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SOME RESULTS ON VALUE DISTRIBUTION OF THE DIFFERENCE OPERATOR

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Abstract. In this article, we consider the uniqueness of the difference monomials $f^n(z)f(z+c)$. Suppose that $f(z)$ and $g(z)$ are transcendental meromorphic functions with finite order and $E_k(1, f^n(z)f(z+c)) = E_k(1, g^n(z)g(z+c))$. Then we prove that if one of the following holds (i) $n \geq 14$ and $k \geq 3$, (ii) $n \geq 16$ and $k = 2$, (iii) $n \geq 22$ and $k = 1$, then $f(z) \equiv t_1 g(z)$ or $f(z)g(z) = t_2$, for some constants t_1 and t_2 that satisfy $t_1^{n+1} = 1$ and $t_2^{n+1} = 1$. We generalize some previous results of Qi et. al.
Keywords: Meromorphic functions, difference equations, uniqueness, finite order.
MSC(2010): Primary: 30D35; Secondary: 39B12

1. **Introduction and main results**

In this article, we assume that the reader is familiar with the fundamental results and the standard notations of the Nevanlinna theory (see, e.g., [8, 18]). Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions in the complex plane. By $S(r, f)$, we denote any quantity satisfying $S(r, f) = o(T(r, f))$ as $r \to \infty$, possibly outside a set of finite logarithmic measure. Then the meromorphic function α is called a small function of $f(z)$, if $T(r, \alpha) = S(r, f)$. If $f(z)-\alpha$ and $g(z)-\alpha$ have same zeros, counting multiplicity (ingoring multiplicity), then we say $f(z)$ and $g(z)$ share the small function α CM (IM).

Let a be a finite complex number, and k be a positive integer. We denote by $N_{k)}(r, \frac{1}{f-a})$ the counting function for the zeros of $f(z)-a$ with multiplicity $\leq k$, and by $\overline{N}_k(r, \frac{1}{f-a})$ the corresponding one for which multiplicity is not counted. Let $N_{(k}(r, \frac{1}{f-a})$ be the counting function for the zeros of $f(z) - a$ with multiplicity $\geq k$, and $\overline{N}_{(k}(r, \frac{1}{f-a})$ be the corresponding one for which multiplicity is

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not counted. Moreover, we set $N_k(r, \frac{1}{f-a}) = \overline{N}(r, \frac{1}{f-a}) + \overline{N}_{(2}(r, \frac{1}{f-a}) + \cdots + \overline{N}_{(k}(r, \frac{1}{f-a})$. In the same way, we can define $N_k(r, f)$.

Currently, Many articles have focused on value distribution in difference analogues of meromorphic functions (see, e.g., [1, 2, 5–7, 9–17, 19]). In particular, there has been an increasing interest in studying the uniqueness problems related to meromorphic functions and their shifts or their difference operators (see, e.g., [1, 9, 11–15]). Our aim in this article is to investigate the uniqueness problems of difference monomials of meromorphic functions.

In 2010, Qi et al. [16] studied the uniqueness of the difference monomials and obtained the following result:

Theorem 1.1. *Let $f(z)$ and $g(z)$ be transcendental entire functions with finite order, c a non-zero complex constant; and $n \geq 6$ an integer. If $E(1, f^n(z)f(z+c)) = E(1, g^n(z)g(z+c))$, then $f(z) \equiv t_1 g(z)$ or $f(z)g(z) = t_2$, for some constants t_1 and t_2 that satisfy $t_1^{n+1} = 1$ and $t_2^{n+1} = 1$.*

In this paper, we consider the case of meromorphic functions of Theorem A. Our result can be stated as follows:

Lemma 1.2. *Let $c \in \mathbb{C} \setminus \{0\}$. Let $f(z)$ and $g(z)$ be two transcendental meromorphic functions with finite order, and $n(\geq 14), k(\geq 3)$ be two positive integers. If $E_k(1, f^n(z)f(z+c)) = E_k(1, g^n(z)g(z+c))$, then $f(z) \equiv t_1 g(z)$ or $f(z)g(z) = t_2$, for some constants t_1 and t_2 that satisfy $t_1^{n+1} = 1$ and $t_2^{n+1} = 1$.*

In the previous theorem we considered the case $k \geq 3$. The following two theorems are about the case $k \leq 2$.

Theorem 1.3. *Let $c \in \mathbb{C}$ and $n \geq 16$ be an integer. Let $f(z)$ and $g(z)$ be two transcendental meromorphic functions with finite order. If $E_2(1, f^n(z)f(z+c)) = E_2(1, g^n(z)g(z+c))$, then $f(z) \equiv t_1 g(z)$ or $f(z)g(z) = t_2$, for some constants t_1 and t_2 that satisfy $t_1^{n+1} = 1$ and $t_2^{n+1} = 1$.*

Theorem 1.4. *Let $c \in \mathbb{C}$ and $n \geq 22$ be an integer. Let $f(z)$ and $g(z)$ be two transcendental meromorphic functions with finite order. If $E_1(1, f^n(z)f(z+c)) = E_1(1, g^n(z)g(z+c))$, then $f(z) \equiv t_1 g(z)$ or $f(z)g(z) = t_2$, for some constants t_1 and t_2 that satisfy $t_1^{n+1} = 1$ and $t_2^{n+1} = 1$.*

2. **Preliminary lemmas**

Before proceeding to the actual proofs, we recall a few lemmas that play an important role in the reasoning.

Lemma 2.1. *[3] Let f and g be two meromorphic functions, and let k be a positive integer. If $E_k(1, f) = E_k(1, g)$, then one of the following cases must occur:*

1.

$$T(r,f)+T(r,g)\le \overline{N}_2(r,f)+\overline{N}_2\Big(r,\frac{1}{f}\Big)+\overline{N}_2(r,g)+\overline{N}_2\Big(r,\frac{1}{g}\Big)$$
$$+\overline{N}\Big(r,\frac{1}{f-1}\Big)+\overline{N}\Big(r,\frac{1}{g-1}\Big)-N_{11}\Big(r,\frac{1}{f-1}\Big)+\overline{N}_{(k+1}\Big(r,\frac{1}{f-1}\Big)$$
$$+\overline{N}_{(k+1}\Big(r,\frac{1}{g-1}\Big)+S(r,f)+S(r,g); \tag{2.1}$$

2.

$$f=\frac{(b+1)g+(a-b-1)}{bg+(a-b)}, \tag{2.2}$$

where $a(\neq 0), b$ are two constants.

Lemma 2.2. *[7] Let $f(z)$ be a nonconstant finite order meromorphic function and let $c\neq 0$ be an arbitrary complex number. Then*

$$T(r,f(z+|c|))=T(r,f(z))+S(r,f).$$

Remark 2.3. *It is shown in [4, p. 66], that for $c\in\mathbb{C}\setminus\{0\}$,*

$$(1+o(1))T(r-|c|,f(z))\le T(r,f(z+c))\le (1+o(1))T(r+|c|,f(z))$$

hold as $r\to\infty$, for a general meromorphic function. By this and Lemma 2.2, we obtain

$$T(r,f(z+c))=T(r,f(z))+S(r,f).$$

Lemma 2.4. *Let $f(z)$ and $g(z)$ be two meromorphic functions with finite order, $n\ge 8$ a positive integer, and let $F=f^n(z)f(z+c)$ and $G=g^n(z)g(z+c)$. If*

$$F=\frac{(b+1)G+(a-b-1)}{bG+(a-b)}, \tag{2.3}$$

where $a(\neq 0), b$ are two constants, then $f(z)\equiv t_1 g(z)$ or $f(z)g(z)=t_2$, for some constants t_1 and t_2 that satisfy $t_1^{n+1}=1$ and $t_2^{n+1}=1$.

Proof of Lemma 2.3. Remark 2.3 yields that

$$T(r,F)=T(r,f^n(z)f(z+c))+S(r,f) \tag{2.4}$$
$$\le T(r,f^n(z))+T(r,f(z+c))+S(r,f) \tag{2.5}$$
$$=(n+1)T(r,f)+S(r,f). \tag{2.6}$$

On the other hand, together the first main Theorem with Remark 2.3, we obtain

$$nT(r,f)=T(r,f^n(z))+S(r,f)$$
$$\le T(r,f^n(z)f(z+c))+T\Big(r,\frac{1}{f(z+c)}\Big)+S(r,f)$$
$$=T(r,f(z))+T(r,F(z))+S(r,f) \tag{2.7}$$

that is,

$$T(r,F) \geq (n-1)T(r,f) + S(r,f) \tag{2.8}$$

Hence, (2.4) and (2.6) yield that

$$S(r,F) = S(r,f). \tag{2.9}$$

Similarly, we obtain

$$T(r,G) \geq (n-1)T(r,g) + S(r,g), \tag{2.10}$$

and

$$S(r,G) = S(r,g). \tag{2.11}$$

Set $I_1 = \{r : T(r,g) \geq T(r,f)\} \subseteq (0,\infty)$, and $I_2 = (0,\infty)\backslash I_1$. Then there is at least one $I_i (i = 1,2)$ such that I_i has infinite logarithmic measure. Without loss of generality, we may suppose that I_1 has infinite logarithmic measure. We break the rest of the proof into three cases. □

Case 1. $b \neq 0, -1$. If $a - b - 1 \neq 0$, then we know from 2.3

$$\overline{N}\Big(r, \frac{1}{F}\Big) = \overline{N}\Big(r, \frac{1}{G - \frac{a-b-1}{b+1}}\Big). \tag{2.12}$$

Together with the first main theorem, the second main theorem with Remark 2.3, 2.8 and 2.12, we obtain

$$\begin{aligned}
(n-1)T(r,g) &\leq T(r,G) + S(r,g) \\
&\leq \overline{N}\Big(r, \frac{1}{G}\Big) + \overline{N}(r,G) + \overline{N}\Big(r, \frac{1}{G - \frac{a-b-1}{b+1}}\Big) + S(r,G) + S(r,g) \\
&= \overline{N}\Big(r, \frac{1}{G}\Big) + \overline{N}(r,G) + \overline{N}\Big(r, \frac{1}{F}\Big) + S(r,g) \\
&\leq \overline{N}\Big(r, \frac{1}{g}\Big) + \overline{N}\Big(r, \frac{1}{g(z+c)}\Big) + \overline{N}(r,g) + \overline{N}(r, g(z+c)) \\
&\quad + \overline{N}\Big(r, \frac{1}{f}\Big) + \overline{N}\Big(r, \frac{1}{f(z+c)}\Big) + S(r,g) \\
&\leq 4T(r,g) + 2T(r,f) + S(r,g) && (2.13) \\
&\leq 6T(r,g) + S(r,g), \quad r \in I_1 && (2.14)
\end{aligned}$$

which is impossible, since $n \geq 8$. Hence, we obtain $a - b - 1 = 0$, so

$$F(z) = \frac{(b+1)G(z)}{bG(z)+1}.$$

Using the similar method as above, we obtain

$$\begin{aligned}(n-1)T(r,g) &\le T(r,G)+S(r,g)\\ &\le \overline{N}\Big(r,\frac{1}{G}\Big)+\overline{N}(r,G)+\overline{N}\Big(r,\frac{1}{G+\frac{1}{b}}\Big)+S(r,G)\\ &= \overline{N}\Big(r,\frac{1}{G}\Big)+\overline{N}(r,G)+\overline{N}(r,F)+S(r,G)\\ &\le 6T(r,g)+S(r,g),\quad r\in I_1\end{aligned}$$

which is a contradiction, since $n\ge 8$.
Case 2. $b=-1, a\ne -1$. By 2.3, we have

$$F=\frac{a}{a+1-G}. \tag{2.15}$$

Similarly, we get a contradiction, hence, we obtain $a=-1$. So, we get $FG=1$, that is $f^n(z)f(z+c)g^n(z)g(z+c)=1$. Set $H(z)=f(z)g(z)$. Suppose that $H(z)$ is not a constant. Then we obtain

$$H^n(z)H(z+c)=1. \tag{2.16}$$

Remark 2.3, the first main Theorem and 2.16 imply that

$$nT(r,H(z))=T(r,H^n(z))=T\Big(r,\frac{1}{H(z+c)}\Big)=T(r,H(z))+S(r,H). \tag{2.17}$$

Hence $H(z)$ must be a nonzero constant, since $n\ge 8$. Set $H(z)=t_1$, by 2.16, we know $t_1^{n+1}=1$. Thus, $f(z)g(z)=t_1$, where $t_1^{n+1}=1$.
Case 3. $b=0, a\ne 1$. By 2.3, we obtain

$$F=\frac{G+a-1}{a}.$$

Similarly, we get a contradiction, hence we obtain $a=1$, so we get $F=G$, that is

$$f^n(z)f(z+c)=g^n(z)g(z+c).$$

Let $H(z)=\frac{f(z)}{g(z)}$, using the similar method as above, we also obtain that $H(z)$ must be a nonzero constant. Thus, we have $f=t_2g$, where $t_2^{n+1}=1$.

3. **Proof of Theorem 1.1**

Let $F(z)=f^n(z)f(z+c)$ and $G(z)=g^n(z)g(z+c)$. Since $k\ge 3$, we have

$$\begin{aligned}&\overline{N}\Big(r,\frac{1}{F-1}\Big)+\overline{N}\Big(r,\frac{1}{G-1}\Big)-N_{11}\Big(r,\frac{1}{F-1}\Big)+\overline{N}_{(k+1}\Big(r,\frac{1}{F-1}\Big)\\ &\quad+\overline{N}_{(k+1}\Big(r,\frac{1}{G-1}\Big)\\ &\le \frac{1}{2}N\Big(r,\frac{1}{F-1}\Big)+\frac{1}{2}N\Big(r,\frac{1}{G-1}\Big)\\ &\le \frac{1}{2}T(r,F)+\frac{1}{2}T(r,G)+S(r,f)+S(r,g).)\end{aligned} \tag{3.1}$$

2.1 and 3.1 give that

$$T(r,F)+T(r,G)\le 2\Big\{N_2\Big(r,\frac{1}{F}\Big)+N_2(r,F)+N_2\Big(r,\frac{1}{G}\Big)+N_2(r,G)\Big\}+S(r,f)+S(r,g). \tag{3.2}$$

Together the definition of F, the first main Theorem with Remark 2.3, we have

$$\begin{aligned} N_2\Big(r,\frac{1}{F}\Big) &\le 2\overline{N}\Big(r,\frac{1}{f}\Big)+N\Big(r,\frac{1}{f(z+c)}\Big)+S(r,f)\\ &\le 3T(r,f)+S(r,f). \end{aligned} \tag{3.3}$$

Similarly,

$$N_2\Big(r,\frac{1}{G}\Big)\le 3T(r,g)+S(r,f), \tag{3.4}$$

$$N_2(r,F)\le 3T(r,f)+S(r,f), \tag{3.5}$$

$$N_2(r,G)\le 3T(r,g)+S(r,f). \tag{3.6}$$

(3.2)-(3.6) yield that

$$T(r,F)+T(r,G)\le 12(T(r,f)+T(r,g))+S(r,f)+S(r,g). \tag{3.7}$$

Then, by (2.6), (2.8) and (3.7), we obtain

$$(n-13)[T(r,f)+T(r,g)]\le S(r,f)+S(r,g), \tag{3.8}$$

which is a contradiction since $n\ge 14$. Hence, by Lemma 2.1, we have $F=(b+1)G+\frac{(a-b-1)}{bG+a-b}$, where $a\ne 0, b$ are two constants. By Lemma 2.3, we get $f(z)\equiv t_1g(z)$ or $f(z)g(z)=t_2$, for some constants t_1 and t_2 that satisfy $t_1^{n+1}=1$ and $t_2^{n+1}=1$.

4. **Proof of Theorem 1.2**

Note that

$$\begin{aligned} &\overline{N}\Big(r,\frac{1}{F-1}\Big)+\overline{N}\Big(r,\frac{1}{G-1}\Big)-N_{11}\Big(r,\frac{1}{F-1}\Big)+\frac{1}{2}\overline{N}_{(3}\Big(r,\frac{1}{F-1}\Big)\\ &\quad+\frac{1}{2}\overline{N}_{(3}\Big(r,\frac{1}{G-1}\Big)\\ &\le \frac{1}{2}N\Big(r,\frac{1}{F-1}\Big)+\frac{1}{2}N\Big(r,\frac{1}{G-1}\Big)\\ &\le \frac{1}{2}T(r,F)+\frac{1}{2}T(r,G)+S(r,f)+S(r,g). \end{aligned} \tag{4.1}$$

Then we obtain from 2.1 and 4.1

$$\begin{aligned} T(r,F)+T(r,G) &\le 2\Big\{N_2\Big(r,\frac{1}{F}\Big)+N_2(r,F)+N_2\Big(r,\frac{1}{G}\Big)+N_2(r,G)\Big\}\\ &\quad+\overline{N}_{(3}\Big(r,\frac{1}{F-1}\Big)+\overline{N}_{(3}\Big(r,\frac{1}{G-1}\Big)+S(r,f)+S(r,g) \end{aligned}$$

Obviously, combining the first main Theorem and Remark 2.3, we have

$$\begin{aligned}
\overline{N}_{(3}\Big(r,\frac{1}{F-1}\Big) &\leq \frac{1}{2}N\Big(r,\frac{F}{F'}\Big)\\
&= \frac{1}{2}N\Big(r,\frac{F'}{F}\Big)+S(r,f)\\
&\leq \frac{1}{2}\overline{N}(r,F)+\frac{1}{2}\overline{N}\Big(r,\frac{1}{F}\Big)+S(r,f)\\
&\leq \frac{1}{2}\Big[\overline{N}(r,f(z))+\overline{N}(r,f(z+c))+\overline{N}\Big(r,\frac{1}{f(z)}\Big)+\overline{N}\Big(r,\frac{1}{f(z+c)}\Big)\Big]\\
&\quad +S(r,f)\leq 2T(r,f)+S(r,f). \qquad (4.2)
\end{aligned}$$

Similarly, we obtain

$$\overline{N}_{(3}\Big(r,\frac{1}{G-1}\Big)\leq 2T(r,g)+S(r,f) \qquad (4.3)$$

Suppose that

$$\begin{aligned}
T(r,F)+T(r,G) &\leq 2\Big\{N_2\Big(r,\frac{1}{F}\Big)+N_2(r,F)+N_2\Big(r,\frac{1}{G}\Big)+N_2(r,G)\Big\}\\
&+ \overline{N}_{(3}\Big(r,\frac{1}{F-1}\Big)+\overline{N}_{(3}\Big(r,\frac{1}{G-1}\Big)+S(r,f)+S(r,g). \qquad (4.4)
\end{aligned}$$

Then we have from 2.8, 2.10, 3.3-3.6 and 4.2-4.4

$$\begin{aligned}
(n-1)T(r,f)+(n-1)T(r,g) &\leq T(r,F)+T(r,G)\\
&\leq 14T(r,f)+14T(r,g)+S(r,f)+S(r,g),
\end{aligned}$$

which is a contradiction, since $n\geq 16$. By Lemma 2.1, we obtain that $F=(b+1)G+\frac{(a-b-1)}{bG+a-b}$, where $a\neq 0,b$ are two constants. By Lemma 2.3, we get $f(z)\equiv t_1g(z)$ or $f(z)g(z)=t_2$, for some constants t_1 and t_2 that satisfy $t_1^{n+1}=1$ and $t_2^{n+1}=1$.

5. **Proof of Theorem 1.3**

Since

$$\begin{aligned}
&\overline{N}\Big(r,\frac{1}{F-1}\Big)+\overline{N}\Big(r,\frac{1}{G-1}\Big)-N_{11}\Big(r,\frac{1}{F-1}\Big)\\
\leq\;& \frac{1}{2}N\Big(r,\frac{1}{F-1}\Big)+\frac{1}{2}N\Big(r,\frac{1}{G-1}\Big)\\
\leq\;& \frac{1}{2}T(r,F)+\frac{1}{2}T(r,G)+S(r,f)+S(r,g). \qquad (5.1)
\end{aligned}$$

Then (2.1) becomes

$$\begin{aligned}
T(r,F)+T(r,G) &\leq 2\Big\{N_2\Big(r,\frac{1}{F}\Big)+N_2(r,F)+N_2\Big(r,\frac{1}{G}\Big)+N_2(r,G)\\
&\quad +\overline{N}_{(2}\Big(r,\frac{1}{F-1}\Big)+\overline{N}_{(2}\Big(r,\frac{1}{G-1}\Big)\Big\}+S(r,f)+S(r,g).
\end{aligned}$$

Combining the first main Theorem and Remark 2.3, we obtain

$$\begin{aligned}\overline{N}_{(2}\Big(r,\frac{1}{F-1}\Big) &\leq N\Big(r,\frac{F}{F'}\Big)\\ &= N\Big(r,\frac{F'}{F}\Big)+S(r,f)\\ &\leq \overline{N}(r,F)+\overline{N}\Big(r,\frac{1}{F}\Big)+S(r,f)\\ &\leq \overline{N}(r,f(z))+\overline{N}(r,f(z+c))+\overline{N}\Big(r,\frac{1}{f(z)}\Big)+\overline{N}\Big(r,\frac{1}{f(z+c)}\Big)\\ &\quad +S(r,f)\\ &\leq 4T(r,f)+S(r,f). \qquad (5.2)\end{aligned}$$

Similarly, we get

$$\overline{N}_{(2}\Big(r,\frac{1}{G-1}\Big)\leq 4T(r,g)+S(r,f). \qquad (5.3)$$

Suppose that

$$\begin{aligned}T(r,F)+T(r,G) \quad &\leq \quad 2\Big\{N_2\Big(r,\frac{1}{F}\Big)+N_2(r,F)+N_2\Big(r,\frac{1}{G}\Big)+N_2(r,G)\\ &+ \quad \overline{N}_{(2}\Big(r,\frac{1}{F-1}\Big)+\overline{N}_{(2}\Big(r,\frac{1}{G-1}\Big)\Big\}+S(r,f)+S(r,g). \qquad (5.4)\end{aligned}$$

Then we obtain from 2.8, 2.10, 3.3-3.6 and 5.2-5.4

$$\begin{aligned}(n-1)T(r,f)+(n-1)T(r,g) \quad &\leq \quad T(r,F)+T(r,G)\\ &\leq \quad 20T(r,f)+20T(r,g)+S(r,f)+S(r,g),\end{aligned}$$

which is impossible, since $n \geq 22$. By Lemma 2.1, we obtain that $F = (b+1)G+\frac{(a-b-1)}{bG+a-b}$, where $a \neq 0, b$ are two constants. By Lemma 2.3, we get $f(z) \equiv t_1 g(z)$ or $f(z)g(z) = t_2$, for some constants t_1 and t_2 that satisfy $t_1^{n+1} = 1$ and $t_2^{n+1} = 1$.

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