Proof. First, from (2.12), we have

$$\begin{aligned}\frac{1}{2}\triangle|B|^2 &= \frac{1}{2}\triangle\sum_{i,j} h_{ij}^2 = \sum_{i,j} h_{ij}\triangle h_{ij} + \sum_{i,j,k} h_{ijk}^2 \\ &= \sum_{i,j,k} h_{ijk}^2 + n\sum_{i,j} h_{ij}H_{ij} + n(|B|^2 - nH^2) + nHf_3 - |B|^4,\end{aligned}$$

where $f_3 = \sum_{i,j,k} h_{ij}h_{jk}h_{ki}$. On the other hand, from Gauss equation and $r = aH + b$, we have

$$\begin{aligned}\triangle|B|^2 &= \triangle(n^2H^2 - n(n-1)(r-1)) \\ &= \triangle(n^2H^2 - n(n-1)(aH+b-1)) \\ &= \triangle(n^2H^2 - (n-1)anH) = \triangle(nH - \frac{1}{2}(n-1)a)^2.\end{aligned} \tag{2.17}$$

Then from (2.17) and Okumura's inequality [10], we get

$$\begin{aligned}
\Box(nH) &= \sum_{i,j}((nH - \frac{1}{2}(n-1)a)\delta_{ij} - h_{ij})(nH)_{ij} \\
&= (nH - \frac{1}{2}(n-1)a)\triangle(nH) - \sum_{i,j} h_{ij}(nH)_{ij} \\
&= (nH - \frac{1}{2}(n-1)a)\triangle(nH - \frac{1}{2}(n-1)a) - \sum_{i,j} h_{ij}(nH)_{ij} \\
&= \frac{1}{2}\triangle(nH - \frac{1}{2}(n-1)a)^2 - |\nabla(nH - \frac{1}{2}(n-1)a)|^2 \\
&\qquad - \sum_{i,j} h_{ij}(nH)_{ij} \\
&= \frac{1}{2}\triangle(nH - \frac{1}{2}(n-1)a)^2 - n^2|\nabla H|^2 - \sum_{i,j} h_{ij}(nH)_{ij} \\
&= \frac{1}{2}\triangle|B|^2 - n^2|\nabla H|^2 - \sum_{i,j} h_{ij}(nH)_{ij} \\
&= \sum_{i,j,k} h_{ijk}^2 - n^2|\nabla H|^2 + n(|B|^2 - nH^2) + nHf_3 - |B|^4 \\
&\geq |\nabla B|^2 - n^2|\nabla H|^2 \\
&\qquad + |\phi|^2\Big(- |\phi|^2 - \frac{n(n-2)}{\sqrt{n(n-1)}}|H||\phi| + n(1+H^2)\Big). \qquad (2.18)
\end{aligned}$$

$\Box$

On the other hand, we choose $e_1, \ldots, e_n$ such that $h_{ij} = \lambda_i\delta_{ij}$, then we can deduce from (2.9) and (2.12) that

$$\frac{1}{2}\triangle|B|^2 = \sum_{i,j,k} h_{ijk}^2 + \sum_{i,j} h_{ij}(nH)_{ij} + \frac{1}{2}R_{ijij}(\lambda_i - \lambda_j)^2 \qquad (2.19)$$

and, as (2.18),

$$\begin{aligned}
\Box(nH) &= \frac{1}{2}\triangle|B|^2 - n^2|\nabla H|^2 - \sum_{i,j} h_{ij}(nH)_{ij} \\
&= \sum_{i,j,k} h_{ijk}^2 - n^2|\nabla H|^2 + \frac{1}{2}R_{ijij}(\lambda_i - \lambda_j)^2 \qquad (2.20) \\
&\geq |\nabla B|^2 - n^2|\nabla H|^2 + \frac{1}{2}R_{ijij}(\lambda_i - \lambda_j)^2.
\end{aligned}$$

Proposition 2.3. *Let M^n be a complete hypersurface of $S^{n+1}(1)$ with bounded mean curvature. If $r = aH + b, b \geq 1$, then there is a sequence of points*

$\{p_k\} \in M^n$ *such that*

$$\lim_{k\to\infty} nH(p_k) = n\sup H; \quad \lim_{k\to\infty} |\nabla(nH)(p_k)| = 0; \quad \limsup_{k\to\infty}(\Box(nH)(p_k)) \le 0.$$

Proof. Choose a local orthonormal frame field $e_1, \dots, e_n$ at a neighbourhood of $p \in M^n$ such that $h_{ij} = \lambda_i\delta_{ij}$. Thus

$$\Box(nH) = \sum_i [(nH - \frac{1}{2}(n-1)a) - \lambda_i](nH)_{ii}. \tag{2.21}$$

If $H \equiv 0$, the proposition is obvious. Let us suppose that H is not identically zero. By changing the orientation of M^n if necessary, we may assume $\sup H > 0$. From Gauss equation, we have

$$\begin{aligned} \lambda_i^2 \le |B|^2 &= n^2H^2 - n(n-1)(r-1) \\ &= n^2H^2 - n(n-1)(aH + b - 1) \\ &= (nH)^2 - (n-1)a(nH) - n(n-1)(b-1) \\ &= (nH - \frac{1}{2}(n-1)a)^2 - \frac{1}{4}(n-1)((n-1)a^2 - 4n + 4nb) \\ &\le (nH - \frac{1}{2}(n-1)a)^2, \quad (i = 1, 2, \dots, n). \end{aligned} \tag{2.22}$$

Therefore

$$|\lambda_i| \le \left|nH - \frac{1}{2}(n-1)a\right|, \quad (i = 1, 2, \dots, n). \tag{2.23}$$

and

$$R_{ijij} = 1 + \lambda_i\lambda_j \ge 1 - (nH - \frac{1}{2}(n-1)a)^2. \tag{2.24}$$

Because H is bounded, it follows from (2.24) that the sectional curvatures are bounded from below. Therefore we may apply generalized maximum principle from [11, 12] to nH, obtaining a sequence of points $\{p_k\} \in M^n$ such that

$$\lim_{k\to\infty} nH(p_k) = n\sup H; \quad \lim_{k\to\infty} |\nabla nH(p_k)| = 0; \quad \limsup_{k\to\infty}((nH)_{ii}(p_k)) \le 0. \tag{2.25}$$

From Gauss equation, we have

$$|B|^2 = n^2H^2 + n(n-1)(1 - aH - b), \tag{2.26}$$

and then

$$nH[nH - (n-1)a] = n(n-1)(b-1) + |B|^2 \ge 0,$$

if $b \ge 1$. If $|B|^2 = 0$, from $|B|^2 \ge nH^2$, we deduce that $H = 0$ which is the trivial case. So $b > 1$ or $|B|^2 \ne 0$ which implies $H \ne 0$. When $H \ne 0$, we can obtain from (2.26) that

$$-\frac{n-1}{2}a = \frac{1}{2nH}\left(|B|^2 - n^2H^2 + n(n-1)(b-1)\right).$$

Therefore,

$$
\begin{aligned}
& nH - \frac{n-1}{2}a - \lambda_i \\
& = nH - \lambda_i + \frac{1}{2nH}\big(|B|^2 - n^2H^2 + n(n-1)(b-1)\big) \\
& = \frac{n-1}{2H}(b-1) + \frac{1}{2nH}\big(|B|^2 + n^2H^2 - 2nH\lambda_i\big). \qquad (2.27)
\end{aligned}
$$

Observe now that

$$
\begin{aligned}
|B|^2 + n^2H^2 - 2nH\lambda_i &= \sum_{j=1}^{n}\lambda_j^2 + \Big(\sum_{j=1}^{n}\lambda_j\Big)^2 - 2\Big(\sum_{j=1}^{n}\lambda_j\Big)\lambda_i \\
&= \sum_{j=1, j\neq i}^{n}\lambda_j^2 + \Big(\sum_{j=1, j\neq i}^{n}\lambda_j\Big)^2 \geq 0. \qquad (2.28)
\end{aligned}
$$

So (2.27), (2.28) and $b \geq 1$ imply that

$$
nH - \frac{n-1}{2}a - \lambda_i \geq 0. \qquad (2.29)
$$

On the other hand, from (2.23) we get

$$
\begin{aligned}
& nH(p_k) - \frac{1}{2}(n-1)a - \lambda_i(p_k) \\
& \leq nH(p_k) - \frac{1}{2}(n-1)a + |\lambda_i(p_k)| \\
& \leq 2nH(p_k) - (n-1)a. \qquad (2.30)
\end{aligned}
$$

Using once more the fact that H is bounded, from (2.29) and (2.30), we infer that $nH(p_k) - \frac{1}{2}(n-1)a - \lambda_i(p_k)$ is nonnegative and bounded. By applying $\Box(nH)$ at p_k, taking the limit and using (2.25), we have

$$
\begin{aligned}
& \limsup_{k\to\infty}(\Box(nH)(p_k)) \\
& \leq \sum_i \limsup_{k\to\infty}[(nH - \frac{1}{2}(n-1)a) - \lambda_i](p_k)(nH)_{ii}(p_k) \leq 0.
\end{aligned}
$$

□

3. Proof of the main result

Proof of Theorem 1.4. If M^n is minimal, i.e., if $H \equiv 0$, then $r = b \geq 1$. So, from Gauss equation, we have $|B|^2 \equiv 0$ and M^n is totally geodesic. Let us suppose that H is not identically zero. In this case, by Proposition 2.3 it is possible to obtain a sequence of points $\{p_k\} \in M^n$ such that

$$
\limsup_{k\to\infty}(\Box(nH)(p_k)) \leq 0, \quad \lim_{k\to\infty} H(p_k) = \sup H > 0. \qquad (3.1)
$$

Moreover, using the Gauss equation, we have that

$$|\phi|^2 = |B|^2 - nH^2 = n(n-1)(H^2 - aH - b + 1). \tag{3.2}$$

In view of $\lim_{k\to\infty} H(p_k) = \sup H$ and $a \le 0$, (3.2) implies that $\lim_{k\to\infty} |\phi|^2(p_k) = \sup|\phi|^2$. Now we consider the following polynomial given by

$$P_{\sup H}(x) = -x^2 - \frac{n(n-2)}{\sqrt{n(n-1)}} \sup Hx + n(1 + \sup H^2). \tag{3.3}$$

Because the discriminant of $P_{\sup H}(x)$ is always positive, the smallest root of $P_{\sup H}(x)$ is negative and the biggest root μ of $P_{\sup H}(x)$ is positive. We claim that $P_{\sup H}(\sup|\phi|) \ge 0$. It's easy to check that $\sup|\phi|^2 - \mu^2 \le 0$ provided $b \ge \frac{n-2}{n-1}$. In fact,

$$\begin{aligned}(\sup|\phi|)^2 = \sup|\phi|^2 &= n(n-1)(\sup H^2 - a\sup H - b + 1)\\ &\le n(n-1)(\sup H^2 - b + 1),\end{aligned} \tag{3.4}$$

it is straightforward to verify that

$$\begin{aligned}\sup|\phi|^2 - \mu^2 \le \frac{n-2}{2(n-1)}\Big(&n^2\sup H^2 - n\sup H\sqrt{n^2\sup H^2 + 4(n-1)}\\ &+ \frac{2n(n-1)}{n-2}(-(n-1)b + (n-2))\Big).\end{aligned} \tag{3.5}$$

It can be easily seen that $\sup|\phi|^2 - \mu^2 \le 0$ if and only if

$$\begin{aligned}&n^2\sup H^2 - n\sup H\sqrt{n^2\sup H^2 + 4(n-1)}\\ &\qquad - \frac{2n(n-1)}{n-2}((n-1)b - (n-2)) \le 0.\end{aligned}$$

So, if $b > 1$, the last inequality is true. We deduce that $\sup|\phi| \le \mu$ and $P_{\sup H}(\sup|\phi|) \ge 0$.

Using Lemma 2.1 and evaluating (2.16) at the points p_k of the sequence, taking the limit and using (3.1), we obtain that

$$0 \ge \limsup_{k\to\infty}(\square(nH)(p_k)) \ge \sup|\phi|^2 P_{\sup H}(\sup|\phi|) \ge 0,$$

so $\sup|\phi|^2 P_{\sup H}(\sup|\phi|) = 0$. Then we have either $\sup|\phi|^2 = 0$ which shows that M^n is totally umbilical or $P_{\sup H}(\sup|\phi|) = 0$. So (2.16) is also equality, then the Okumura's formula implies that M^n has two distinct principal curvatures one of which is simple and $M^n = S^1(c) \times S^{n-1}(\sqrt{1-c^2})$. □

Remark 3.1. *Using our method, we can easily generalize Theorem 1.2 and Theorem 1.3 to the complete case. But we don't state it here.*

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