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A UNIFORM APPROXIMATION METHOD TO SOLVE ABSOLUTE VALUE EQUATION

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Abstract. In this paper, we propose a parametric uniform approximation method to solve NP-hard absolute value equation. For this, we uniformly approximate absolute value in such a way that the nonsmooth absolute value equation can be formulated as a smooth nonlinear equation. By solving the parametric smooth nonlinear equation using Newton method, for a decreasing sequence of parameters, we can get the solution of absolute value equation. It is proved that the method is globally convergent under some weaker conditions with respect to existing methods. Moreover, preliminary numerical results indicate effectiveness and robustness of our method to solve absolute value equation.
Keywords: Absolute value equation, Uniform approximation, Newton method.
MSC(2010): Primary: 65K99; Secondary: 65K05; 90C99.

1. Introduction

In this article, we consider the Absolute Value Equation (AVE):

$$f(x) = Ax - |x| - b = 0, \tag{1.1}$$

in which $A \in \mathbb{R}^{n\times n}$ and $b \in \mathbb{R}^n$ are known. Here, $|x|$ denotes a vector with components $|x_i|$, $i = 1, 2, \dots, n$. Notice that AVE (1.1) is a nonsmooth nonlinear equation due to the non-differentiability of absolute value function.

The significance of absolute value equation (1.1) arises from the fact that the general NP-hard linear complementarity problem (LCP) [2–4], which subsumes many mathematical programming problems, can be formulated as an AVE (1.1). This implies that AVE (1.1) is NP-hard in its general form [7, 10, 11]. There are some interesting results about the AVE (1.1), which we list as follows:

- Determining the existence of a solution to (1.1) is NP-hard [7];

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- AVE (1.1) is equivalent to LCP [7, 10, 11];
- If solvable, AVE (1.1) can have either unique solution or multiple solutions [10];
- AVE (1.1) is uniquely solvable for any b if $\|A^{-1}\| < 1$, or the singular values of A exceed 1 [10];
- If 1 is not an eigenvalue of A, the singular values of A are merely greater than or equal to 1 and $\{x|(A+I)x - b \leq 0,\ (A-I)x - b \geq 0\} \neq \emptyset$, then, AVE (1.1) is solvable [10];
- If $b < 0$ and $\|A\| < \gamma/2$, where $\gamma = \frac{\min_i |b_i|}{\max_i |b_i|}$, then AVE (1.1) has exactly 2^n distinct solutions, each of which has no zero components and a different sign pattern [10];
- Various characterizations of the solution set and the minimal norm solution to (1.1) are given in [5, 14];

Recently, some computational methods have been presented for AVE (1.1) (see [1, 5–9, 13, 16, 17]). In [9], a generalized Newton algorithm is proposed for AVE (1.1). It is proved that the generalized Newton iterates are well defined and bounded when the singular values of A exceed 1. However, the global linear convergence of the method is only guaranteed under more stringent condition $\|A^{-1}\| < \frac{1}{4}$ rather than the singular values of A exceeding 1.

Another approach for solving (1.1) is the smoothing method (see [12] and references therein). The feature of smoothing method is to construct a smoothing approximation function $f_\mu : \mathbb{R}^n \times \mathbb{R} \mapsto \mathbb{R}^n$ of f such that for any $\mu > 0$, f_μ is continuously differentiable and

$$\lim_{\mu \to 0^+} \|f(x) - f_\mu(x)\| = 0, \quad \forall\, x \in \mathbb{R}^n,$$

and then to find a solution of (1.1) by (inexactly) solving the following problems for a given positive sequence $\{\mu_k\}$, $k = 0, 1, 2, \ldots,$

$$f_{\mu_k}(x) = 0. \tag{1.2}$$

If $x(\mu_k)$ denotes the approximate solution of (1.2), we expect that $x(\mu_k)$ converge to x^*, the solution of (1.1), as $\mu_k \to 0^+$. To get this aim, we must prove that:

- The equation (1.2) has a unique solution $x(\mu_k)$, for all $\mu_k > 0$,
- $x(\mu_k) \to x^*$, when $\mu_k \to 0^+$.

The merits of the smoothing method are global convergence and convenience in handling smooth functions instead of nonsmooth functions. However, (1.2), which needs to be solved at each step, is nonlinear in general.

The smoothing Newton method [12] can be regarded as a variant of the smoothing method. It uses the derivative of f_μ with respect to x in Newton method, namely

$$x_{k+1} = x_k - \alpha_k \nabla f_{\mu_k}(x_k)^{-1} f(x_k) \tag{1.3}$$

where $\mu_k > 0$, $\nabla f_{\mu_k}(x_k)$ denotes the Jacobian of f_μ with respect to x at (x_k, μ_k) and $\alpha_k > 0$ is the step size. The smoothing Newton method (1.3) for solving nonsmooth equation (1.1) has been studied for decades in different areas. In some previous papers, method (1.3) was called a splitting method because f is split into a smooth part f_μ and a nonsmooth part $f - f_\mu$. The global and linear convergence of (1.3) has been discussed, but so far no superlinear convergence result has been obtained.

In [17], authors established global and finite convergence of a generalized Newton method proposed for AVE (1.1). Their method utilizes both the semismooth and the smoothing Newton steps, in which the semismooth Newton step guarantees the finite convergence and the smoothing Newton step contributes to the global convergence.

In this paper, to solve AVE (1.1), we approximate the nonsmooth function $f(x)$ with a parametric accurate smoothing approximation function $f_{\mu_k}(x)$ having good properties, and solve the equation (1.2) using Newton method for a fast convergent sequence of parameters $\{\mu_k\}$. We will prove that the equation (1.2) has a unique solution $x(\mu_k)$, for all $\mu_k > 0$, and Newton method, initiated from $x(\mu_{k-1})$, is well defined. Moreover, we prove that $x(\mu_k) \to x^*$, when $\mu_k \to 0^+$, in which x^* is the unique solution of (1.1). This algorithm is proved to be globally convergent under the condition that the singular values of A exceed 1. This condition is weaker than the one used in [9]. Preliminary numerical results given in Sec. 3 show that our method is very promising.

2. Uniform smooth approximation method for AVE

In this section, we propose a uniform smooth approximation method to solve AVE (1.1). We begin with the following definitions.

Definition 2.1. *([12]) A function $S_\mu : \mathbb{R}^n \mapsto \mathbb{R}^m$ is called a smoothing function of a nonsmooth function $S : \mathbb{R}^n \mapsto \mathbb{R}^m$ if, for any $\mu > 0$, S_μ is continuously differentiable and, for any $x \in \mathbb{R}^n$,*

$$\lim_{\mu \downarrow 0,\ z \to x} S_\mu(z) = S(x).$$

Definition 2.2. *([12]) Let $S : \mathbb{R}^n \mapsto \mathbb{R}^m$ be a locally Lipschitz continuous function and $S_\mu : \mathbb{R}^n \mapsto \mathbb{R}^m$, $\mu \in \mathbb{R}$, continuously differentiable. Then, S_μ is called a regular smoothing function of S if for any compact set $D \subseteq \mathbb{R}^n$ and $\overline{\mu} > 0$, there exists a constant $L > 0$ such that, for any $x \in D$ and $\mu \in (0, \overline{\mu}]$*

$$\|S_\mu(x) - S(x)\| \leq L\mu.$$

Note that AVE (1.1) is nonsmooth due to the non-differentiability of absolute value function. So, to smooth approximation of $f(x)$, it is sufficient to smooth approximation of absolute value function. On the other hand, absolute value function $|x_i|$ can be regarded as the integral of signum function defined by:

$$sign(x_i) = \begin{cases} -1 & x_i < 0 \\ 0 & x_i = 0 \\ 1 & x_i > 0. \end{cases}$$

Hence, to smooth approximation of absolute value function, first of all, we need to smooth approximation of the signum function. Suppose that $\mu > 0$ is a parameter and consider the following function:

$$t_\mu(x_i) = \begin{cases} -1 & x_i \le -\mu \\ \dfrac{2(x_i/\mu)}{1+(x_i/\mu)^2} & -\mu \le x_i \le \mu \\ 1 & x_i \ge \mu. \end{cases}$$

It can be observed that the $t_\mu(x_i)$ is continuously differentiable and satisfies

$$|t_\mu(x_i) - sign(x_i)| \le \mu, \quad \forall\, \mu > 0.$$

Indeed, according to Definition 2.2, $t_\mu(x_i)$ is a regular smoothing approximation to the signum function.

Using integration of $t_\mu(x_i)$ with respect to x_i, we can obtain the following regular smoothing function to the absolute value function:

$$\phi_\mu(x_i) = \begin{cases} -x_i & x_i \le -\mu \\ \mu \ln\left(1+(x_i/\mu)^2\right) + \mu(1-\ln 2) & -\mu \le x_i \le \mu \\ x_i & x_i \ge \mu. \end{cases}$$

Notice that the $\phi_\mu(x_i)$ is continuously differentiable and satisfies

$$0 \le \phi_\mu(x_i) - |x_i| \le \mu(1-\ln 2), \quad \forall\, \mu > 0.$$

Moreover, as Figure 1 indicates, it is a very accurate uniform smooth approximation to $|x_i|$, especially for small μ.

Based on $|x_i| = \max\{x_i, -x_i\}$, Yong and etc [16] adopted the aggregate function introduced by Li [6] to smooth the max function. The smoothing approximation function to absolute function $|x_i|$ is then derived as:

$$\eta_\mu(x_i) = \mu \ln\left[e^{x_i/\mu} + e^{-x_i/\mu}\right].$$

It is shown that

$$0 \le \eta_\mu(x_i) - |x_i| \le \mu \ln 2.$$

As we note, the approximation function $\phi_\mu(x_i)$ to $|x_i|$ is better than $\eta_\mu(x_i)$ because $1 - \ln 2 = 0.3069 < 0.6931 = \ln 2$.

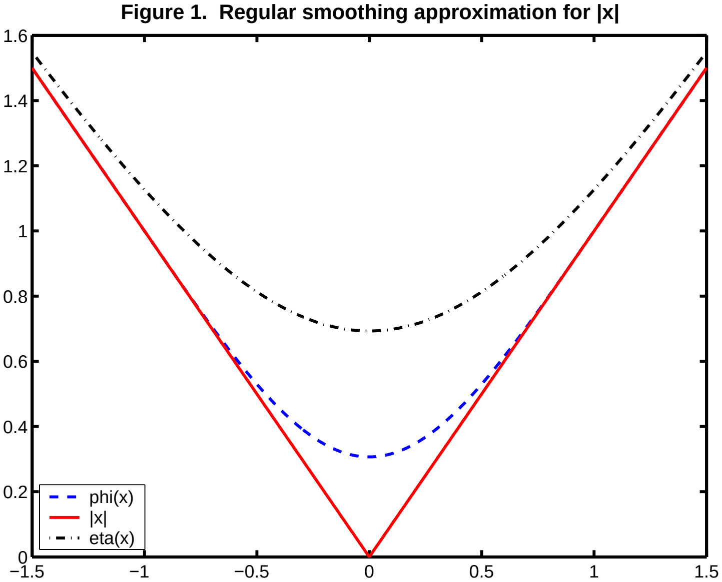
Figure 1 demonstrates functions $|x_i|$ (solid line), $\phi_\mu(x_i)$ (dotted line), and $\eta_\mu(x_i)$ (dashed line) over the interval [-1.5,1.5] for $\mu = 1$. It show that our smooth approximation $\phi_\mu(x_i)$ matches well with $|x_i|$.

Now, let's define the functions $t_\mu, \phi_\mu, f_\mu : \mathbb{R}^n \mapsto \mathbb{R}^n$ by

$$\begin{aligned} t_\mu(x) &= [t_\mu(x_1), \ldots, t_\mu(x_n)]^T, \\ \phi_\mu(x) &= [\phi_\mu(x_1), \ldots, \phi_\mu(x_n)]^T, \\ f_\mu(x) &= Ax - \phi_\mu(x) - b. \end{aligned} \tag{2.1}$$

The following Lemma denotes that $f_\mu(x)$ is a regular smoothing approximation for $f(x)$ in (1.1).

Figure 1. Regular smoothing approximation for |x|



Lemma 2.3. *(i) For any $x \in \mathbb{R}^n$ and $\mu > 0$,*

$$\|f_\mu(x) - f(x)\| \leq \sqrt{n}\,\mu(1 - \ln 2).$$

(ii) For any $\mu > 0$, $f_\mu(x)$ is continuously differentiable on $\mathbb{R}^n$ and

$$\nabla f_\mu(x) = A - diag(t_\mu(x_1), \ldots, t_\mu(x_n)).$$

Proof. (i) For any $\mu > 0$ and $x_i \in \mathbb{R}$, we have $0 \leq \phi_\mu(x_i) - |x_i| \leq \mu(1 - \ln 2)$. So,

$$\|f_\mu(x) - f(x)\| = \|\phi_\mu(x) - |x|\| = \sqrt{\sum_{i=1}^{n} (\phi_\mu(x_i) - |x_i|)^2} \leq \sqrt{n}\, \mu(1 - \ln 2).$$

(ii) Because of separability of $\phi_\mu(x)$, we have

$$\begin{aligned} \nabla f_\mu(x) &= A - \nabla \phi_\mu(x) \\ &= A - diag(\phi'_\mu(x_1), \ldots, \phi'_\mu(x_n)) \\ &= A - diag(t_\mu(x_1), \ldots, t_\mu(x_n)). \end{aligned}$$

□

Lemma 2.4. *([15]) Suppose that A is nonsingular. If matrix E is such that $\|A^{-1}E\| < 1$, then $A + E$ is nonsingular and*

$$\frac{\|A^{-1} - (A+E)^{-1}\|}{\|A^{-1}\|} \leq \frac{\|A^{-1}E\|}{1 - \|A^{-1}E\|}.$$

Lemma 2.5. *Take $D = diag(d_1, \ldots, d_n)$ with $d_i \in [-1, 1]$, $i = 1, \ldots, n$. Suppose that $\|A^{-1}\| < 1$. Then, $A + D$ is nonsingular.*

Proof. Inequalities $\|A^{-1}D\| \leq \|A^{-1}\|\,\|D\| < \|D\| \leq 1$ and Lemma 2.4 imply that the matrix $A + D$ is nonsingular. □

Now we investigate the nonsingularity of the Jacobian matrix $\nabla f_\mu(x)$.

Lemma 2.6. *For all $x \in \mathbb{R}^n$, $\nabla f_\mu(x)$ is nonsingular if $\|A^{-1}\| < 1$.*

Proof. According to the definition of $t_\mu(x_i)$, we have $|t_\mu(x_i)| \leq 1$, for all $i = 1, \ldots, n$. So, $\|diag(t_\mu(x_1), \ldots, t_\mu(x_n))\| \leq 1$. Since $\|A^{-1}\| < 1$, the result can be obtained from the previous Lemma. □

The following lemma gives the boundedness of the inverse matrix of $\nabla f_\mu(x)$.

Lemma 2.7. *Suppose that $\|A^{-1}\| < 1$. Then, there exists a constant $M > 0$ such that for any $\mu > 0$ and any $x \in \mathbb{R}^n$*

$$\|[\nabla f_\mu(x)]^{-1}\| \leq M.$$

Proof. Take $E = -diag(t_\mu(x_1), \dots, t_\mu(x_n))$. Then, from Lemma 2.4 we have

$$\|[\nabla f_\mu(x)]^{-1}\| = \|(A+E)^{-1}\| \le \|(I+F)A^{-1}\| \le (1+\|F\|)\|A^{-1}\|,$$

where $\|F\| \le \frac{\|A^{-1}E\|}{1-\|A^{-1}E\|}$. Furthermore, from the fact that $\|E\| \le 1$, we have

$$\begin{aligned}\|(A+E)^{-1}\| &\le \left(1+\frac{\|A^{-1}\|\,\|E\|}{1-\|A^{-1}\|\,\|E\|}\right)\|A^{-1}\| \\ &\le \left(1+\frac{\|A^{-1}\|}{1-\|A^{-1}\|}\right)\|A^{-1}\| \\ &= \frac{\|A^{-1}\|}{1-\|A^{-1}\|}.\end{aligned}$$

By setting $M := \frac{\|A^{-1}\|}{1-\|A^{-1}\|}$, we can get the desired result. □

Lemma 2.8. *Suppose that* $\|A^{-1}\| < 1$ *and* μ *is fixed. Then, the equation* $f_\mu(x) = 0$ *has a unique solution.*

Proof. If $\overline{x}$ and $\tilde{x}$ are two different solutions, then, using the mean value theorem, we have

$$0 = f_\mu(\overline{x}) - f_\mu(\tilde{x}) = \nabla f_\mu(z)(\overline{x} - \tilde{x}),$$

in which $z = \tilde{x} + t(\overline{x} - \tilde{x})$ and $t \in [0,1]$. This means that $\nabla f_\mu(z)$ is singular, which is a contradiction to Lemma 2.6. □

Now, consider the sequence

$$\mu_0 = 1, \quad \mu_{k+1} = \mu_k - 1 + e^{-\mu_k}, \quad k \ge 0 \tag{2.2}$$

that is Newton method for solving the equation $e^\mu = 1$. The sequence $\{\mu_k\}$ is quadratically convergent to zero. Suppose that $x(\mu_0)$ is given and $x(\mu_k)$, $k \ge 1$, is the solution of the equation

$$f_{\mu_k}(x) = 0,$$

obtained by Newton method, initiated from $x(\mu_{k-1})$. Notice that the function $f_{\mu_k}(x)$ and the sequence $\{\mu_k\}$ are defined by relations (2.1) and (2.2), respectively. In the following theorem, we show that $\lim\limits_{k\to\infty} f(x(\mu_k)) = 0$ which means that $x^* = \lim\limits_{k\to\infty} x(\mu_k)$ exists and is a solution of AVE (1.1).

Theorem 2.9. *Suppose that* $\|A^{-1}\| < 1$ *and* $x(\mu_k)$ *is the unique solution of* $f_{\mu_k}(x) = 0$*, in which* μ_k *is defined by* (2.2)*. Then,* $x^* = \lim\limits_{k\to\infty} x(\mu_k)$ *exists and is a solution of AVE* (1.1)*.*

Proof. First of all, AVE (1.1) has a unique solution because $\|A^{-1}\| < 1$. On the other hand, from $f_{\mu_k}(x(\mu_k)) = 0$ and Lemma 2.3, we have

$$\|f(x(\mu_k))\| = \|f(x(\mu_k)) - f_{\mu_k}(x(\mu_k))\| \le \sqrt{n}\,\mu_k(1 - \ln 2),$$

that denotes $\lim_{k\to\infty} f(x(\mu_k)) = 0$. Continuity of $f(x)$ implies that

$$f(\lim_{k\to\infty} x(\mu_k)) = \lim_{k\to\infty} f(x(\mu_k)) = 0.$$

Since the equation $f(x) = 0$ has a unique solution, $x^* = \lim_{k\to\infty} x(\mu_k)$ exists and is the unique solution of AVE (1.1). □

Corollary 2.10. *Suppose that* $\|A^{-1}\| < 1$ *and* $x(\mu_k)$ *is the unique solution of* $f_{\mu_k}(x) = 0$*, in which* μ_k *is defined by* (2.2)*. Then, the sequence* $\{\|f(x(\mu_k))\|\}$ *is linearly convergent to zero.*

In the following algorithm to solve AVE (1.1), we use the above ideas. The algorithm is initialized from an arbitrary point $x_0 \in \mathbb{R}^n$, $\mu = 1$, and $k = 0$. It stops when

$$\|f(x_k)\| \le \varepsilon \tag{2.3}$$

in which ε is user's precision. In the beginning of an iteration, we check (2.3). If it holds, x_k is accepted solution of AVE (1.1). Otherwise, we approximately solve the smooth nonlinear equation $f_\mu(x) = 0$, initiated from x_k, using Newton iteration $x_{k+1} := x_k - [\nabla f_\mu(x_k)]^{-1} f_\mu(x_k)$ until $\|f_\mu(x_k)\| \le \delta$. Then, the parameter μ is updated according to (2.2). We note that, according to Lemmas 2.3, 2.8, and Theorem 2.9, the algorithm is well defined and convergent to the unique solution of AVE (1.1).

Algorithm 1.
1. Take $k := 0$ and $\mu := 1$. Select an initial point x_0.
2. **While** $\|f(x_k)\| > \varepsilon$ **do**
3. Solve the nonlinear equation $f_\mu(y) = 0$, initiated from x_k, using Newton method:
   While $\|f_\mu(x_k)\| > \delta$ **do**
   Take $x_{k+1} := x_k - [\nabla f_\mu(x_k)]^{-1} f_\mu(x_k)$.
   Set $x_k := x_{k+1}$ and $k := k + 1$.
   End While.
4. Set $\mu := \mu - 1 + e^{-\mu}$.
5. **End While**.

To prove that Algorithm 1 is bounded, we need the following theorem.

Theorem 2.11. *Suppose that $\|A\|^{-1} < 1$. Then, the set*

$$L = \{x \in \mathbb{R}^n \mid \|f(x)\| \leq \alpha\}$$

is bounded for any $\alpha > 0$.

Proof. Since $\|A^{-1}\| < 1$, then $\sigma_{\min}(A) > 1$, in which $\sigma_{\min}(A)$ denotes the smallest singular value of A. Using the fact that $\|Ax\| \geq \sigma_{\min}(A)\|x\|$, we have

$$\begin{aligned}\|Ax - |x| - b\| &\geq \|Ax\| - \| \, |x| \, \| - \|b\| \\ &= \|Ax\| - \|x\| - \|b\| \\ &\geq (\sigma_{\min}(A) - 1)\|x\| - \|b\|.\end{aligned}$$

Thus, for any $x \in L$, we have

$$(\sigma_{\min}(A) - 1)\|x\| - \|b\| \leq \alpha,$$

that is,

$$\|x\| \leq \frac{\alpha + \|b\|}{\sigma_{\min}(A) - 1}.$$

This denotes that the set L is bounded. □

Using the two above theorems, we conclude that all iterates x_k, $k \geq 1$, generated by Algorithm 1, belong to the set L, with $\alpha = \sqrt{n}(1 - \ln 2)$. This proves that Algorithm 1 is bounded and convergent.

Theorem 2.12. *If $\|A^{-1}\| < 1$, then the sequence $\{x_k\}_{k \geq 1}$ generated by Algorithm 1 is well defined and bounded, while belonging to the compact set L, with $\alpha = \sqrt{n}(1 - \ln 2)$. So, there exists an accumulation point x^* such that $f(x^*) = 0$.*

3. Computational results

In this section, we report some numerical results of our algorithm for solving AVE (1.1) compared with [16]. All experiments are done using a PC with CPU of 2.6 GHz and RAM 2 GB, and all codes are finished in MATLAB 7.1. Specially, each required inverse is computed by the MATLAB command "inv".

In order to generate a random solvable AVE with fully dense matrix $A \in \mathbb{R}^{n\times n}$, we take

$$A = 10 * randn(n, n) - 10 * randn(n, n),$$

$$x = randn(n, 1) - randn(n, 1),$$

and set

$$b := Ax - |x|.$$

Following the procedure in [8], we ensured that $\|A^{-1}\| < 1$ by actually computing $\sigma = \sigma_{\min}(A)$ and rescaling A by $A := A/(\tau\sigma)$, in which τ is a nonzero random number in the interval [0,1]. We choose the starting point $x_0 = 0$ and the maximum number of iterations is set to be 50. Also, we have $\varepsilon = 10^{-6}$ and $\delta = 10^{-1}$. For each $n = 100, 200, 300, 400, 500$, we generated 10 random test problems. The numerical results are listed in Table 1, where n denotes the dimension of the testing problem; for the problem of every size, among ten tests, AIt, ACPU, MaxIt, MinIt, ARes, and AF denote the average value of the iteration numbers, the average value of the CPU time in seconds, the maximal value of the iteration numbers, the minimal value of the iteration numbers, the average value of those of $\|f(x_k)\|$ when Algorithm 1 stops, and the number of tests where the algorithm fails, respectively.

Numerical results are summarized in Table 1. For any dimension n, the first line is related to our smooth approximation function $\phi_\mu(x)$ and another is for aggregate function $\eta_\mu(x)$ from [16]. From Table 1, it is easy to see that all randomly generated problems can be solved with few number of iterations and short CPU time; and the numerical results are very stable such that the iteration number does not change when the size of the problem varies. However, our smooth approximation function gives better numerical results with respect to aggregate function. Thus, Algorithm 1 based on smooth approximation $\phi_\mu(x)$ is very effective to solve AVE (1.1).

Table 1. The numerical results of Algorithm 1

n	AIt	ACPU	MaxIt	MinIt	ARes	AF
100	5.1	0.0157	6	5	6.1e-12	0
	5.1	0.0158	6	5	6.1e-12	0
200	5.2	0.0610	6	5	5.6e-11	0
	5.3	0.0630	7	5	5.6e-10	0
300	5.3	0.1687	6	5	2.9e-10	0
	5.5	0.1798	7	5	2.5e-9	0
400	5.3	0.3468	6	5	3.9e-8	0
	5.6	0.3675	8	6	2.7e-7	0
500	5.4	0.6534	6	5	1.0e-7	0
	5.7	0.7506	8	6	1.0e-6	0

4. **Conclusion**

In this paper, we proposed an accurate smooth approximation method to solve the NP-hard absolute value equation $Ax - |x| = b$ under certain assumptions on A. We proved that our method is well defined, bounded, and convergent. Numerical experiments over random solvable AVEs showed that our method works very well.

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