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APPROXIMATE MULTI-ADDITIVE MAPPINGS IN 2-BANACH SPACES

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ABSTRACT. A mapping $f : V^n \longrightarrow W$, where V is a commutative semigroup, W is a linear space and n is a positive integer, is called multi-additive if it is additive in each variable. In this paper we prove the Hyers-Ulam stability of multi-additive mappings in 2-Banach spaces. The corollaries from our main results correct some outcomes from [W.-G. Park, Approximate additive mappings in 2-Banach spaces and related topics, *J. Math. Anal. Appl.* **376** (2011) 193–202].
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MSC(2010): Primary: 39B82; Secondary: 39B52, 41A65.

1. **Introduction**

Let us recall that a mapping $f : V^n \longrightarrow W$, where V is a commutative semigroup, W is a linear space and $n \in \mathbb{N}$ (here and subsequently, $\mathbb{N}$ stands for the set of all positive integers), is called *multi-additive* or *n-additive* if it is additive (satisfies Cauchy's functional equation) in each variable, that is

$$f(x_1, \dots, x_{i-1}, x_i + x_i', x_{i+1}, \dots, x_n) = f(x_1, \dots, x_n)$$
$$+f(x_1, \dots, x_{i-1}, x_i', x_{i+1}, \dots, x_n),$$
$$i \in \{1, \dots, n\},\ x_1, \dots, x_{i-1}, x_i, x_i', x_{i+1}, \dots, x_n \in V.$$

Some basic facts on such mappings can be found for instance in [23], where their application to the representation of polynomial functions is also presented (see also [24, 25]).

The below lemma from [10] reduces the original system of n Cauchy equations to a single functional equation.

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Lemma 1.1. *Let V be a commutative semigroup with an identity element and W be a linear space. A mapping $f : V^n \longrightarrow W$ is multi-additive if and only if it satisfies the equation*

$$f(x_{11}+x_{12},...,x_{n1}+x_{n2}) = \sum_{i_1,\dots,i_n\in\{1,2\}} f(x_{1i_1},...,x_{ni_n}), \tag{1.1}$$

$$(x_{11},...,x_{n1}),\ (x_{12},...,x_{n2}) \in V^n.$$

Let us also recall (see for instance [9,16,26]) that by a *linear 2-normed space* we mean a pair $(X, \|\cdot,\cdot\|)$ such that X is at least a two-dimensional real linear space and $\|\cdot,\cdot\| : X \times X \longrightarrow \mathbb{R}$ is a function (called the *2-norm*) satisfying the following conditions:

$$\|x,y\| = 0 \quad \text{if and only if} \quad x \text{ and } y \text{ are linearly dependent}, \qquad x,y \in X,$$

$$\|x,y\| = \|y,x\|, \qquad x,y \in X,$$

$$\|\alpha x,y\| = |\alpha|\|x,y\|, \qquad \alpha \in \mathbb{R},\ x,y \in X$$

and

$$\|x,y+z\| \le \|x,y\| + \|x,z\|, \qquad x,y,z \in X.$$

If $(X, \|\cdot,\cdot\|)$ is a linear 2-normed space, then the function $\|\cdot,\cdot\|$ is non-negative and

$$\|\sum_{i=1}^{n} x_i, y\| \le \sum_{i=1}^{n} \|x_i, y\|, \qquad x_i, y \in X,\ i \in \{1,...,n\},\ n \in \mathbb{N}. \tag{1.2}$$

A sequence $(x_n)_{n\in\mathbb{N}}$ of elements of a linear 2-normed space $(X, \|\cdot,\cdot\|)$ is called a *Cauchy sequence* if there are linearly independent $y, z \in X$ such that

$$\lim_{n,m\longrightarrow\infty} \|x_n - x_m, y\| = 0 = \lim_{n,m\longrightarrow\infty} \|x_n - x_m, z\|,$$

whereas $(x_n)_{n\in\mathbb{N}}$ is said to be *convergent* if there exists an $x \in X$ (called the *limit* of this sequence) with

$$\lim_{n\longrightarrow\infty} \|x_n - x, y\| = 0, \qquad y \in X.$$

Every convergent sequence has exactly one limit. If x is the limit of the sequence $(x_n)_{n\in\mathbb{N}}$, then we write $\lim_{n\longrightarrow\infty} x_n = x$. For any convergent sequences $(x_n)_{n\in\mathbb{N}}$ and $(y_n)_{n\in\mathbb{N}}$ of elements of X, the sequence $(x_n+y_n)_{n\in\mathbb{N}}$ is convergent and

$$\lim_{n\longrightarrow\infty}(x_n + y_n) = \lim_{n\longrightarrow\infty} x_n + \lim_{n\longrightarrow\infty} y_n.$$

If, moreover, $(\alpha_n)_{n\in\mathbb{N}}$ is a convergent sequence of real numbers, then the sequence $(\alpha_n \cdot x_n)_{n\in\mathbb{N}}$ is also convergent and

$$\lim_{n\longrightarrow\infty}(\alpha_n \cdot x_n) = \lim_{n\longrightarrow\infty} \alpha_n \cdot \lim_{n\longrightarrow\infty} x_n.$$

A linear 2-normed space in which every Cauchy sequence is convergent is called a *2-Banach space*.

In what follows, we shall also use the following lemmas from [26].

Lemma 1.2. *Let $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space and $x \in X$. If*

$$\|x, y\| = 0, \qquad y \in X,$$

then $x = 0$.

Lemma 1.3. *Let $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space and $(x_n)_{n\in\mathbb{N}}$ be a convergent sequence of elements of X. Then*

$$\lim_{n\longrightarrow\infty} \|x_n, y\| = \|\lim_{n\longrightarrow\infty} x_n, y\|, \qquad y \in X.$$

Speaking of the stability of a functional equation we follow the question raised in 1940 by S.M. Ulam: "when is it true that the solution of an equation differing slightly from a given one, must of necessity be close to the solution of the given equation?". The first answer (in the case of Cauchy's functional equation in Banach spaces) to Ulam's question was given by D.H. Hyers (see [21]). After his result a great number of papers (see for instance [2,3,6,7,13,15, 20,22,27,28] and the references given there) on the subject have been published, generalizing Ulam's problem and Hyers's theorem in various directions and to other functional equations (as the words "differing slightly" and "be close" may have various meanings, different kinds of stability can be dealt with). In particular, the stability of multi-additive mapings was investigated in [1,4,10–12,17,29] (see also [8,23]), whereas the stability of some functional equations in 2-Banach spaces was studied for example in [14,18,26].

In this paper we deal with the Hyers-Ulam stability, in the spirit of D.G. Bourgin (see [5]) and P. Găvrută (see [19]), of equation (1.1) in 2-Banach spaces. The corollaries from our main results correct some outcomes from [26].

2. Results

In this section, we prove the Hyers-Ulam stability of equation (1.1).

Theorem 2.1. *Let V be a commutative semigroup with an identity element and W be a 2-Banach space. Assume also that $\varphi : V^{2n} \longrightarrow [0,\infty)$ is a function such that for any $(x_{11}, x_{12}, ..., x_{n1}, x_{n2}) \in V^{2n}$ we have*

$$\widetilde{\varphi}(x_{11}, x_{12}, ..., x_{n1}, x_{n2}) := \sum_{j=0}^{\infty} \frac{1}{2^{n(j+1)}} \varphi(2^j x_{11}, 2^j x_{12}, ..., 2^j x_{n1}, 2^j x_{n2}) < \infty. \tag{2.1}$$

If $f : V^n \longrightarrow W$ is a mapping satisfying

$$\|f(x_{11} + x_{12}, ..., x_{n1} + x_{n2}) - \sum_{i_1,...,i_n\in\{1,2\}} f(x_{1i_1}, ..., x_{ni_n}), y\| \tag{2.2}$$

$$\leq \varphi(x_{11}, x_{12}, ..., x_{n1}, x_{n2}), \qquad (x_{11}, x_{12}, ..., x_{n1}, x_{n2}) \in V^{2n},\ y \in W,$$

then there exists a unique multi-additive mapping $F : V^n \longrightarrow W$ *for which*

$$\|f(x_{11}, ..., x_{n1}) - F(x_{11}, ..., x_{n1}), y\| \leq \widetilde{\varphi}(x_{11}, x_{11}, ..., x_{n1}, x_{n1}), \tag{2.3}$$

$$(x_{11}, ..., x_{n1}) \in V^n,\ y \in W.$$

The mapping F *is given by*

$$F(x_{11}, ..., x_{n1}) \tag{2.4}$$

$$:= \lim_{j \longrightarrow \infty} \frac{1}{2^{nj}} f(2^j x_{11}, ..., 2^j x_{n1}), \qquad (x_{11}, ..., x_{n1}) \in V^n.$$

Proof. Fix $(x_{11}, ..., x_{n1}) \in V^n$, $y \in W$ and $j \in \mathbb{N} \cup \{0\}$. Putting $x_{i2} := x_{i1}$ for $i \in \{1, ..., n\}$ in (2.2) we get

$$\|f(2x_{11}, ..., 2x_{n1}) - 2^n f(x_{11}, ..., x_{n1}), y\| \leq \varphi(x_{11}, x_{11}, ..., x_{n1}, x_{n1}).$$

Dividing both sides of the above inequality by $2^{n(j+1)}$ and replacing x_{i1} by $2^j x_{i1}$ for $i \in \{1, ..., n\}$ we see that

$$\|\frac{1}{2^{n(j+1)}} f(2^{j+1} x_{11}, ..., 2^{j+1} x_{n1}) - \frac{1}{2^{nj}} f(2^j x_{11}, ..., 2^j x_{n1}), y\|$$

$$\leq \frac{1}{2^{n(j+1)}} \varphi(2^j x_{11}, 2^j x_{11}, ..., 2^j x_{n1}, 2^j x_{n1}),$$

whence by (1.2) for any non-negative integers l and m with $l < m$ we obtain

$$\|\frac{1}{2^{nm}} f(2^m x_{11}, ..., 2^m x_{n1}) - \frac{1}{2^{nl}} f(2^l x_{11}, ..., 2^l x_{n1}), y\| \tag{2.5}$$

$$\leq \sum_{j=l}^{m-1} \frac{1}{2^{n(j+1)}} \varphi(2^j x_{11}, 2^j x_{11}, ..., 2^j x_{n1}, 2^j x_{n1}).$$

Therefore from (2.1) it follows that $(\frac{1}{2^{nj}} f(2^j x_{11}, ..., 2^j x_{n1}))_{j \in \mathbb{N}}$ is a Cauchy sequence. Since W is a 2-Banach space, this sequence is convergent and we define $F : V^n \longrightarrow W$ by (2.4). Putting $l = 0$, letting $m \longrightarrow \infty$ in (2.5) and using (2.1) and Lemma 1.3 we see that (2.3) holds.

Next, fix also $(x_{12}, ..., x_{n2}) \in V^n$ and note that according to (2.2) we have

$$\|\frac{1}{2^{nj}} f(2^j (x_{11} + x_{12}), ..., 2^j (x_{n1} + x_{n2}))$$

$$- \sum_{i_1, ..., i_n \in \{1,2\}} \frac{1}{2^{nj}} f(2^j x_{1i_1}, ..., 2^j x_{ni_n}), y\|$$

$$\leq \frac{1}{2^{nj}} \varphi(2^j x_{11}, 2^j x_{12}, ..., 2^j x_{n1}, 2^j x_{n2}).$$

Letting $j \longrightarrow \infty$ in the above inequality and using (2.1) and Lemma 1.3 we see that

$$\|F(x_{11}+x_{12},...,x_{n1}+x_{n2}) - \sum_{i_1,...,i_n\in\{1,2\}} F(x_{1i_1},...,x_{ni_n}), y\| = 0, \tag{2.6}$$

and therefore from Lemma 1.2 it follows that the mapping F satisfies equation (1.1). Lemma 1.1 now shows that F is multi-additive.

Finally, assume that $F' : V^n \longrightarrow W$ is another multi-additive mapping satisfying (2.3) and fix a $k \in \mathbb{N} \cup \{0\}$. Then, using the multi-additivity of F and F', (1.2), (2.3) and (2.1), we have

$$\begin{aligned} &\|F(x_{11},...,x_{n1}) - F'(x_{11},...,x_{n1}), y\| \\ &= \|\frac{1}{2^{nk}}F(2^k x_{11},...,2^k x_{n1}) - \frac{1}{2^{nk}}F'(2^k x_{11},...,2^k x_{n1}), y\| \\ &\le \|\frac{1}{2^{nk}}F(2^k x_{11},...,2^k x_{n1}) - \frac{1}{2^{nk}}f(2^k x_{11},...,2^k x_{n1}), y\| \\ &+\|\frac{1}{2^{nk}}f(2^k x_{11},...,2^k x_{n1}) - \frac{1}{2^{nk}}F'(2^k x_{11},...,2^k x_{n1}), y\| \\ &\le \frac{2}{2^{nk}}\widetilde{\varphi}(2^k x_{11}, 2^k x_{11},...,2^k x_{n1}, 2^k x_{n1}) \\ &= 2\cdot\sum_{j=k}^{\infty}\frac{1}{2^{n(j+1)}}\varphi(2^j x_{11}, 2^j x_{11},...,2^j x_{n1}, 2^j x_{n1}), \end{aligned}$$

whence letting $k \longrightarrow \infty$ we obtain

$$\|F(x_{11},...,x_{n1}) - F'(x_{11},...,x_{n1}), y\| = 0, \tag{2.7}$$

and Lemma 1.2 now shows that $F = F'$. □

Putting in Theorem 2.1, $n := 1$ and

$$\varphi(x_{11}, x_{12}) := \theta\|x_{11}\|^p\|x_{12}\|^q, \qquad x_1, x_2 \in V \tag{2.8}$$

we obtain the following corollary which corrects Theorem 2.1 from [26].

Corollary 2.2. *Let V be a normed linear space and W be a 2-Banach space. Assume also that $\theta \in [0,\infty)$ and $p, q \in (0,\infty)$ are such that $p + q < 1$. If $f : V \longrightarrow W$ is a mapping satisfying*

$$\begin{aligned} &\|f(x_1+x_2) - f(x_1) - f(x_2), y\| \\ &\le \theta\|x_1\|^p\|x_2\|^q, \qquad x_1, x_2 \in V,\ y \in W, \end{aligned} \tag{2.9}$$

then there exists a unique additive mapping $F : V \longrightarrow W$ for which

$$\|f(x) - F(x), y\| \le \frac{\theta\|x\|^{p+q}}{2-2^{p+q}}, \qquad x \in V,\ y \in W.$$

Theorem 2.3. *Let V be a real linear space and W be a 2-Banach space. Assume also that $\varphi : V^{2n} \longrightarrow [0,\infty)$ is a function such that for any $(x_{11}, x_{12}, ..., x_{n1}, x_{n2}) \in V^{2n}$ we have*

$$\begin{aligned}(2.10)\qquad &\widetilde{\varphi}(x_{11}, x_{12}, ..., x_{n1}, x_{n2})\\ &:= \sum_{j=0}^{\infty} 2^{nj}\varphi(\frac{x_{11}}{2^{j+1}}, \frac{x_{12}}{2^{j+1}}, ..., \frac{x_{n1}}{2^{j+1}}, \frac{x_{n2}}{2^{j+1}}) < \infty.\end{aligned}$$

If $f : V^n \longrightarrow W$ satisfies (2.2), then there exists a unique multi-additive mapping $F : V^n \longrightarrow W$ for which (2.3) holds. The mapping F is given by

$$\begin{aligned}(2.11)\qquad &F(x_{11}, ..., x_{n1})\\ &:= \lim_{j\longrightarrow\infty} 2^{nj} f(\frac{x_{11}}{2^j}, ..., \frac{x_{n1}}{2^j}), \qquad (x_{11}, ..., x_{n1}) \in V^n.\end{aligned}$$

Proof. Fix $(x_{11}, ..., x_{n1}) \in V^n$, $y \in W$ and $j \in \mathbb{N} \cup \{0\}$. Since

$$\begin{aligned}&\|2^{nj} f(\frac{x_{11}}{2^j}, ..., \frac{x_{n1}}{2^j}) - 2^{n(j+1)} f(\frac{x_{11}}{2^{j+1}}, ..., \frac{x_{n1}}{2^{j+1}}), y\|\\ &\leq 2^{nj}\varphi(\frac{x_{11}}{2^{j+1}}, \frac{x_{11}}{2^{j+1}}, ..., \frac{x_{n1}}{2^{j+1}}, \frac{x_{n1}}{2^{j+1}}),\end{aligned}$$

for any non-negative integers l and m with $l < m$ we have

$$\begin{aligned}(2.12)\qquad &\|2^{nm} f(\frac{x_{11}}{2^m}, ..., \frac{x_{n1}}{2^m}) - 2^{nl} f(\frac{x_{11}}{2^l}, ..., \frac{x_{n1}}{2^l}), y\|\\ &\leq \sum_{j=l}^{m-1} 2^{nj}\varphi(\frac{x_{11}}{2^{j+1}}, \frac{x_{11}}{2^{j+1}}, ..., \frac{x_{n1}}{2^{j+1}}, \frac{x_{n1}}{2^{j+1}}),\end{aligned}$$

and from (2.10) it follows that $(2^{nj} f(\frac{x_{11}}{2^j}, ..., \frac{x_{n1}}{2^j}))_{j\in\mathbb{N}}$ is a Cauchy sequence. Since W is a 2-Banach space, this sequence is convergent and we define $F : V^n \longrightarrow W$ by (2.11). Putting $l = 0$, letting $m \longrightarrow \infty$ in (2.12) and using (2.10) and Lemma 1.3 we see that (2.3) holds.

Next, fix also $(x_{12}, ..., x_{n2}) \in V^n$ and note that according to (2.2) we have

$$\begin{aligned}&\|2^{nj} f(\frac{x_{11}+x_{12}}{2^j}, ..., \frac{x_{n1}+x_{n2}}{2^j}) - \sum_{i_1,...,i_n\in\{1,2\}} 2^{nj} f(\frac{x_{1i_1}}{2^j}, ..., \frac{x_{ni_n}}{2^j}), y\|\\ &\leq 2^{nj}\varphi(\frac{x_{11}}{2^j}, \frac{x_{12}}{2^j}, ..., \frac{x_{n1}}{2^j}, \frac{x_{n2}}{2^j}).\end{aligned}$$

Letting $j \longrightarrow \infty$ in the above inequality and using (2.10) and Lemma 1.3 we see that (2.6) holds. Lemmas 1.2 and 1.1 now show that F is multi-additive.

Finally, assume that $F' : V^n \longrightarrow W$ is another multi-additive mapping satisfying (2.3) and fix a $k \in \mathbb{N} \cup \{0\}$. Then, using the multi-additivity of F and F', (1.2), (2.3) and (2.10), we have

$$\|F(x_{11}, ..., x_{n1}) - F'(x_{11}, ..., x_{n1}), y\|$$

$$\le \|2^{nk}F(\frac{x_{11}}{2^k},...,\frac{x_{n1}}{2^k}) - 2^{nk}f(\frac{x_{11}}{2^k},...,\frac{x_{n1}}{2^k}),y\|$$
$$+\|2^{nk}f(\frac{x_{11}}{2^k},...,\frac{x_{n1}}{2^k}) - 2^{nk}F'(\frac{x_{11}}{2^k},...,\frac{x_{n1}}{2^k}),y\|$$
$$\le 2\cdot\sum_{j=k}^{\infty}2^{nj}\varphi(\frac{x_{11}}{2^{j+1}},\frac{x_{11}}{2^{j+1}},...,\frac{x_{n1}}{2^{j+1}},\frac{x_{n1}}{2^{j+1}}),$$

whence letting $k \longrightarrow \infty$ we get (2.7), and Lemma 1.2 now shows that $F = F'$. □

Putting $n := 1$ and defining φ by (2.8) in Theorem 2.3 we obtain the following corollary which corrects Theorem 2.2 from [26].

Corollary 2.4. *Let V be a real normed linear space and W be a 2-Banach space. Assume also that $\theta \in [0,\infty)$ and $p, q \in (0,\infty)$ are such that $p+q > 1$. If $f : V \longrightarrow W$ is a mapping satisfying* (2.9)*, then there exists a unique additive mapping $F : V \longrightarrow W$ for which*

$$\|f(x) - F(x), y\| \le \frac{\theta\|x\|^{p+q}}{2^{p+q}-2}, \quad x \in V,\ y \in W.$$

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