ISSN: 1017-060X (Print)



ISSN: 1735-8515 (Online)

Special Issue of the

# Bulletin of the Iranian Mathematical Society

in Honor of Professor Heydar Radjavi's 80th Birthday

### Vol. 41 (2015), No. 7, pp. 1-14

Title:

The existence and uniqueness of Heydar Radjavi

Author(s):

L. Marcoux, M. Omladič, P. Rosenthal, P. Šemrl and B. R. Yahaghi

Published by Iranian Mathematical Society http://bims.ims.ir

Bull. Iranian Math. Soc. Vol. 41 (2015), No. 7, pp. 1–14 Online ISSN: 1735-8515

### THE EXISTENCE AND UNIQUENESS OF HEYDAR RADJAVI

## LAURENT MARCOUX, MATJAŽ OMLADIČ, PETER ROSENTHAL, PETER ŠEMRL AND BAMDAD R. YAHAGHI

And when Thyself with shining Foot shall pass Among the Guests Star-scatter'd on The Grass, And in Thy joyous Errand reach the Spot Where I made one-turn down an empty Glass! (Khayyam, rendered into English verse by Edward Fitzgerald)

In 2004, Linear Algebra and its Applications published a special issue devoted to Heydar Radjavi, celebrating his 70th birthday. We encourage the reader to consult the article [BORŠ] for a survey of his research up until 2004.

With this special issue, the Bulletin of the Iranian Mathematical Society is pleased to honour Heydar Radjavi on the occasion of his 80th birthday, which took place on January 17, 2015. While he was born in what a younger generation might refer to as the "good old days", and while he carries with him much of the virtues which we nostalgically attribute to that period, nevertheless he is firmly rooted in the present. The fact that he has retained so much of his youth and vitality is due in large part to his love of life, to his eternal optimism and to his willingness to always search for the good in others, even when it might be hard to find. He is, quite simply, a joy to be around.

Heydar's passion for mathematics is clear upon first meeting him. It is this which has allowed both the volume and the quality of his work to remain "invariant" under the passage of time. He is a prolific researcher whose research has concentrated mostly in matrix theory and the theory of Hilbert space operators, with 192 papers cited in Math Reviews at the time of writing of this preface. He has devoted a great deal of his mathematical life to the study of invariant subspaces and triangularizations of collections of matrices and operators, and with Peter Rosenthal, he has written two research monographs (and 56 papers!) on this topic. In recent years, with countless colleagues and friends acting as co-authors, he has also spent much time and effort on the study of

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Article electronically published on December 31, 2015.

the structure and irreducibility properties of matrix semigroups. These contributions have had a profound influence upon the area. He is a fountain of knowledge, and always willing to share his ideas with his colleagues and students.

Like all human beings, he does, of course, have many faults. He is, for example, both modest and generous to a fault. He is also irritatingly punctual - his "delay constant" being zero. But these faults are tempered by his qualities: one can always count on him when one needs him, and he is considerate of, and sensitive to, those around him. He is an excellent colleague, a first-rate coauthor, and a wonderful mathematical "father". His wife Ursula and daughters Marjan and Shirin assure us that he is also a great husband and father.

In May of 2011, Heydar ventured into writing prose, on top of maintaining his mathematical research program. The result was a delightful collection of short stories entitled French Hats in Iran, relating personal anecdotes from his youth and eye-witness accounts of life in mid-twentieth century Iran. It was published by Mage Publishers.

On the eve of his 81st birthday, we are pleased to wish him good health, prosperity and longevity, and we look forward to honouring him yet again on his 100th birthday, by which time we expect that he will have published another 50 papers and written at least one more book!

Dear precious Heydar! Stay like mountain and laugh like tulip! Go like wheel and rain like cloud!

حیدر عزیز و گرانسنگ! به سان کوه بپای و به سان لاله بخند! به سان چرخ بتاز و به سان ابر ببار!

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(Laurent Marcoux) DEPARTMENT OF PURE MATHEMATICS, UNIVERSITY OF WATERLOO, WATERLOO, N2L 3G1, ONTARIO, CANADA.

E-mail address: lwmarcoux@uwaterloo.ca

(Matjaž Omladič) Faculty of mathematics, natural sciences and information technologies, University of Primorska, Glagoljaška 8, SI-6000 Koper, Slovenia. *E-mail address*: matjaz.omladic@fmf.uni-lj.si

(Peter Rosenthal) Department of Mathematics, University of Toronto, Toronto, M5S 3G3, Ontario, Canada.

E-mail address: rosent@math.toronto.edu

(Peter Šemrl) Department of Mathematics, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia.

*E-mail address*: peter.semrl@fmf.uni-lj.si

(Bamdad R. Yahaghi) Department of Mathematics, Faculty of Sciences, Golestan University, Gorgan 49138-15759, Iran.

 $E\text{-}mail\ address: \texttt{bamdad5@hotmail.com}$ 

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