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ON THE FISCHER-CLIFFORD MATRICES OF THE NON-SPLIT EXTENSION $2^{6\cdot}G_2(2)$

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(Communicated by Jamshid Moori)

Abstract. The group $2^{6\cdot}G_2(2)$ is a maximal subgroup of the Rudvalis group Ru of index 188500 and has order $774144 = 2^{12}.3^3.7$. In this paper, we construct the character table of the group $2^{6\cdot}G_2(2)$ by using the technique of Fischer-Clifford matrices.

Keywords: Coset analysis, Fischer-Clifford matrices, permutation character.

MSC(2010): Primary: 20C15; Secondary: 20C40.

1. Introduction

The Rudvalis group Ru, founded by Arunas Rudvalis [22] and constructed by Conway and Wales [10], is a sporadic simple group of order $145926144000 = 2^{14}.3^3.5^3.7.13.29$. Ru is one of the six sporadic simple groups known as "pariah groups" as they are not found within the Monster group [14]. Wilson [27] found that the group Ru has 14 conjugacy classes of maximal subgroups as listed in the Atlas of Finite Groups [11]. The non-split extension $2^{6}{\cdot}G_2(2)$, is the second largest maximal subgroup of Ru of index 188500.

Let $\overline{G}= N{\cdot}G$ be a non-split extension of $N \cong 2^6$, the vector space of dimension 6 over $GF(2)$, by $G \cong G_2(2)$ (the adjoint Chevalley group of type G_2 over $GF(2)$). In the present paper, we construct the character table of $2^{6\cdot}G_2(2)$ using the method of the Fischer-Clifford matrices. This method was presented by Bernd Fischer [12] for the construction of the character tables of finite group extensions and extensively used by Moori and his research team (see [1–7, 19, 24, 25] and [28]). Pahlings in [20] also used Fischer-Clifford theory to compute the character table of the non-split extension $2^{1+22\cdot}Co_2$. The method involves the construction of a non-singular matrix $M(g)$, called a

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Fischer-Clifford matrix, for each conjugacy class $[g]$ of $\overline{G}/N \cong G_2(2)$. The character table of $\overline{G}$ can be constructed from these Fischer-Clifford matrices and the character tables of certain subgroups of $G_2(2)$, called inertia factor groups. Our computations were done in the computer algebra systems MAGMA [8] and GAP [23]. We adopt the notation used in the ATLAS [11] for conjugacy classes and permutation characters.

2. **Theory of Fischer-Clifford matrices**

We will see later in Section 6 that for the group $\overline{G} = 2^{6\cdot}G_2$ under discussion in this paper, the projective characters of the inertia factor groups of $\overline{G}$ are not involved in the construction of the character table of $\overline{G}$. Only the ordinary characters of the inertia factors groups are used and therefore we only need the special case of Fischer-Clifford theory [1] (Chapter 5) to compute the character table of $\overline{G}$. In this section, we will give a brief theoretical background of this technique which is covered extensively in [1], [12], [19], [24] [25] and [28].

Let $\overline{G} = N{\cdot}G$ be an extension of N by G and $\theta \in Irr(N)$, where $Irr(N)$ denotes the irreducible characters of N. Define θ^g by $\theta^g(n) = \theta(gng^{-1})$ for $g \in \overline{G}$ and $n \in N$ and $\theta^g \in Irr(N)$. Let $\overline{H} = \left\{x \in \overline{G} | \theta^x = \theta\right\} = I_{\overline{G}}(\theta)$ be the inertia group of θ in $\overline{G}$ then N is normal in $\overline{H}$. We say that θ is extendible to $\overline{H}$ if there exists $\phi \in Irr(\overline{H})$ such that $\phi \downarrow_N = \theta$. If θ is extendible to $\overline{H}$, then by Gallagher [16], we have

$$\left\{\phi | \phi \in Irr(\overline{H}), < \phi \downarrow_N, \theta > \neq 0\right\} = \left\{\beta\phi | \beta \in Irr(\overline{H}/N)\right\}.$$

Let $\overline{G}$ have the property that every irreducible character of N can be extended to its inertia group. Now let $\theta_1 = 1_N, \theta_2, \cdots, \theta_t$ be representatives of the orbits of $\overline{G}$ on $Irr(N)$, $\overline{H}_i = I_{\overline{G}}(\phi_i)$, $1 \leq i \leq t$, $\phi_i \in Irr(\overline{H}_i)$ be an extension of θ_i to $\overline{H}_i$ and $\beta \in Irr(\overline{H}_i)$ such that $N \subseteq ker(\beta)$. Then it can be shown that

$$Irr(\overline{G}) = \bigcup_{i=1}^{t} \{(\beta\,\phi_i)^{\overline{G}} \,|\, \beta \in Irr(\overline{H}_i), N \subseteq ker(\beta)\} = \bigcup_{i=1}^{t} \{(\beta\,\phi_i)^{\overline{G}} \,|\, \beta \in Irr(\overline{H}_i/N)\}$$

Hence the irreducible characters of $\overline{G}$ will be divided into blocks, where each block corresponds to an inertia group $\overline{H}_i$.

Let H_i be the inertia factor group and ϕ_i be an extension of θ_i to $\overline{H}_i$. Take $\theta_1 = 1_N$ as the identity character of N, then $\overline{H}_1 = \overline{G}$ and $H_1 \cong G$. Let $X(g) = \{x_1, x_2, \cdots, x_{c(g)}\}$ be a set of representatives of the conjugacy classes of $\overline{G}$ from the coset $N\overline{g}$ whose images under the natural homomorphism $\overline{G} \longrightarrow G$ are in $[g]$ and we take $x_1 = \bar{g}$. We define

$$R(g) = \{(i, y_k) \,|\, 1 \leq i \leq t, H_i \cap [g] \neq \emptyset, 1 \leq k \leq r\}$$

and we note that y_k runs over representatives of the conjugacy classes of elements of H_i which fuse into $[g]$ in G. Let $\{y_{l_k}\}$ be the representatives of conjugacy classes of $\overline{H}_i$ that contain y_k. Then we define the Fischer-Clifford matrix $M(g)$ by $M(g) = (a^j_{(i,y_k)})$, where

$$a^j_{(i,y_k)} = \sum_l{}' \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H_i}}(y_{l_k})|}\phi_i(y_{l_k}),$$

with columns indexed by $X(g)$ and rows indexed by $R(g)$ and where $\sum_l'$ is the summation over all l for which $y_{l_k} \sim x_j$ in $\overline{G}$. Then the partial character table of $\overline{G}$ on the classes $\{x_1, x_2, \cdots, x_{c(g)}\}$ is given by $\begin{bmatrix} C_1(g)\, M_1(g) \\ C_2(g)\, M_2(g) \\ \vdots \\ C_t(g)\, M_t(g) \end{bmatrix}$ where the Fischer-Clifford matrix $M(g) = \begin{bmatrix} M_1(g) \\ M_2(g) \\ \vdots \\ M_t(g) \end{bmatrix}$ is divided into blocks $M_i(g)$ with each block corresponding to an inertia group $\overline{H}_i$ and $C_i(g)$ is the partial character table of H_i consisting of the columns corresponding to the classes that fuse into $[g]$ in G. Hence the full character table of $\overline{G}$ will be $\begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_t \end{bmatrix}$, where $\Delta_i = [C_i(1)M_i(1)|C_i(g_2)M_i(g_2)|...|C_i(g_k)M_i(g_k)]$ with $\{1, g_1, g_2, ..., g_k\}$ the representatives of conjugacy classes of G. We can also observe that $|Irr(\overline{G})|$ $= |Irr(H_1)| + |Irr(H_2)| + ... + |Irr(H_t)|$.

Let $x_j \in X(g)$ and define $m_j = [C_{\overline{g}}{:}C_{\overline{G}}(x_j)]$, where $C_{\overline{g}} = \{x \in \overline{G} | x(N\overline{g}) = (N\overline{g})x\}$ is the set stabilizer of $N\overline{g}$ in $\overline{G}$ under the action by conjugation of $\overline{G}$ on $N\overline{g}$. Hence $C_{\overline{g}} \leq \overline{G}$ and it can be shown that N is normal in $C_{\overline{g}}$. The Fischer-Clifford matrix $M(g)$ is partitioned row–wise into blocks, where each block corresponds to an inertia group. The columns of $M(g)$ are indexed by $X(g)$ and for each $x_j \in X(g)$, at the top of the columns of $M(g)$, we write $|C_{\overline{G}}(x_j)|$ and at the bottom we write m_j. The rows of $M(g)$ are indexed by $R(g)$ and on the left of each row we write $|C_{H_i}(y_k)|$, where y_k fuses into $[g]$ in G. Then in general we can write $M(g)$ with corresponding weights for rows and columns as follows, where blocks corresponding to the inertia groups are separated by horizontal lines.

$$
\begin{array}{c}
\\
|C_G(g)| \\
|C_{H_2}(y_1)| \\
|C_{H_2}(y_2)| \\
\vdots \\
|C_{H_i}(y_1)| \\
|C_{H_i}(y_2)| \\
\vdots \\
|C_{H_t}(y_1)| \\
|C_{H_t}(y_2)| \\
\vdots \\
\\
\end{array}
\begin{array}{cccc}
|C_{\overline{G}}(x_1)| & |C_{\overline{G}}(x_2)| & \cdots & |C_{\overline{G}}(x_{c(g)})| \\
\hline
a^1_{(1,g)} & a^2_{(1,g)} & \cdots & a^{c(g)}_{(1,g)} \\
\hline
a^1_{(2,y_1)} & a^2_{(2,y_1)} & \cdots & a^{c(g)}_{(2,y_1)} \\
a^1_{(2,y_2)} & a^2_{(2,y_2)} & \cdots & a^{c(g)}_{(2,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\
\hline
a^1_{(i,y_1)} & a^2_{(i,y_1)} & \cdots & a^{c(g)}_{(i,y_1)} \\
a^1_{(i,y_2)} & a^2_{(i,y_2)} & \cdots & a^{c(g)}_{(i,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\
\hline
a^1_{(t,y_1)} & a^2_{(t,y_1)} & \cdots & a^{c(g)}_{(t,y_1)} \\
a^1_{(t,y_2)} & a^2_{(t,y_2)} & \cdots & a^{c(g)}_{(t,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\
\hline
m_1 & m_2 & \cdots & m_{c(g)}
\end{array}
$$

The Fischer-Clifford matrix $M(g)$ satisfies the following properties [19]:

(a) $a^j_{(1,g)} = 1$ for all $j = \{1, 2, .., c(g)\}$.

(b)$|X(g)| = |R(g)|$.

(c) $\sum_{j=1}^{c(g)} m_j \, a^j_{(i,y_k)} \, \overline{a^j_{(i',y'_k)}} = \delta_{(i,y_k),(i',y'_k)} \frac{|C_G(g)|}{|C_{H_i}(y_k)|} \, |N|$.

(d) $\sum_{(i,y_k)\in R(g)} a^j_{(i,y_k)} \, \overline{a^{j'}_{(i,y_k)}} \, |C_{H_i}(y_k)| = \delta_{jj'} \, |C_{\overline{G}}(x_j)|$.

(e) $M(g)$ is square and nonsingular.

If N is elementary abelian, then we obtain the following additional properties of $M(g)$:

(f) $a^1_{(i,y_k)} = \frac{|C_G(g)|}{|C_{H_i}(y_k)|}$.

(g) $|a^1_{(i,y_k)}| \geq |a^j_{(i,y_k)}|$.

(h) $a^j_{(i,y_k)} \equiv a^1_{(i,y_k)} (\mathrm{mod} p)$, if $|N| = p^n$, for p a prime and $n \in \mathbb{N}$.

3. The group $\overline{G} = 2^{6\cdot} G_2(2)$

In the construction of $2^{6\cdot}G_2(2)$, $G_2(2)$ acts on the elementary abelian group 2^6. The action on 2^6 is multiplication on the right of the six dimensional row vector space $N = 2^6$. This requires $2^6{\cdot}G_2(2)$ to be represented as a matrix group of dimension six over a finite field of two elements. In this section we will construct $G_2(2)$ as a 6×6 matrix group representation over $GF(2)$. All our computations were done in MAGMA .

We represented the Rudvalis group Ru and $M = 2^{6\cdot} G_2(2)$ as permutations on 4060 points in MAGMA, by making use of Wilson's online ATLAS of Group

Representations [26]. The command "Ismaximal(Ru, M)" in MAGMA [8] confirms that M is a maximal subgroup of Ru. By checking all the indices of maximal subgroups of Ru in the Atlas [11], we deduce that the maximal subgroup $\overline{G}$ of index 188500 in Ru is indeed our group M. Using the commands "a,b:= ChiefSeries(M)", "N:= a[3]", "IsNormal(M, N)", "IsElementaryAbelian(N)" and "Complements(M, N)" in MAGMA, we are able to represent $2^6 \cong N$ as a permutation group on 4060 points inside Ru. We obtain the group $M/N \cong G_2(2)$, represented as a matrix group of dimension 6 over $GF(2)$, as the result of the action of the generators of M on the generators of $a[3]$ by conjugation. The generators g_1 and g_2 of orders 2 and 3, respectively, for the group $G \cong G_2(2)$ are as follows:

$$g_1 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad g_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

4. The action of $G_2(2)$ on 2^6 and $Irr(2^6)$

When $G_2(2)$ acts on the conjugacy classes of elements of 2^6, we obtain two orbits of lengths 1 and 63. The orbits have the representatives $(0,0,0,0,0,0)$ and $(1,0,0,0,0,0)$ with corresponding point stabilizers $G_2(2)$ and $4^2{:}D_{12}$ of orders 12096 and 192, respectively. Let $\chi(G_2(2)|2^6)$ be the permutation character of $G_2(2)$ on 2^6. Then, from methods that were developed by Mpono [19], we obtain that $\chi(G_2(2)|2^6) = 2 \times 1a + 14a + 21a + 27b$, which is the sum of the identity characters of the point stabilizers induced to $G_2(2)$. Therefore $\chi(G_2(2)|2^6)$ will give the number k of points of 2^6 fixed by each $g \in G_2(2)$ such that $k = 2^n$, where $n \in \{0,1,2,3,4,5,6\}$. These values of k are found in Table 2.

Since G has two orbits on N of lengths 1 and 63 respectively, then by Brauer's Theorem [13] G acts on $Irr(N)$ with the same number of orbits. Hence the lengths of these orbits will also be 1 and 63 with corresponding point stabilizers H_1 and H_2 as subgroups of G such that $[G : H_1]$ =1 and $[G : H_2]$ = 63. By checking the indices of all the maximal subgroups of $G_2(2) \cong U_3(3){:}2$ in the ATLAS [11], we found there are two maximal subgroups $M_1 = 4^2{:}D_{12}$ and $M_2 = (4^{\cdot}S_4){:}2$ with indices of 63. M_1 and M_2 have 14 and 17 conjugacy classes of elements, respectively. Let T be the matrix group of dimension 6 over $GF(2)$ formed by the transpose of the generators of $G_2(2)$. The action of T on the classes of $N = 2^6$ is the equivalent of $G_2(2)$ acting on $Irr(N)$. The action of T on N has orbits of lengths 1 and 63 with point stabilizers T and $4^2{:}D_{12}$, respectively. Therefore, the orbits of lengths 1 and 63 resulted from the action of $G_2(2)$ on $Irr(G)$ will also have point stabilizers $H_1 = G_2(2) \cong T$ and

$H_2 \cong 4^2{:}D_{12}$, respectively. Hence the action of $\overline{G}$ on $Irr(N)$ determined two inertia groups $\overline{H}_i = 2^{6\cdot}H_i$ in $2^{6\cdot}G_2(2)$, $i \in \{1, 2\}$, with corresponding inertia factor groups $H_1 = G_2(2)$ and $H_2 = 4^2{:}D_{12}$.

We represented the group H_2 as permutations on 63 points in MAGMA (by making use of Wilson's online ATLAS of Group Representations [26]) as follows:

H_2:= PermutationGroup< 63|(1, 62, 52, 30, 61, 4, 2, 17)(3, 27)(5, 16, 33, 31, 63, 32, 13, 14)(6, 19, 8, 21, 38, 18, 44, 49)(7, 54, 55, 53, 43, 59, 9, 46)(10, 22, 12, 25, 45, 15, 48, 41)(11, 29, 28, 57, 35, 26, 20, 56)(24, 39, 36, 58)(34, 37, 50, 47)(40, 51, 42, 60), (1, 6)(2, 35, 55, 8, 62, 49)(3, 47, 50)(4, 28, 59, 11, 30, 26)(5, 14, 36, 25, 12, 24)(7, 20, 52, 19, 17, 44)(9, 29, 43, 21, 54, 18)(10, 40, 15, 16, 58, 13)(22, 32, 42, 33, 45, 39)(27, 37, 34)(31, 51, 41, 48, 60, 63)(38, 53, 57, 61, 56, 46)>;

We construct all of the normal subgroups of H_2 within MAGMA using the command "NormalSubgroups(H_2)". We found that there is only one normal subgroup N_1 that has order 16 and therefore N_1 must be the group 4^2. The command "K:=Complements(H_2, N_1)" returns us one copy of a group of order 12. We check that the group "K[1]" is indeed a complement for N_1 using the command "IsTrivial(N_1 meet K[1])". This is a confirmation that H_2 is a split extension of N_1 by "K[1]". Using the command "IsIsomorphic(K[1],DihedralGroup(6))" confirms that the group "K[1]" is isomorphic to the dihedral group D_{12}. Note that the dihedral group D_{12} of order 12 can be represented as a permutation group acting on 6 points using the MAGMA command "DihedralGroup(6)". Hence the structure of the inertia factor group H_2 is identified as $4^2{:}D_{12}$. The group $4^2{:}D_{12}$ is constructed from elements within $G_2(2)$ and the generators are as follows:

$4^2{:}D_{12} = \langle \alpha_1, \alpha_2 \rangle$, $\alpha_1 \in 2A$, $\alpha_2 \in 8B$ where

$$\alpha_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} , \quad \alpha_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

We obtain the fusion of the inertia factor $4^2{:}D_{12}$ into $G_2(2)$ by using direct matrix conjugation in $G_2(2)$ and the permutation character of the inertia factor group in $G_2(2)$ of degree 63. MAGMA was used for the various computations. The fusion map of $4^2{:}D_{12}$ into $G_2(2)$ is shown in Table 1.

Table 1. The fusion of $4^2{:}D_{12}$ into $G_2(2)$

$[h]_{4^2:D_{12}}$ $\longrightarrow$	$[g]_{G_2(2)}$	$[h]_{4^2:D_{12}}$ $\longrightarrow$	$[g]_{G_2(2)}$
$1A$	$1A$	$4A$	$4A$
$2A$	$2A$	$4B$	$4C$
$2B$	$2B$	$4C$	$4B$
$2C$	$2B$	$4D$	$4C$
$2D$	$2A$	$6A$	$6B$
$2E$	$2B$	$8A$	$8A$
$3A$	$3B$	$8B$	$8B$

5. The conjugacy classes of $2^{6\cdot}G_2(2)$

In Section 3, the group $2^6{\cdot}G_2(2)$ was constructed as a permutation group on 4060 points inside Ru. We obtained that $2^6{\cdot}G_2(2)$ has exactly 30 conjugacy classes of elements, using direct computation in MAGMA. Again, using direct computation in MAGMA, we are able to determine the fusion of the conjugacy classes of $\overline{G}$ into the classes of Ru. We rearrange the conjugacy classes of $\overline{G}$ into the form normally obtained by the technique of coset-analysis and are listed in Table 2. In Section 4 we computed the values of k with the aid of the permutation character $\chi(G_2(2)|2^6)$. We used Programme A [1] written in MAGMA to calculate the f_j's. The order of the centralizer $C_{\overline{G}}(x)$ for each element $x \in \overline{G}$ in a conjugacy class $[x]_{\overline{G}}$ is given by $|C_{\overline{G}}(x)| = \frac{k|C_G(g)|}{f_j}$, where $C_G(g)$ is the centralizer for $g \in G_2(2)$. The reader is referred to [1], [17], [18], [19], [21] and [25] for detailed information about coset analysis and the descriptors of the parameters used in Table 2.

Table 2. The conjugacy classes of elements of $2^{6\cdot}G_2(2)$

$[g]_{G_2(2)}$	k	f_j	$\lvert[x]_{2^6\cdot G_2(2)}\rvert$	$\lvert C_{2^6\cdot G_2(2)}(x)\rvert$	$\to Ru$
$1A$	64	$f_1 = 1$	$1A$	774144	$1A$
		$f_2 = 63$	$2A$	12288	$2A$
$2A$	16	$f_1 = 1$	$2B$	3072	$2A$
		$f_2 = 3$	$2C$	1024	$2A$
		$f_3 = 12$	$4A$	256	$4A$
$2B$	8	$f_1 = 1$	$2D$	384	$2A$
		$f_2 = 1$	$4B$	384	$4A$
		$f_3 = 3$	$4C$	128	$4D$
		$f_4 = 3$	$4D$	128	$4C$
$3A$	1	$f_1 = 1$	$3A$	216	$3A$
$3B$	4	$f_1 = 1$	$3B$	72	$3A$
		$f_2 = 3$	$6A$	24	$6A$
$4A$	4	$f_1 = 1$	$4E$	384	$4A$
		$f_2 = 3$	$4F$	128	$4D$
$4B$	4	$f_1 = 1$	$4G$	192	$4B$
		$f_2 = 3$	$4H$	64	$4C$
$4C$	4	$f_1 = 1$	$4I$	128	$4D$
		$f_2 = 1$	$4J$	128	$4D$
		$f_3 = 2$	$4K$	64	$4C$
$6A$	1	$f_1 = 1$	$6B$	24	$6A$
$6B$	2	$f_1 = 1$	$6C$	12	$6A$
		$f_2 = 1$	$12A$	12	$12A$
$7A$	1	$f_1 = 1$	$7A$	7	$7A$
$8A$	2	$f_1 = 1$	$8A$	16	$8A$
		$f_2 = 1$	$8B$	16	$8C$
$8B$	2	$f_1 = 1$	$8C$	16	$8C$
		$f_2 = 1$	$8D$	16	$8C$
$12A$	1	$f_1 = 1$	$12B$	12	$12B$
$12B$	1	$f_1 = 1$	$12C$	12	$12A$
$12C$	1	$f_1 = 1$	$12D$	12	$12B$

Let $\chi(Ru|2^{6\cdot}G_2(2))$ be the permutation character of Ru on the cosets of $2^{6\cdot}G_2(2)$ of degree 188500. Having obtained the fusion of $2^{6\cdot}G_2(2)$ into Ru and thus the values of the permutation character of Ru on the classes of $2^{6\cdot}G_2(2)$, we will proceed to compute $\chi(Ru|2^{6\cdot}G_2(2)$ in terms of irreducible characters of Ru. From the ATLAS [11], we only need to restrict $\psi_i \in Irr(Ru)$, $i \in \{1, 2, 3, ..., 16\}$, to $2^{6\cdot}G_2(2)$. Let γ_1 be the identity character of $2^{6\cdot}G_2(2)$, then we compute the inner product of each ψ_i with γ_1 . The values of the inner product $\langle\psi_i, \gamma_1\rangle$ are given in Table 3.

Table 3. The values of $\langle\psi_i, \gamma_1\rangle$

	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7	ψ_8	ψ_9	ψ_{10}	ψ_{11}	ψ_{12}	ψ_{13}	ψ_{14}	ψ_{15}	ψ_{16}
$\langle\psi_i, \gamma_1\rangle$	1	0	0	0	0	2	1	0	0	0	1	1	1	1	1	1

Using Table 3 and the Frobenius-Reciprocity Theorem (Theorem 3.4.3 in [19]), we obtain that $\chi(Ru|2^{6\cdot}G_2) =$ 1a + 2× 3276a + 3654a + 27000abc +27405a + 34944ab.

6. The Fischer-Clifford matrices of $2^{6\cdot}G_2(2)$

In Section 5, we obtained that $\overline{G}$ has 30 conjugacy classes and hence we have to find 30 irreducible characters for $\overline{G}$. From Section 2, these 30 characters are distributed into two blocks Δ_1 and Δ_2 corresponding to the inertia factor groups H_1 and H_2, respectively. $H_1 = G$ contributes 16 characters towards the character table of $\overline{G}$ which are coming from the ordinary irreducible character table of G (see Note 5.3.1 in [6]). If the character $\Psi = \sum_{i=2}^{64} \theta_i$, where θ_i's are the non-trivial linear characters of $N = 2^6$, is extendable to an ordinary character of its inertia group $\overline{H}_2$, then we will use the ordinary character table of H_2 to complete the character table of $\overline{G}$. Otherwise, we have to use the appropriate projective character table of H_2 with associated factor set α^{-1} (see [1], [6] and [24]). In Section 4 we found that H_2 has 14 conjuacy classes and thus we deduce that $|Irr(H_2)| = 14$. Since $|Irr(\overline{G})| = |Irr(G)| + |IrrProj(H_2, \alpha^{-1})| = 16 + |IrrProj(H_2, \alpha^{-1})| = 30$ [7] (Section 5.3, equation 5.7) then it follows that the inertia factor H_2 must contribute with 14 irreducible projective characters with associated factor set α^{-1} to complete the ordinary character table of $\overline{G}$. $IrrProj(H_2, \alpha^{-1})$ denotes the set of all irreducible projective characters of H_2 with associated factor set α^{-1}.

The first step to find all the projective character tables of H_2 with their corresponding factor sets is to compute the Schur multiplier $M(H_2)$ of H_2. We represented the group H_2 as permutations on 63 points in MAGMA (see Section 4). The sequence of Magma commands found in [6] (Section 4 , page 52) is used to compute the Schur multiplier $M(H_2)$ of H_2 and also the ordinary character table of the full covering group $C = M(H_2)\cdot H_2$ of H_2. We found that $M(H_2) \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \cong 2^2$ so that there are 3 sets of projective characters of H_2 with non-trivial factor sets β_i^{-1}, $i = 1, 2, 3$, such that $\beta_i^2 \sim 1$. We obtained that $|Irr(M(H_2){\cdot}H_2| = 36$, where 14 of these are the ordinary characters of H_2 and so we deduce that $\sum_{i=1}^{3} |IrrProj(H_2, \beta_i^{-1})| = 22$.

Haggarty and Humphreys [15] show that is possible to determine the projective characters of H_2 with a given factor set $\beta_i^{-1}, i = 1, 2, 3$, without the full representation group $2^2{\cdot}H_2$ of H_2. We proceed computationally in MAGMA by first computing the center Z of $2^2{\cdot}H_2$. We obtained that $Z \cong 2^2 \cong M(H_2)$. Next, we compute the three non-conjugate subgroups P_i of Z, $i = 1, 2, 3$, of order two. The command "$R_i := C/P_i$" resulted in a qoutient group $R_i \cong 2_i{\cdot}H_2$ of $M(H_2){\cdot}H_2$ and any projective representation of H_2 with factor set β_i^{-1} can be lifted to an ordinary representation of R_i. Thus the projective characters of H_2 with factor set β_i^{-1} can be determined from the ordinary character table of R_i. We compute the character tables of the groups R_i and found that $|Irr(R_1)| = |Irr(R_2)| = 21$ and $|Irr(R_3)| = 22$, where 14 of these in each group are the ordinary irreducible characters of H_2. Thus the number of projective characters of H_2 associated with each non-trivial factor set β_1^{-1}, β_2^{-1} and β_3^{-1} is 7,7 and 8, respectively. This shows that we should use the set $Irr(H_2)$ to construct the ordinary character table of $\overline{G}$. Therefore, Ψ is extendable to an ordinary character of $\overline{H}_2$ and hence we will use the ordinary character tables of the inertia factor groups $G_2(2)$ and $4^2{:}D_{12}$ to obtain the irreducible characters of $2^{6\cdot}G_2(2)$. This implies that every coset corresponding to a conjugacy class of $G_2(2)$ is a split coset and therefore by Ali and Moori (Section 2 in [3]) the shapes of the Fischer-Clifford matrices of $\overline{G}$ are forced.

Having obtained the fusions of the inertia factors into $G_2(2)$ and the conjugacy classes of $G_2(2)$ in coset-analysis form (Table 2), we are now able to compute the Fischer-Clifford matrices of the group $2^{6\cdot}G_2(2)$. We will use the theory and properties discussed in Section 2 and [3] to help us in the construction of these matrices. The fusion of $\overline{G}$ into Ru together with the restriction of characters of Ru to $\overline{G}$ forces the signs of the Fischer-Clifford matrices and the orders of the elements of $\overline{G}$. Note that all the relations hold since 2^6 is an elementary abelian group.

For example, consider the conjugacy class $2B$ of $G_2(2)$. Then we obtain that $M(2B)$ has the following form with corresponding weights attached to the rows and columns:

$$M(2B) = \begin{array}{c} \\ 48 \\ 48 \\ 16 \\ 16 \\ . \end{array} \begin{array}{c} \begin{array}{cccc} 384 & 384 & 128 & 128 \end{array} \\ \left(\begin{array}{cccc} a & e & i & m \\ b & f & j & n \\ c & g & k & o \\ d & h & l & p \\ & & & \end{array} \right) \\ \begin{array}{cccc} 8 & 8 & 24 & 24 \end{array} \end{array}$$

By properties (a) and (f) of the Fischer-Clifford matrix $M(g)$ in Section 2, we have $a = e = i = m = 1$, $b = 1, c = d = 3$. Thus we get the following form

$$M(2B) = \begin{array}{c} \\ 48 \\ 48 \\ 16 \\ 16 \\ . \end{array} \begin{array}{c} \begin{array}{cccc} 384 & 384 & 128 & 128 \end{array} \\ \left(\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & f & j & n \\ 3 & g & k & o \\ 3 & h & l & p \\ & & & \end{array} \right) \\ \begin{array}{cccc} 8 & 8 & 24 & 24 \end{array} \end{array}$$

By the orthogonality relations for columns and rows (properties (c) and (d) in Section 2) we obtained the equations $f + g + h = -1$,$3f^2 + g^2 + h^2 = 21$, $j+k+l = -1$, $3j^2+k^2+l^2 = 5$, $n+o+p = -1$, $3n^2+o^2+p^2 = 5$, $f+3j+3n = -1$, $f^2 + 3j^2 + 3n^2 = 7$, $g + 3k + 3o = -3$, $g^2 + 3k^2 + 3o^2 = 15$, $h + 3l + 3p = -3$ and $h^2 + 3l^2 + 3p^2 = 15$. Solving the above equations simultaneously and using the remaining properties discussed in Section 2, we obtained that

$$M(2B)=\left(\begin{array}{rrrr} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 3 & 3 & -1 & -1 \\ 3 & -3 & 1 & -1 \end{array} \right)$$

Let $2D$, $4B$, $4C$, and $4D$ be the conjugacy classes of $\overline{G}$, obtained from the coset corresponding to the class $2B$ of $G_2(2)$. Suppose that the above matrix is the Fischer-Matrix $M(2B)$ obtained from the coset $2B \in G_2(2)$. Then by considering the restriction of $\psi_4 \in Irr(Ru)$ [11] to $\overline{G}$, we observe that there will be no fusion from $2D \in \overline{G}$ into $2A \in Ru$. Hence this is not the required Fischer-Clifford matrix and therefore the sign of the rows has to be changed. Now we multiply each of rows 2 and 3 by -1 , then we obtain the proper Fischer-Clifford matrix $M(2B)$ for $\overline{G}$. Hence

$$M(2B)=\left(\begin{array}{rrrr} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ 3 & 3 & -1 & -1 \\ -3 & 3 & -1 & 1 \end{array} \right).$$

We use a similar type of argument as in the case of $M(2B)$ to construct a Fischer-Clifford matrix $M(g)$ for each class representative $g \in G_2(2)$ which are listed in Table 4.

Table 4. The Fischer-Clifford Matrices of $2^{6\cdot}G_2(2)$

$M(g)$	$M(g)$
$M(1A) = \begin{pmatrix} 1 & 1 \\ 63 & -1 \end{pmatrix}$	$M(2A) = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 3 & -1 \\ 12 & -4 & 0 \end{pmatrix}$
$M(2B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ 3 & 3 & -1 & -1 \\ -3 & 3 & -1 & 1 \end{pmatrix}$	$M(3A) = \begin{pmatrix} 1 \end{pmatrix}$
$M(3B) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$	$M(4A) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$
$M(4B) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$	$M(4C) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$
$M(6A) = \begin{pmatrix} 1 \end{pmatrix}$	$M(6B) = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$
$M(7A) = \begin{pmatrix} 1 \end{pmatrix}$	$M(8A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(8B) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(12A) = \begin{pmatrix} 1 \end{pmatrix}$
$M(12B) = \begin{pmatrix} 1 \end{pmatrix}$	$M(12C) = \begin{pmatrix} 1 \end{pmatrix}$

7. **Character Table and Power maps of** $2^{6\cdot}G_2(2)$

We use the Fischer-Clifford matrices of $2^{6\cdot}G_2(2)$ and the ordinary character tables of H_1 and H_2 together with the fusions of H_2 into H_1 to obtain the character table of $2^{6\cdot}G_2(2)$.

For example, we calculate the partial character table of $2^{6\cdot}G_2(2)$ corresponding to the coset of $2B \in G_2(2)$. From the Fischer-Clifford matrix $M(2B)$ we obtain that

$$M_1(2B) = (1\ 1\ 1\ 1) \text{ and } M_2(2B) = \begin{pmatrix} -1 & 1 & 1 & -1 \\ 3 & 3 & -1 & -1 \\ -3 & 3 & -1 & 1 \end{pmatrix}.$$

Let $C_1(2B)$ and $C_2(2B)$ be the partial character tables of the inertia factors for the classes which fuse to $2B \in G_2(2)$. Then the partial character table of $2^{6\cdot}G_2(2)$ on the classes $\{2D, 4B, 4C, 4D\}$ is given by:

$$C_1(2B)M_1(2B) = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 1 \\ -1 \\ 0 \\ -2 \\ 2 \\ 3 \\ -3 \\ -3 \\ 3 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix} = \begin{array}{cccc} 2D & 4B & 4C & 4D \\ \end{array} \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ -2 & -2 & -2 & -2 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ -3 & -3 & -3 & -3 \\ -3 & -3 & -3 & -3 \\ 3 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C_2(2B)M_2(2B)=\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & -1 \\ -2 & -2 & 0 \\ 2 & 2 & 0 \\ -3 & 1 & -1 \\ 3 & -1 & -1 \\ -3 & 1 & 1 \\ 3 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} -1 & 1 & 1 & -1 \\ 3 & 3 & -1 & -1 \\ -3 & 3 & -1 & 1 \end{pmatrix} = \begin{array}{c} \begin{matrix} 2D & 4B & 4C & 4D \end{matrix} \\ \begin{pmatrix} -1 & 7 & -1 & -1 \\ -5 & -1 & -1 & 3 \\ 1 & -7 & 1 & 1 \\ 5 & 1 & 1 & -3 \\ -4 & -8 & 0 & 4 \\ 4 & 8 & 0 & -4 \\ 9 & -3 & -3 & 1 \\ -3 & -3 & 5 & -3 \\ 3 & 3 & -5 & 3 \\ -9 & 3 & 3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 6 & -6 & 2 & -2 \\ -6 & 6 & -2 & 2 \end{pmatrix} \end{array}$$

Similarly for all the other classes of $\overline{G}$, we can compute the partial character tables of $\overline{G}$. Altogether we obtain 30 irreducible characters of $\overline{G}$. The set of irreducible characters of $2^{6\cdot}G_2(2)$ will be partioned into two blocks $\triangle_1$ and $\triangle_2$ corresponding to the inertia factor groups H_1 and H_2, respectively. In fact, $\triangle_1 = \{\chi_j | 1 \leq j \leq 16\}$ and $\triangle_2 = \{\chi_j | 17 \leq j \leq 30\}$, where $\chi_i \in Irr(\overline{G})$ such that $Irr(\overline{G}) = \triangle_1 \cup \triangle_2$. The character table of $2^{6\cdot}G_2(2)$ is given in Table 5. The consistency and accuracy of the character table of $2^{6\cdot}G_2(2)$ have been tested by using Programme E [24] written in GAP.

Table 5. Character table of $G = 2^{6\cdot}G_2(2)$

	$1A$		$2A$			$2B$				$3A$	$3B$		$4A$	
	$1A$	$2A$	$2B$	$2C$	$4A$	$2D$	$4B$	$4C$	$4D$	$3A$	$3B$	$6A$	$4E$	$4F$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	1
χ_3	6	6	-2	-2	-2	0	0	0	0	-3	0	0	-2	-2
χ_4	6	6	-2	-2	-2	0	0	0	0	-3	0	0	-2	-2
χ_5	7	7	-1	-1	-1	1	1	1	1	-2	1	1	3	3
χ_6	7	7	-1	-1	-1	-1	-1	-1	-1	-2	1	1	3	3
χ_7	14	14	6	6	6	0	0	0	0	-4	2	2	-2	-2
χ_8	14	14	-2	-2	-2	-2	-2	-2	-2	5	-1	-1	2	2
χ_9	14	14	-2	-2	-2	2	2	2	2	5	-1	-1	2	2
χ_{10}	21	21	5	5	5	3	3	3	3	3	0	0	1	1
χ_{11}	21	21	5	5	5	-3	-3	-3	-3	3	0	0	1	1
χ_{12}	27	27	3	3	3	-3	-3	-3	-3	0	0	0	3	3
χ_{13}	27	27	3	3	3	3	3	3	3	0	0	0	3	3
χ_{14}	42	42	2	2	2	0	0	0	0	6	0	0	-6	-6
χ_{15}	56	56	-8	-8	-8	0	0	0	0	2	2	2	0	0
χ_{16}	64	64	0	0	0	0	0	0	0	-8	-2	-2	0	0
χ_{17}	63	-1	15	-1	-1	-1	7	-1	-1	0	3	-1	3	-1
χ_{18}	63	-1	-9	7	-1	-5	-1	-1	3	0	3	-1	3	-1
χ_{19}	63	-1	15	-1	-1	1	-7	1	1	0	3	-1	3	-1
χ_{20}	63	-1	-9	7	-1	5	1	1	-3	0	3	-1	3	-1
χ_{21}	126	-2	6	6	-2	-4	-8	0	4	0	-3	1	6	-2
χ_{22}	126	-2	6	6	-2	4	8	0	-4	0	-3	1	6	-2
χ_{23}	189	-3	21	5	-3	9	-3	-3	1	0	0	0	-3	1
χ_{24}	189	-3	-3	13	-3	-3	-3	5	-3	0	0	0	-3	1
χ_{25}	189	-3	-3	13	-3	3	3	-5	3	0	0	0	-3	1
χ_{26}	189	-3	21	5	-3	-9	3	3	-1	0	0	0	-3	1
χ_{27}	378	-6	18	-14	2	0	0	0	0	0	0	0	-6	2
χ_{28}	378	-6	-30	2	2	0	0	0	0	0	0	0	-6	2
χ_{29}	378	-6	-6	-6	2	6	-6	2	-2	0	0	0	6	-2
χ_{30}	378	-6	-6	-6	2	-6	6	-2	2	0	0	0	6	-2

Table 5 continued

	4B		4C			6A	6B		7A	8A		8B		12A	12B	12C
	4G	4H	4I	4J	4K	6B	6C	12A	7A	8A	8B	8C	8D	12B	12C	12D
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	1	1	1	1	-1	-1	1	1	1	-1	-1	1	-1	-1
χ_3	0	0	2	2	2	1	0	0	-1	0	0	0	0	1	$-i3$	$i3$
χ_4	0	0	2	2	2	1	0	0	-1	0	0	0	0	1	$i3$	$-i3$
χ_5	-3	-3	-1	-1	-1	2	1	1	0	-1	-1	-1	-1	0	0	0
χ_6	3	3	-1	-1	-1	2	-1	-1	0	-1	-1	1	1	0	0	0
χ_7	0	0	2	2	2	0	0	0	0	0	0	0	0	-2	0	0
χ_8	2	2	2	2	2	1	1	1	0	0	0	0	0	-1	-1	-1
χ_9	-2	-2	2	2	2	1	-1	-1	0	0	0	0	0	-1	1	1
χ_{10}	-1	-1	1	1	1	-1	0	0	0	-1	-1	1	1	1	-1	-1
χ_{11}	1	1	1	1	1	-1	0	0	0	-1	-1	-1	-1	1	1	1
χ_{12}	-3	-3	-1	-1	-1	0	0	0	-1	1	1	1	1	0	0	0
χ_{13}	3	3	-1	-1	-1	0	0	0	-1	1	1	-1	-1	0	0	0
$\chi_1 4$	0	0	-2	-2	-2	2	0	0	0	0	0	0	0	0	0	0
χ_{15}	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0
χ_{16}	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
χ_{17}	3	-1	3	-1	-1	0	-1	1	0	1	-1	1	-1	0	0	0
χ_{18}	3	-1	-1	3	-1	0	1	-1	0	-1	1	1	-1	0	0	0
χ_{19}	-3	1	3	-1	-1	0	1	-1	0	1	-1	-1	1	0	0	0
χ_{20}	-3	1	-1	3	-1	0	-1	1	0	-1	1	-1	1	0	0	0
χ_{21}	0	0	2	2	-2	0	-1	1	0	0	0	0	0	0	0	0
χ_{22}	0	0	2	2	-2	0	1	-1	0	0	0	0	0	0	0	0
χ_{23}	-3	1	1	-3	1	0	0	0	0	-1	1	1	-1	0	0	0
χ_{24}	-3	1	-3	1	1	0	0	0	0	1	-1	1	-1	0	0	0
χ_{25}	3	-1	-3	1	1	0	0	0	0	1	-1	-1	1	0	0	0
χ_{26}	3	-1	1	-3	1	0	0	0	0	-1	1	-1	1	0	0	0
χ_{27}	0	0	-2	6	-2	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	0	0	6	-2	-2	0	0	0	0	0	0	0	0	0	0	0
χ_{29}	6	-2	-2	-2	2	0	0	0	0	0	0	0	0	0	0	0
χ_{30}	-6	2	-2	-2	2	0	0	0	0	0	0	0	0	0	0	0

We can use GAP to compute possible power maps from the character table of $\overline{G}$. The Programme E in [24] produces unique p-power maps for our Table 5 and are listed in Table 6.

Table 6. The power maps of the elements of $2^{6\cdot}G_2(2)$

$[g]_{G_2(2)}$	$[x]_{2^{6\cdot}G_2(2)}$	2	3	7	$[g]_{G_2(2)}$	$[x]_{2^{6\cdot}G_2(2)}$	2	3	7
1A	1A				2A	2B	1A		
	2A	1A				2C	1A		
						4A	2A		
2B	2D	1A			3A	3A		1A	
	4B	2A							
	4C	2A							
	4D	2A							
3B	3B		1A		4A	4E	2B		
	6A	3B	2A			4F	2C		
4B	4G	2B			4C	4I	2B		
	4H	2C				4J	2C		
						4K	2C		
6A	6B	3A	2B		6B	6C	3B	2D	
						12A	6A	4B	
7A	7A			1A	8A	8A	4E		
						8B	4F		
8B	8C	4I			12A	12B	6B	4E	
	8D	4J							
12B	12C	6B	4G		12C	12D	6B	4G	

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