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EXPLICIT MULTIPLE SINGULAR PERIODIC SOLUTIONS AND SINGULAR SOLITON SOLUTIONS TO KDV EQUATION

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Abstract. Based on some stationary periodic solutions and stationary soliton solutions, one studies the general solution for the relative lax system, and a number of exact solutions to the Korteweg-de Vries (KdV) equation are first constructed by the known Darboux transformation, these solutions include double and triple singular periodic solutions as well as singular soliton solutions whose amplitude depend on some rational functions.
Keywords: Lax system, KdV equation, Darboux transformation, singular periodic solution, singular periodic solution.
MSC(2010): Primary: 35Q51; Secondary: 35A.

1. Introduction

The KdV equation

$$u_t + 6uu_x + u_{xxx} = 0 \tag{1.1}$$

is a classical soliton equation, it and its various generalized versions with time-dependent coefficients have been studied by many authors, one of the main interesting and most widely studies aspects is the soliton structure, some exact solitary wave solutions together with periodic wave solutions are obtained [1–7, 10–17, 19]. However, the results associated with exact multi-periodic solutions and multi-soliton solutions are few, the famous two-soliton and three-soliton solutions are stemmed from [1].

The Darboux transformation method is one of the powerful and direct methods to construct exact multiple waves solution for the nonlinear evolution equation [9, 18, 20, 21], but it should be noted that the Lax system is an over-determined linear partial differential equations, when its coefficients depend on the independent variable x and t, finding the exact solutions of the system

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may be difficult or even impossible. The research has usually relied on some nontrivial methods, which restrict the range of applications, but enable one to solve the more sophisticated multiple waves solution.

2. **Solution for the Lax system**

According to [8], one knows that Equation (1.1) is the compatibility condition of the following Lax system

$$\Phi_x = \begin{pmatrix} 0 & 1 \\ \lambda - u & 0 \end{pmatrix} \Phi, \tag{2.1a}$$

$$\Phi_t = \begin{pmatrix} u_x & -(4\lambda + 2u) \\ -(4\lambda + 2u)(\lambda - u) + u_{xx} & -u_x \end{pmatrix} \Phi, \tag{2.1b}$$

where λ is the spectral parameter, when $\Phi = \Phi(x, t, \lambda) = \binom{\varphi_1}{\varphi_2}$ is the general solution to the system (2.1), the form

$$u' = 2\lambda_1 - u - 2\sigma_1^2 \tag{2.2}$$

is called Darboux transformation, where u' is a new solution generated from the seed solution u, $\sigma_1 = \frac{\varphi_2}{\varphi_1}|_{\lambda=\lambda_1}$. In particular,

$$\tilde{\Phi}(x, t, \lambda) = \begin{pmatrix} -\sigma_1 & 1 \\ \lambda - \lambda_1 + \sigma_1^2 & -\sigma_1 \end{pmatrix} \Phi(x, t, \lambda), \tag{2.3}$$

is the general solution to the system (2.1) based on u', and

$$u'' = 2\lambda_2 - u' - 2\sigma_2^2 \tag{2.4}$$

is a new solution generated from u', where $\sigma_2 = \frac{\tilde{\varphi}_2}{\tilde{\varphi}_1}|_{\lambda=\lambda_2}$.

The first step for constructing new solutions from some stationary solutions is to find the solutions of the system (2.1) associated with the stationary solutions.

Without any loss of generality it is assumed that $u = u(x)$ is the stationary formal solution to (1.1), thus, $6uu_x + u_{xxx} = 0$, integrating this gives

$$u_{xx} = a - 3u^2, \tag{2.5}$$

multiplying (2.5) by u and integrating it, further gives

$$u_x^2 = b + 2au - 2u^3, \tag{2.6}$$

where a, b are arbitrary constants. One can assert that every solution to (2.5) is a stationary solution to (1.1), so is each solution to (2.6). At the same time, (2.1b) is written as

$$\begin{cases} \varphi_{1t} = u_x\varphi_1 - (4\lambda + 2u)\varphi_2, \\ \varphi_{2t} = [(4\lambda + 2u)(u - \lambda) + u_{xx}]\varphi_1 - u_x\varphi_2. \end{cases} \tag{2.7}$$

As $u = u(x)$, the above system and

$$\varphi_{1tt} = \{u_x^2 - [(4\lambda + 2u)^2(u - \lambda) + (4\lambda + 2u)u_{xx}]\}\varphi_1, \tag{2.8a}$$

$$\varphi_2 = \frac{u_x\varphi_1 - \varphi_{1t}}{(4\lambda+2u)}, \tag{2.8b}$$

possess the same solution, substituting (2.5) and (2.6) into (2.8a) leads to the simpler system

$$\begin{cases} \varphi_{1tt} = (16\lambda^3 - 4\lambda a + b)\varphi_1, \\ \varphi_2 = \frac{u_x\varphi_1 - \varphi_{1t}}{(4\lambda+2u)}. \end{cases}$$

when $b = 4\lambda a - 16\lambda^3$, the general solution to the above system is given by

$$\begin{pmatrix} t & 1 \\ \frac{tu_x - 1}{4\lambda+2u} & \frac{u_x}{4\lambda+2u} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}.$$

Obviously

$$\Phi(x, t, \lambda) = \begin{pmatrix} t & 1 \\ \frac{tu_x - 1}{4\lambda+2u} & \frac{u_x}{4\lambda+2u} \end{pmatrix} \begin{pmatrix} c_1(x) \\ c_2(x) \end{pmatrix}. \tag{2.9}$$

is a solution of (2.1b), where $c_1(x), c_2(x)$ are arbitrary functions. If (2.9) is also solution of (2.1a), then substituting it into the system (2.1a) yields

$$\begin{cases} c_1'(x) = \frac{u_x}{4\lambda+2u}c_1(x), \\ c_2'(x) = -\frac{1}{4\lambda+2u}c_1(x) + \frac{u_x}{4\lambda+2u}c_2(x), \end{cases} \tag{2.10}$$

solving $c_1(x), c_2(x)$ from (2.10) and substituting them into (2.9) gives the general solution to the Lax system (2.1)

$$\Phi(x, t, \lambda) = \begin{pmatrix} \sqrt{|4\lambda + 2u|}(t - \int \frac{\mathrm{d}x}{4\lambda+2u}) & \sqrt{|4\lambda + 2u|} \\ \frac{\sqrt{|4\lambda+2u|}}{4\lambda+2u}(tu_x - 1 - u_x \int \frac{\mathrm{d}x}{4\lambda+2u}) & \frac{u_x\sqrt{|4\lambda+2u|}}{4\lambda+2u} \end{pmatrix} \begin{pmatrix} k_1 \\ \mu_1 \end{pmatrix}, \tag{2.11}$$

where k_1, μ_1 are arbitrary constants.

3. Explicit singular solutions to the KdV equation

From (2.11), one gets

$$\sigma_1 = \frac{\frac{u_x}{4\lambda+2u}(k_1 t - k_1 \int \frac{\mathrm{d}x}{4\lambda+2u} + \mu_1) - \frac{k_1}{4\lambda+2u}}{k_1 t - k_1 \int \frac{\mathrm{d}x}{4\lambda+2u} + \mu_1}\Big|_{\lambda=\lambda_1}, \tag{3.1}$$

inserting σ_1 into (2.2) gives the new formal solution

$$u' = 2\lambda_1 - u - 2\frac{[\frac{u_x}{4\lambda+2u}(k_1 t - k_1 \int \frac{\mathrm{d}x}{4\lambda+2u} + \mu_1) - \frac{k_1}{4\lambda+2u}]^2}{(k_1 t - k_1 \int \frac{\mathrm{d}x}{4\lambda+2u} + \mu_1)^2}\Big|_{\lambda=\lambda_1}. \tag{3.2}$$

Substituting (2.11) into (2.3) gives the general solution to the Lax system associated with u'

$$\tilde{\Phi}(x,t,\lambda) = \sqrt{|4\lambda+2u|} \times$$
$$\begin{pmatrix} (\frac{u_x}{4\lambda+2u}-\sigma_1)(t-\int\frac{\mathrm{d}x}{4\lambda+2u})-\frac{1}{4\lambda+2u} & \frac{u_x}{4\lambda+2u}-\sigma_1 \\ (\lambda-\lambda_1+\sigma_1^2-\frac{\sigma_1 u_x}{4\lambda+2u})(t-\int\frac{\mathrm{d}x}{4\lambda+2u})+\frac{\sigma_1}{4\lambda+2u} & \lambda-\lambda_1+\sigma_1^2-\frac{\sigma_1 u_x}{4\lambda+2u} \end{pmatrix} K,$$

with $K = \begin{pmatrix} k_2 \\ \mu_2 \end{pmatrix}$, k_2, μ_2 arbitrary constants, which leads to

$$\sigma_2 = -\sigma_1 + \frac{(\lambda-\lambda_1)(k_2 t - k_2\int\frac{\mathrm{d}x}{4\lambda+2u}+\mu_2)}{(\frac{u_x}{4\lambda+2u}-\sigma_1)(k_2 t-k_2\int\frac{\mathrm{d}x}{4\lambda+2u}+\mu_2)-\frac{k_2}{4\lambda+2u}}|_{\lambda=\lambda_2}. \tag{3.3}$$

Substituting (2.2) into (2.4) yields

$$u'' = u + 2(\lambda_2-\lambda_1+\sigma_1^2-\sigma_2^2), \tag{3.4}$$

then substituting (3.3) into (3.4) gives the new solution

$$u'' = u + 2(\lambda_2-\lambda_1)\times \frac{[\lambda_1-\lambda+(\frac{u_x}{4\lambda+2u})^2-\sigma_1^2)]\Gamma^2-2k_2\Gamma\frac{u_x}{(4\lambda+2u)^2}+\frac{k_2^2}{(4\lambda+2u)^2}}{[\Gamma(\frac{u_x}{4\lambda+2u}-\sigma_1)-\frac{k_2}{4\lambda+2u}]^2}\Big|_{\lambda=\lambda_2}, \tag{3.5}$$

where $\Gamma = k_2 t - k_2\int\frac{\mathrm{d}x}{4\lambda+2u}+\mu_2|_{\lambda=\lambda_2}$.

When $b = 4\lambda a - 16\lambda^3$, (2.6) reduces to

$$u_x^2 = 4\lambda a - 16\lambda^3 + 2au - 2u^3. \tag{3.6}$$

In view of the case for a in (3.6), there are four cases to be considered in the rest section. For simplicity, first setting $\alpha = \sqrt{-3\lambda}$, $\beta = \frac{\sqrt{6\lambda}}{2}$, $\gamma = \sqrt{3\lambda}$, $\omega = \frac{\sqrt{-6\lambda}}{2}$, $\alpha_i = \alpha(\lambda_i)$, $\beta_i = \beta(\lambda_i)$, $\gamma_i = \gamma(\lambda_i)$, $\omega_i = (\lambda_i)$, $\xi_i = k_i\alpha_i x + 8k_i\alpha_i^3 t + 8\mu_i\alpha_i^3$, $\eta_i = 4k_i\beta_i^3 t - k_i\beta_i x + 4\mu_i\beta_i^3$, $\zeta_i = k_i\gamma_i x - 8k_i\gamma_i^3 t - 8\mu_i\gamma_i^3$, $\theta_i = 4k_i\omega_i^3 t + k_i\omega_i x + 4\mu_i\omega_i^3$, where $i = 1, 2$.

Case 1. Taking $a = \frac{4}{3}\alpha^4$ in (3.6) gives the stationary periodic solutions to (1.1)

$$u_1 = -2\alpha^2\tan^2\alpha x - \frac{4}{3}\alpha^2, \tag{3.7}$$

and

$$u_2 = -2\alpha^2\cot^2\alpha x - \frac{4}{3}\alpha^2. \tag{3.8}$$

Substituting the seed solution (3.7) into (3.1) gives

$$\sigma_1 = \alpha_1\tan\alpha_1 x + \frac{2k_1\alpha_1\cos\alpha_1 x}{\xi_1\sec\alpha_1 x + k_1\sin\alpha_1 x}. \tag{3.9}$$

From (3.2), the double singular periodic solution is given by

$$u_1' = -u_1 + 2\lambda_1 - 2\alpha_1^2 \tan^2 \alpha_1 x - \frac{8k_1\alpha_1^2(k_1 + \xi_1 \tan \alpha_1 x)}{(\xi_1 \sec \alpha_1 x + k_1 \sin \alpha_1 x)^2}.$$

Substituting (3.7), (3.9) into (3.5) and collecting, one can obtain the triple singular periodic solution

$$u_1'' = u_1 + 2(\lambda_2 - \lambda_1) \times \frac{(\lambda_1 - \lambda_2 + \alpha_2^2 \tan^2 \alpha_2 x - \alpha_1^2 \tan^2 \alpha_1 x)A_1^2 A_2^2 - B_1 A_2^2 + B_2 A_1^2}{[(\alpha_2 \tan \alpha_2 x - \alpha_1 \tan \alpha_1 x)A_1 A_2 - 2k_1\alpha_1 A_2 \cos \alpha_1 x + 2k_2\alpha_2 A_1 \cos \alpha_2 x]^2},$$

where $A_1 = \xi_i \sec \alpha_i x + k_i \sin \alpha_i x,\ B_i = 4k_i\alpha_i^2\xi_i \tan \alpha_i x + 4k_i^2\alpha_i^2,\ i = 1, 2.$ Similarly, substituting (3.8) into (3.2) and (3.5), respectively, one gets

$$u_2' = -u_2 + 2\lambda_1 - 2\alpha_1^2 \cot^2 \alpha_1 x - \frac{8k_1\alpha_1^2(k_1 - \xi_1 \cot \alpha_1 x)}{(\xi_1 \csc \alpha_1 x - k_1 \cos \alpha_1 x)^2}$$

and

$$u_2'' = u_2 + 2(\lambda_2 - \lambda_1) \times \frac{(\lambda_1 - \lambda_2 + \alpha_2^2 \cot^2 \alpha_2 x - \alpha_1^2 \cot^2 \alpha_1 x)C_1^2 C_2^2 + D_1 C_2^2 - D_2 C_1^2}{[(\alpha_1 \cot \alpha_1 x - \alpha_2 \cot \alpha_2 x)C_1 C_2 - 2k_1\alpha_1 C_2 \sin \alpha_1 x + 2k_2\alpha_2 C_1 \sin \alpha_2 x]^2},$$

respectively, where $C_i = \xi_i \csc \alpha_i x - k_i \cos \alpha_i x$, $D_i = 4k_i\alpha_i^2\xi_i \cot \alpha_i x - 4k_i^2\alpha_i^2, i = 1, 2.$

Case 2. Taking $a = \frac{4}{3}\beta^4$ in (3.6) yields another set of the seed solutions

$$u_3 = -2\beta^2 \sec^2 \beta x - \frac{2}{3}\beta^2 \tag{3.10}$$

and

$$u_4 = -2\beta^2 \csc^2 \beta x - \frac{2}{3}\beta^2. \tag{3.11}$$

Substituting (3.10) into (3.2) and (3.5), respectively, one can obtain

$$u_3' = -u_3 + 2\lambda_1 - 2\beta_1^2 \frac{(\eta_1 \sec^2 \beta_1 x - k_1 \tan \beta_1 x)^2}{(\eta_1 \tan \beta_1 x - k_1)^2},$$

and

$$u_3'' = u_3 + 2(\lambda_2 - \lambda_1)\frac{(\lambda_1 - \lambda_2)E_1^2 E_2^2 - \beta_1^2 F_1^2 E_2^2 + \beta_2^2 F_2^2 E_1^2}{(\beta_1 F_1 E_2 - \beta_2 F_2 E_1)^2},$$

respectively, where $E_i = \eta_i \tan \beta_i x - k_i,\ F_i = \eta_i \sec^2 \beta_i x - k_i \tan \beta_i x$. Repeating the same procedure gives

$$u_4' = -u_4 + 2\lambda_1 - 2\beta_1^2 \frac{(\eta_1 \csc^2 \beta_1 x + k_1 \cot \beta_1 x)^2}{(\eta_1 \cot \beta_1 x + k_1)^2},$$

and

$$u_4'' = u_4 + 2(\lambda_2 - \lambda_1)\frac{(\lambda_1 - \lambda_2)G_1^2G_2^2 - \beta_1^2H_1^2G_2^2 + \beta_2^2H_2^2G_1^2}{(\beta_1 H_1 G_2 - \beta_2 H_2 G_1)^2},$$

respectively, where $G_i = \eta_i \cot\beta_i x + k_i,\ H_i = \eta_i \csc^2\beta_i x + k_i \cot\beta_i x, i = 1, 2.$

Case 3. Taking $a = \frac{4}{3}\gamma^4$ in (3.6) leads to the stationary soliton solutions

$$u_5 = -2\gamma^2 \tanh^2\gamma x + \frac{4}{3}\gamma^2$$

and

$$u_6 = -2\gamma^2 \coth^2\gamma x + \frac{4}{3}\gamma^2.$$

At the same time, one constructs multiple singular soliton solutions

$$u_5' = -u_5 + 2\lambda_1 - 2\gamma_1^2\tanh^2\gamma_1 x - \frac{8k_1\gamma_1^2(k_1 - \zeta_1\tanh\gamma_1 x)}{(\zeta_1 \mathrm{sech}\gamma_1 x + k_1\sinh\gamma_1 x)^2},$$

$$u_6' = -u_6 + 2\lambda_1 - 2\gamma_1^2\coth^2\gamma_1 x + \frac{8k_1\gamma_1^2(k_1 - \zeta_1\coth\gamma_1 x)}{(\zeta_1 \mathrm{csch}\gamma_1 x - k_1\cosh\gamma_1 x)^2},$$

$$u_5'' = u_5 + 2(\lambda_2 - \lambda_1)\times \frac{(\lambda_1 - \lambda_2 + \gamma_2^2\tanh^2\gamma_2 x - \gamma_1^2\tanh^2\gamma_1 x)I_1^2I_2^2 + J_1I_2^2 - J_2I_1^2}{[(\gamma_2\tanh\gamma_2 x - \gamma_1\tanh\gamma_1 x)I_1I_2 + 2k_1\gamma_1 I_2\cosh\gamma_1 x - 2k_2\gamma_2 I_1\cosh\gamma_2 x]^2},$$

and

$$u_6'' = u_6 + 2(\lambda_2 - \lambda_1)\times \frac{(\lambda_1 - \lambda_2 + \gamma_2^2\coth^2\gamma_2 x - \gamma_1^2\coth^2\gamma_1 x)M_1^2M_2^2 - N_1M_2^2 + N_2M_1^2}{[(\gamma_1\coth\gamma_1 x - \gamma_2\coth\gamma_2 x)M_1M_2 + 2k_1\gamma_1 M_2\sinh\gamma_1 x - 2k_2\gamma_2 M_1\sinh\gamma_2 x]^2},$$

where $I_i = \zeta_i \mathrm{sech}\gamma_i x + k_i\sinh\gamma_i x$, $J_i = 4k_i\gamma_i^2\zeta_i\tanh\gamma_i x - 4k_i^2\gamma_i^2$, $M_i = \zeta_i\mathrm{csch}\gamma_i x - k_i\cosh\gamma_i x$, $N_i = 4k_i\gamma_i^2\zeta_i\coth\gamma_i x - 4k_i^2\gamma_i^2, i = 1, 2.$

Case 4. Taking $a = \frac{4}{3}\omega^4$ in (3.6) gives another group of stationary soliton solutions

$$u_7 = 2\omega^2\tanh^2\omega x - \frac{2}{3}\omega^2$$

and

$$u_8 = -2\omega^2\coth^2\omega x - \frac{2}{3}\omega^2.$$

Similarly, one obtains

$$u_7' = -u_7 + 2\lambda_1 - 2\omega_1^2\frac{(\theta_1 \mathrm{sech}^2\omega_1 x - k_1\tanh\omega_1 x)^2}{(\theta_1\tan\omega_1 x - k_1)^2},$$

$$u_8' = -u_8 + 2\lambda_1 - 2\omega_1^2\frac{(\theta_1 \mathrm{csch}^2\omega_1 x - k_1\coth\omega_1 x)^2}{(\theta_1\coth\omega_1 x - k_1)^2},$$

$$u_7'' = u_7 + 2(\lambda_2 - \lambda_1)\frac{(\lambda_1 - \lambda_2)P_1^2P_2^2 - \omega_1^2Q_1^2P_2^2 + \omega_2^2Q_2^2P_1^2}{(\omega_1Q_1P_2 - \omega_2Q_2P_1)^2}$$

along with

$$u_8'' = u_8 + 2(\lambda_2 - \lambda_1)\frac{(\lambda_1 - \lambda_2)R_1^2R_2^2 - \omega_1^2S_1^2R_2^2 + \omega_2^2S_2^2R_1^2}{(\omega_1S_1R_2 - \omega_2S_2R_1)^2},$$

where $P_i = \theta_i \tanh \omega_i x - k_i,\ Q_i = \theta_i \mathrm{sech}^2\omega_i x + k_i \tanh \omega_i x,\ R_i = \theta_i \coth \omega_i x - k_i,\ S_i = \theta_i \mathrm{csch}^2\omega_i x - k_i \coth \omega_i x, i = 1, 2.$

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