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ERRATUM: COUPLED FIXED POINT RESULTS FOR WEAKLY RELATED MAPPINGS IN PARTIALLY ORDERED METRIC SPACES

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(Communicated by Hamid Reza Ebrahimi Vishki)

Abstract. In this note we point out and rectify some errors in a recently published paper "N. Singh, R. Jain: Coupled Fixed Point Results For Weakly Related Mappings in Partially Ordered Metric Spaces, *Bull. Iranian Math. Soc.* 40 (2014), no. 1, 29–40".
Keywords: Coupled fixed point, common coupled fixed point, partially ordered space, weakly related mappings.
MSC(2010): Primary: 47H10; Secondary: 54H25.

In [1], the authors showed the existence of coupled fixed points for the non-decreasing mappings in partially ordered complete metric space using a partial order induced by an appropriate function ϕ. The reader should consult [1] for terms not specifically defined in this note.

Remark 1. The authors in [1] claimed that Example 2.4 supports Theorem 2.3. In Theorem 2.3, the function ϕ is considered to be bounded from above but in Example 2.4, the function $\phi : X(= [0, +\infty)) \to R$ defined by $\phi(x) = 2x$ for $x \in X$ is not bounded from above. Further, the order relation "$\preccurlyeq$' must be induced by ϕ but in Example 2.4 it is considered to be the usual ordering.

We now rectify Example 2.4 as follows:

Example 2. Let $X = [0, 1]$ and $d(x, y) = |x - y|$, then (X, d) is a complete metric space. Let $\phi : X \to R$ be the mapping defined by $\phi(x) = -2x$ for all $x \in X$. Define the relation "$\preccurlyeq$" on X as follows:

$$x \preccurlyeq y \quad \text{iff} \quad d(x, y) \leq \phi(y) - \phi(x).$$

Then "$\preccurlyeq$" is partial order induced by ϕ.

Clearly, ϕ is bounded from above on X.

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Define $F : X \times X \to X$ by $F(x, y) = \dfrac{x(1+y)}{2}$ for all $x, y \in X$.

Then F is non-decreasing function on X.

Let $x_0 = 0$, $y_0 = 1$, then $F(x_0, y_0) = \dfrac{x_0(1+y_0)}{2} = 0$ and

$F(y_0, x_0) = \dfrac{y_0(1+x_0)}{2} = \dfrac{1}{2}$.

Finally, we check that $x_0 \preccurlyeq F(x_0, y_0)$ and $y_0 \preccurlyeq F(y_0, x_0)$.

Now,

$$\begin{aligned} x_0 \preccurlyeq F(x_0, y_0) \ &\text{iff} \ d(x_0, F(x_0, y_0)) \le \phi(F(x_0, y_0)) - \phi(x_0) \\ &\text{iff} \ d(0,0) = 0 \le \phi(0) - \phi(0) = 0, \ \text{which is true.} \end{aligned}$$

Also,

$$\begin{aligned} y_0 \preccurlyeq F(y_0, x_0) \ &\text{iff} \ d(y_0, F(y_0, x_0)) \le \phi(F(y_0, x_0)) - \phi(y_0) \\ &\text{iff} \ d\left(1, \frac{1}{2}\right) \le \phi\left(\frac{1}{2}\right) - \phi(1) \\ &\text{iff} \ \frac{1}{2} \le (-2)\left(\frac{1}{2}\right) - (-2)(1) = 1, \ \text{which is again true.} \end{aligned}$$

Hence all the conditions of Theorem 2.3 are satisfied. Applying Theorem 2.3, $(0, 0)$ is the coupled fixed point of F.

Remark 3. (i) In Example 2, the partial order "$\preccurlyeq$" induced by ϕ is not the usual ordering "$\le$". Clearly, $1 \preccurlyeq \frac{1}{2}$ but $1 \not\le \frac{1}{2}$.

(ii) Again, the authors in [1] claimed that Example 3.3 supports Theorem 3.2. In Theorem 3.2, the function ϕ is considered to be bounded from above but in Example 3.3, the function $\phi : X(= [0, \infty)) \to R$ defined by $\phi(x) = 2x$ for $x \in X$ is not bounded from above. Further, the order relation "$\preccurlyeq$" must be induced by ϕ but in Example 3.3 it is considered to be the usual ordering.

We now rectify Example 3.3 as follows:

Example 4. Let $X = [0, 1]$ and $d(x, y) = |x - y|$, then (X, d) is a complete metric space. Let $\phi : X \to R$ be the mapping defined by $\phi(x) = -2x$ for all $x \in X$. Define the relation "$\preccurlyeq$" on X as follows:

$$x \preccurlyeq y \quad \text{iff} \quad d(x, y) \le \phi(y) - \phi(x).$$

Then "$\preccurlyeq$" is partial order induced by ϕ.

Since $1 \preccurlyeq \frac{1}{2}$, so "$\preccurlyeq$" is not the usual order "$\le$".

Clearly, ϕ is bounded from above on X.

Define $F : X \times X \to X$ by $F(x,y) = \dfrac{x(1+y)}{4}$ for all $x, y \in X$ and $G : X \to X$ by $G(x) = \frac{x}{2}$ for $x \in X$.

Now, $GF(x,y) = \dfrac{x(1+y)}{8}$, $F(Gx,Gy) = F\left(\dfrac{x}{2}, \dfrac{y}{2}\right) = \dfrac{x(2+y)}{16}$,

$GF(y,x) = \dfrac{y(1+x)}{8}$, $F(Gy,Gx) = F\left(\dfrac{y}{2}, \dfrac{x}{2}\right) = \dfrac{y(2+x)}{16}$ for $x, y \in X$.

We first show that the pair $\{F, G\}$ is weakly related.

We consider the following:

(i) $F(x,y) \preccurlyeq GF(x,y)$

$$\begin{aligned}
&\text{iff} \quad d(F(x,y), GF(x,y)) \leq \phi(GF(x,y)) - \phi(F(x,y)) \\
&\text{iff} \quad \left|\frac{x(1+y)}{4} - \frac{x(1+y)}{8}\right| \leq \phi\left(\frac{x(1+y)}{8}\right) - \phi\left(\frac{x(1+y)}{4}\right) \\
&\text{iff} \quad \frac{x(1+y)}{8} \leq \frac{-x(1+y)}{4} + \frac{x(1+y)}{2} \\
&\text{iff} \quad \frac{x(1+y)}{8} \leq \frac{x(1+y)}{4}, \text{ which is true for } x, y \in X.
\end{aligned}$$

(ii) $Gx \preccurlyeq F(Gx, Gy)$

$$\begin{aligned}
&\text{iff } d(Gx, F(Gx,Gy)) \leq \phi(F(Gx,Gy)) - \phi(Gx) \\
&\text{iff } \left|\frac{x}{2} - \frac{x(2+y)}{16}\right| \leq \phi\left(\frac{x(2+y)}{16}\right) - \phi\left(\frac{x}{2}\right) \\
&\text{iff } \left|\frac{6x - xy}{16}\right| \leq \frac{-x(2+y)}{8} + x \\
&\text{iff } \left|\frac{6x - xy}{16}\right| \leq \frac{6x - xy}{8}, \text{ which is true for } x, y \in X.
\end{aligned}$$

Similarly, we show that $F(y,x) \preccurlyeq GF(y,x)$ and $Gy \preccurlyeq F(Gy,Gx)$ for all $x, y \in X$. Hence the pair $\{F, G\}$ is weakly related.

Let $x_0 = 0$, $y_0 = 1$, then $F(x_0, y_0) = 0$ and $F(y_0, x_0) = \dfrac{1}{4}$.

Finally, we check that $x_0 \preccurlyeq F(x_0, y_0)$ and $y_0 \preccurlyeq F(y_0, x_0)$.

$$\begin{aligned}
x_0 \preccurlyeq F(x_0,y_0) &\text{ iff } d(x_0, F(x_0,y_0)) \leq \phi(F(x_0,y_0)) - \phi(x_0) \\
&\text{ iff } d(0,0) \leq \phi(0) - \phi(0), \text{ which is true.} \\
y_0 \preccurlyeq F(y_0,x_0) &\text{ iff } d(y_0, F(y_0,x_0)) \leq \phi(F(y_0,x_0)) - \phi(y_0) \\
&\text{ iff } d\left(1, \frac{1}{4}\right) \leq \phi\left(\frac{1}{4}\right) - \phi(1) \\
&\text{ iff } \frac{3}{4} \leq (-2)\left(\frac{1}{4}\right) - (-2)(1) \\
&\text{ iff } \frac{3}{4} \leq -\frac{1}{2} + 2 = \frac{3}{2}, \text{ which is true.}
\end{aligned}$$

Hence all the conditions of Theorem 3.2 are satisfied. Applying Theorem 3.2, $(0, 0)$ is the common coupled fixed point of the pair $\{F, G\}$.

References

[1] N. Singh and R. Jain, Coupled fixed point results for weakly related mappings in partially ordered metric spaces, *Bull. Iranian Math. Soc.* **40** (2014), no. 1, 29–40.

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