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COMMON FIXED POINT THEOREMS FOR FOUR MAPPINGS IN COMPLETE METRIC SPACES

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ABSTRACT. In this paper, we prove some common fixed point theorems for four maps in complete metric spaces. These theorems are versions of some known results in ordinary metric spaces.

1. **Introduction and preliminaries**

In the present work, we introduce a new binary operation which is a probable modification of the definition of ordinary metric. In section 1, we give some properties about this operation metric. In section 2, we prove two common fixed point theorems for four weakly compatible maps in complete metric spaces. In section 3, we prove a fixed point theorem for compatible mappings satisfying a new general contractive type condition.

In what follows, $\mathbf{N}$ is the set of all natural numbers and $\mathbf{R}^+$ is the set of all positive real numbers.

Let $\diamond : \mathbf{R}^+ \times \mathbf{R}^+ \longrightarrow \mathbf{R}^+$ be a binary operation satisfyng the following conditions:

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(i) $\diamond$ is associative and commutative,
(ii) $\diamond$ is continuous.

Five typical examples of $\diamond$ are:
$a \diamond b = \max\{a, b\}$, $a \diamond b = a + b$, $a \diamond b = ab$, $a \diamond b = ab + a + b$ and
$a \diamond b = \dfrac{ab}{\max\{a, b, 1\}}$ for each $a, b \in \mathbf{R}^+$.

Definition 1.1. The binary operation $\diamond$ is said to satisfy *α-property* if there exists a positive real number α such that

$$a \diamond b \leq \alpha \max\{a, b\}$$

for all $a, b \in \mathbf{R}^+$.

Example 1.2. (1) If $a \diamond b = a + b$, for each $a, b \in \mathbf{R}^+$, then for $\alpha \geq 2$, we have $a \diamond b \leq \alpha \max\{a, b\}$.

(2) If $a \diamond b = \dfrac{ab}{\max\{a, b, 1\}}$, for each $a, b \in \mathbf{R}^+$, then for $\alpha \geq 1$, we have $a \diamond b \leq \alpha \max\{a, b\}$.

In 1996 Jungck [4] introduced the concept of weakly compatible mappings and proved some common fixed point theorems using this concept on ordinary metric spaces. After then, many fixed point results have been obtained using weakly compatible mappings on ordinary metric spaces (see [1], [2], [3], [6]).

Definition 1.3. Let A and S be mappings from a metric space (X, d) into itself. A and S are said to be *weakly compatible* if they commute at their coincidence points, that is, $Ax = Sx$ for some $x \in X$ implies that $ASx = SAx$.

2. Main results

Theorem 2.1. *Let (X, d) be a complete metric space such that $\diamond$ satisfies α-property with $\alpha > 0$. Let A, B, S and T be self mappings of X into itself satisfying the following conditions*

(i) *$A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$ and $T(X)$ or $S(X)$ is a closed subset of X,*
(ii) *the pairs (A, S) and (B, T) are weakly compatible,*

(iii) *for all* $x, y \in X$,

$$\begin{aligned} d(Ax, By) \leq\ & k_1(d(Sx, Ty) \diamond d(Ax, Sx)) + k_2(d(Sx, Ty) \diamond d(By, Ty)) \\ & + k_3(d(Sx, Ty) \diamond \frac{d(Sx, By) + d(Ax, Ty)}{2}), \end{aligned}$$

where $k_1, k_2, k_3 > 0$ *and* $0 < \alpha(k_1 + k_2 + k_3) < 1$.

Then, A, B, S *and* T *have a unique common fixed point in* X.

Proof. Let x_0 be an arbitrary point in X. By (i), we can define inductively a sequence $\{y_n\}$ in X such that $y_{2n} = Ax_{2n} = Tx_{2n+1}$ and $y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$, for $n = 0, 1, 2, \cdots$. We claim that the sequence $\{y_n\}$ is a Cauchy sequence.

Using (iii), we have

$$\begin{aligned} & d(y_{2n}, y_{2n+1}) \\ =\ & d(Ax_{2n}, Bx_{2n+1}) \\ \leq\ & k_1(d(Sx_{2n}, Tx_{2n+1}) \diamond d(Ax_{2n}, Sx_{2n})) \\ & + k_2(d(Sx_{2n}, Tx_{2n+1}) \diamond d(Bx_{2n+1}, Tx_{2n+1})) \\ & + k_3(d(Sx_{2n}, Tx_{2n+1}) \diamond \frac{d(Sx_{2n}, Bx_{2n+1}) + d(Ax_{2n}, Tx_{2n+1})}{2}) \\ =\ & k_1(d(y_{2n-1}, y_{2n}) \diamond d(y_{2n}, y_{2n-1})) \\ & + k_2(d(y_{2n-1}, y_{2n}) \diamond d(y_{2n+1}, y_{2n})) \\ & + k_3(d(y_{2n-1}, y_{2n}) \diamond \frac{d(y_{2n-1}, y_{2n+1}) + d(y_{2n}, y_{2n})}{2}). \end{aligned}$$

Set $d_n = d(y_n, y_{n+1})$. Using the above inequality, we get

$$d_{2n} \leq k_1(d_{2n-1} \diamond d_{2n-1}) + k_2(d_{2n-1} \diamond d_{2n}) + k_3(d_{2n-1} \diamond \frac{d(y_{2n-1}, y_{2n+1})}{2}).$$

Hence,

$$d_{2n} \leq k_1 \alpha d_{2n-1} + k_2 \alpha \max\{d_{2n-1}, d_{2n}\} + k_3 \alpha \max\{d_{2n-1}, \frac{d_{2n-1} + d_{2n}}{2}\}.$$

If $d_{2n} > d_{2n-1}$, we obtain

$$d_{2n} \leq k_1 \alpha d_{2n} + k_2 \alpha d_{2n} + k_3 \alpha d_{2n} < d_{2n},$$

which is a contradiction. Hence $d_{2n} \leq d_{2n-1}$. Similarly it is easy to see that $d_{2n+1} \leq d_{2n}$. Therefore, $d_n \leq d_{n-1}$, for $n = 1, 2, \cdots$.

Using the above inequality we get

$$d_n \leq \alpha(k_1 + k_2 + k_3) d_{n-1} = k d_{n-1},$$

where $\alpha(k_1 + k_2 + k_3) = k < 1$. So

$$d_n \leq kd_{n-1} \leq k^2 d_{n-2} \leq \cdots \leq k^n d_0.$$

That is,

$$d(y_n, y_{n+1}) \leq k^n d(y_0, y_1) \longrightarrow 0 \text{ as } n \to \infty.$$

If $m > n$ then

$$\begin{aligned} d(y_n, y_m) &\leq d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \cdots + d(y_{m-1}, y_m) \\ &\leq k^n d(y_0, y_1) + k^{n+1} d(y_0, y_1) \cdots + k^{m-1} d(y_0, y_1) \\ &= \frac{k^n}{1-k} d(y_0, y_1) \longrightarrow 0 \end{aligned}$$

as $n, m \to \infty$. It follows that the sequence $\{y_n\}$ is a Cauchy sequence and by the completeness of X, $\{y_n\}$ converges to $y \in X$. Therefore,

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} Ax_{2n} = \lim_{n\to\infty} Bx_{2n+1} = \lim_{n\to\infty} Sx_{2n+2} = \lim_{n\to\infty} Tx_{2n+1} = y.$$

Assume that $T(X)$ is a closed subset of X. Then there exists $v \in X$ such that $Tv = y$.

If $Bv \neq y$ then by using (iii), we obtain

$$\begin{aligned} d(Ax_{2n}, Bv) \leq\ & k_1(d(Sx_{2n}, Tv) \diamond d(Ax_{2n}, Sx_{2n})) \\ &+ k_2(d(Sx_{2n}, Tv) \diamond d(Bv, Tv)) \\ &+ k_3(d(Sx_{2n}, Tv) \diamond \frac{d(Sx_{2n}, Bv) + d(Ax_{2n}, Tv)}{2}). \end{aligned}$$

As $n \to \infty$, we get

$$\begin{aligned} d(y, Bv) &\leq k_1(d(y, Tv) \diamond d(y, y)) + k_2(d(y, Tv) \diamond d(Bv, Tv)) \\ &\quad + k_3(d(y, Tv) \diamond \frac{d(y, Bv) + d(y, Tv)}{2}) \\ &\leq k_1 \alpha \max\{d(y, Tv), 0\} + k_2 \alpha \max\{0, d(Bv, y)\} \\ &\quad + k_3 \alpha \max\{0, \frac{d(y, Bv) + 0}{2}\} \\ &< d(y, Bv). \end{aligned}$$

It follows that $Bv = y = Tv$. Since B and T are weakly compatible, we have $BTv = TBv$ and so $By = Ty$.

If $y \neq By$, by (iii), we get

$$\begin{aligned}\lim_{n\to\infty} d(Ax_{2n}, By) \quad &\leq \quad \lim_{n\to\infty}[k_1(d(Sx_{2n}, Ty) \diamond d(Ax_{2n}, Sx_{2n}))\\ &+k_2(d(Sx_{2n}, Ty) \diamond d(By, Ty))\\ &+k_3(d(Sx_{2n}, Ty) \diamond \frac{d(Sx_{2n}, By) + d(Ax_{2n}, Ty)}{2})].\end{aligned}$$

Hence,

$$\begin{aligned}d(y, By) \quad &\leq \quad k_1(d(y, Ty) \diamond d(y, y)) + k_2(d(y, Ty) \diamond d(By, Ty))\\ &+k_3(d(y, Ty) \diamond \frac{d(y, By) + d(y, Ty)}{2})\\ &\leq \quad k_1\alpha \max\{d(y, Ty), d(y, y)\} + k_2\alpha \max\{d(y, Ty), d(By, Ty)\\ &+k_3\alpha \max\{d(y, Ty), \frac{d(y, By) + d(y, Ty)}{2}\}\\ &< \quad d(y, By),\end{aligned}$$

and so $By = y$.

Since $B(X) \subseteq S(X)$, there exists $w \in X$ such that $Sw = y$.

If $Aw \neq y$, by (iii), we have

$$\begin{aligned}d(Aw, By) \quad &\leq \quad k_1(d(Sw, Ty) \diamond d(Aw, Sw)) + k_2(d(Sw, Ty) \diamond d(By, Ty))\\ &+k_3(d(Sw, Ty) \diamond \frac{d(Sw, By) + d(Aw, Ty)}{2}),\end{aligned}$$

and it follows that

$$\begin{aligned}d(Aw, y) \quad &\leq \quad k_1(d(Sw, y) \diamond d(Aw, Sw)) + k_2(d(Sw, y) \diamond d(y, y))\\ &+k_3(d(Sw, y) \diamond \frac{d(Sw, y) + d(Aw, y)}{2})\\ &\leq \quad k_1\alpha \max\{d(Sw, y), d(Aw, Sw)\} + k_2\alpha \max\{d(Sw, y), d(y, y)\}\\ &+k_3\alpha \max\{d(Sw, y), \frac{d(Sw, y) + d(Aw, y)}{2}\}\\ &< \quad d(Aw, y).\end{aligned}$$

This implies that $Aw = y$ and hence $Aw = Sw = y$. Since A and S are weakly compatible, $ASw = SAw$ and so $Ay = Sy$.

If $Ay \neq y$ then by (iii), we get

$$\begin{aligned} d(Ay,y) &= d(Ay,By) \\ &\leq k_1(d(Sy,Ty) \diamond d(Ay,Sy)) + k_2 d(Sy,Ty) \diamond d(By,Ty)) \\ &\quad + k_3(d(Sy,Ty) \diamond \frac{d(Sy,By)+d(Ay,Ty)}{2}) \\ &= k_1(d(Sy,y) \diamond d(Ay,Sy)) + k_2(d(Sy,y) \diamond d(y,y)) \\ &\quad + k_3(d(Sy,y) \diamond \frac{d(Sy,y)+d(Ay,y)}{2}) \\ &\leq k_1\alpha \max\{d(Sy,y), d(Ay,Sy)\} + k_2\alpha \max\{d(Sy,y), d(y,y)\} \\ &\quad + k_3\alpha \max\{d(Sy,y), \frac{d(Sy,y)+d(Ay,y)}{2}\} \\ &< d(Ay,y), \end{aligned}$$

and so $Ay = y$. Thus, $Ay = Sy = By = Ty = y$, that is, y is a common fixed point for A, B, S and T.

The proof is similar when $S(X)$ is assumed to be a closed subset of X.

The uniqueness of y follows from (iii). □

Corollary 2.2. *Let (X,d) be a complete metric space. Let A, B, S and T be self mappings of X into itself satisfying the following conditions*

(i) *$A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$ and $T(X)$ or $S(X)$ is a closed subset of X,*

(ii) *the pairs (A,S) and (B,T) are weakly compatible,*

(iii) *for all $x, y \in X$,*

$$\begin{aligned} d(Ax,By) &\leq k_1(d(Sx,Ty) + d(Ax,Sx)) + k_2(d(Sx,Ty) + d(By,Ty)) \\ &\quad + k_3(d(Sx,Ty) + \frac{d(Sx,By)+d(Ax,Ty)}{2}), \end{aligned}$$

where $k_1, k_2, k_3 > 0$ and $0 < k_1 + k_2 + k_3 < \frac{1}{2}$.

Then A, B, S and T have a unique common fixed point in X.

Proof. Define $a \diamond b = a + b$ for each $a, b \in \mathbf{R}^+$. Then for $\alpha \geq 2$, we have $a \diamond b \leq \alpha \max\{a, b\}$. Putting $\alpha = 2$, we get $0 < \alpha(k_1 + k_2 + k_3) < 1$, and hence all conditions of Theorem 2.1 hold. Therefore A, B, S and T have a unique common fixed point in X. □

3. A further generalization of a contraction principle

In what follows we deal with the class Ψ of all functions $\psi : [0,\infty)^6 \longrightarrow \mathbf{R}$ with the following properties:

(1) (ψ_1): $\psi(u,v,v,u,u+v,0) \le 0$ or $\psi(u,v,u,v,0,u+v) \le 0$ for every $v > 0$ implies that $u < v$ and $v = 0$ implies that $u = 0$;
(2) (ψ_2): ψ is non-increasing in variables t_5 and t_6;
(3) (ψ_3): $\psi(u,u,0,0,u,u) \le 0$ implies that $u = 0$;
(4) (ψ_4): ψ is continuous in each coordinate variable.

Examples of ψ are
$\psi(t_1,t_2,t_3,t_4,t_5,t_6) = t_1 - p\max\{t_2,t_3,t_4,\frac{1}{2}t_5,\frac{1}{2}t_6\}$, $0 < p < 1$;
and
$\psi(t_1,t_2,t_3,t_4,t_5,t_6) = \int_0^{t_1}\phi(s)ds - h\max\{\int_0^{t_i}\phi(s)ds\}$, $i = 2,3,4$, where $0 < h < 1$ and $\phi : \mathbf{R}^+ \longrightarrow \mathbf{R}^+$ is a Lebesgue integrable mapping which is summable non-negative and such that

$$\int_0^{\epsilon}\phi(t)dt > 0 \quad \text{for each} \quad \epsilon > 0.$$

Theorem 3.1. *Let (X,d) be a metric space and $f,g,S,T : X \longrightarrow X$ be mappings such that*

(i) *$f(X) \subseteq T(X)$, $g(X) \subseteq S(X)$, and $E = \{d(fx,Sx)|\ x \in X\}$ is a closed subset of $[0,\infty)$,*

(ii) *the pairs (f,S) and (g,T) are weakly compatible,*

(iii) $\psi\begin{pmatrix} d(fx,gy), d(Sx,Ty), d(Sx,fx), \\ d(gy,Ty), d(Sx,gy), d(Ty,fx) \end{pmatrix} \le 0,$

for all $x,y \in X$ and $\psi \in \Psi$.
Then f,g,S and T have a unique fixed point in X.

Proof. Since E is nonempty and a lower bounded subset of $[0,\infty)$, putting $\alpha = \inf E$, we have $\alpha \in \overline{E} = E$. That is, there exists $u \in X$ such that $\alpha = d(fu,Su)$. Since $fu \in f(X) \subseteq T(X)$, there exists $v \in X$ such that $fu = Tv$. Thus

$$\alpha = d(fu,Su) = d(Tv,Su).$$

We prove that $\alpha = 0$. On letting $\alpha > 0$, by (iii), we get

$$\psi\begin{pmatrix} d(fu,gv), d(Su,Tv), d(Su,fu), \\ d(gv,Tv), d(Su,gv), d(Tv,fu) \end{pmatrix} \le 0.$$

Since $d(Su, gv) \leq d(Su, fu) + d(fu, gv)$, by the above inequality, it follows that

$$\psi\begin{pmatrix} d(fu, gv), \alpha, \alpha, \\ d(gv, fu), \alpha + d(fu, gv), 0 \end{pmatrix} \leq 0,$$

and (ψ_1) implies that $d(fu, gv) < \alpha = d(fu, Su)$. Since $gv \in g(X) \subseteq S(X)$, there exists $w \in X$ such that $Sw = gv$. Similarly, using (iii), we obtain

$$\psi\begin{pmatrix} d(fw, gv), d(Sw, Tv), d(Sw, fw), \\ d(gv, Tv), d(Sw, gv), d(Tv, fw) \end{pmatrix} \leq 0.$$

As $d(fw, Tv) \leq d(fw, Sw) + d(Sw, Tv)$, by the above inequality, we have

$$\psi\begin{pmatrix} d(fw, Sw), d(gv, Tv), d(Sw, fw), \\ d(gv, Tv), 0, d(fw, Sw) + d(gv, Tv) \end{pmatrix} \leq 0.$$

If $d(gv, Tv) = 0$, by (ψ_1), we get $d(fw, Sw) = 0$. Thus

$$\alpha = d(fu, Su) \leq d(fw, Sw) = 0,$$

a contradiction. So $d(gv, Tv) > 0$, and by (ψ_1), we get $d(fw, Sw) < d(gv, Tv)$. Thus

$$\begin{aligned} \alpha = d(fu, Su) &\leq d(fw, Sw) \\ &< d(gv, Tv) \\ &< d(fu, Su) = \alpha \end{aligned}$$

a contradiction. Hence $\alpha = 0$ which implies that $fu = Su = Tv$. If $gv \neq Tv$, by (iii), we get

$$\begin{aligned} & \psi\begin{pmatrix} d(fu, gv), d(Su, Tv), d(Su, fu), \\ d(gv, Tv), d(Su, gv), d(Tv, fu) \end{pmatrix} \\ = \; & \psi\begin{pmatrix} d(Tv, gv), 0, 0, \\ d(gv, Tv), d(Tv, gv), 0 \end{pmatrix} \\ \leq \; & 0. \end{aligned}$$

From (ψ_1) it follows that $gv = Tv$. Hence, $Tv = gv = fu = Su = p$.

By weak compatibility of the pairs (g, T) and (f, S), we have $gp = Tp$ and $fp = Sp$. We now prove that $fp = p$. In fact, if $p \neq fp$, by using (iii), we have

$$
\begin{aligned}
& \psi\begin{pmatrix} d(fp,gv), d(Sp,Tv), d(Sp,fp), \\ d(gv,Tv), d(Sp,gv), d(Tv,fp) \end{pmatrix} \\
= \ & \psi\begin{pmatrix} d(fp,p), d(fp,p), 0, \\ 0, d(fp,p), d(p,fp) \end{pmatrix} \\
\leq \ & 0,
\end{aligned}
$$

and (ψ_3) implies that $p = fp = Sp$. We next prove that $gp = p$. Indeed, if $p \neq gp$, by using (iii), we obtain

$$
\begin{aligned}
\leq \ & \psi\begin{pmatrix} d(fp,gp), d(Sp,Tp), d(Sp,fp), \\ d(gp,Tp), d(Sp,gp), d(Tp,fp) \end{pmatrix} \\
= \ & \psi\begin{pmatrix} d(p,gp), d(p,gp), 0, \\ 0, d(p,gp), d(p,gp) \end{pmatrix} \\
\leq \ & 0,
\end{aligned}
$$

and (ψ_3) implies that $p = gp = Tp$. Therefore, p is a common fixed point of f, g, S and T.

The uniqueness of p follows from (iii).

Example 3.2. Let (X,d) be a metric space with $d(x,y) = |x-y|$. Define the self-maps f, g, S and T on X by

$$fx = gx = 1/2, \ Sx = \frac{x+1}{3} \text{ and } Tx = \frac{2x+1}{4},$$

for all $x \in X$. Hence

$$0 = d(fx,gx) \leq d(Sx,Tx),$$

for every x in X . If we define $\psi(t_1,t_2,t_3,t_4,t_5,t_6) = t_1 - t_2$, it is easy to see that all conditions of Theorem 3.1 hold and there exists a unique $z = 1/2$ such that

$$f(1/2) = g(1/2) = S(1/2) = T(1/2) = 1/2.$$

Corollary 3.3. *Let f_i, g_j, T and S be self-mappings of a complete metric space (X,d) satisfying the following conditions:*

(i) *there exists $i_0, j_0 \in \mathbf{N}$ such that $f_{i_0}(X) \subseteq T(X)$, $g_{j_0}(X) \subseteq S(X)$and $E = \{d(f_{i_0}x, Sx)|\ x \in X\}$ is a closed subset of $[0,\infty)$,*

(ii) *the pairs (f_{i_0}, S) and (g_{j_0}, T) are weakly compatible,*

(iii) $\psi\left(\begin{array}{l} d(f_ix,g_jy),d(Sx,Ty),d(Sx,f_ix), \\ d(g_jy,Ty),d(Sx,g_jy),d(Ty,f_ix) \end{array}\right)\leq 0,$

for all $x,y\in X$, $\psi\in\Psi$ *and* $i,j=1,2,\cdots$.

Then, f_i,g_j,S *and* T *have a unique common fixed point in* X *for all* $i,j=1,2,\cdots$.

Proof. By Theorem 3.1, S,T and f_{i_0} and g_{j_0}, for some i_0, $j_0\in\mathbf{N}$, have a unique common fixed point in X. That is, there exists a unique $z\in X$ such that

$$S(z)=T(z)=f_{i_0}(z)=g_{j_0}(z)=z.$$

Suppose there exists $j\in\mathbf{N}$ such that $j\neq j_0$. Then by (iii) we have

$$\begin{aligned}\psi&\left(\begin{array}{l} d(f_{i_0}z,g_jz),d(Sz,Tz),d(Sz,f_{i_0}z), \\ d(g_jz,Tz),d(Sz,g_jz),d(Tz,f_{i_0}z) \end{array}\right)\\ &=\psi\left(\begin{array}{l} d(z,g_jz),0,0, \\ d(g_jz,z),d(z,g_jz),0 \end{array}\right)\leq 0.\end{aligned}$$

By $(\psi 3)$, it follows that $d(g_jz,z)=0$. Hence for every $j\in\mathbf{N}$, we have $g_j(z)=z$. Similarly for every $i\in\mathbf{N}$, we get $f_iz=z$. Therefore for every $i,j\in\mathbf{N}$, we have

$$f_iz=g_jz=Sz=Tz=z.$$

□

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