ISSN: 1017-060X (Print)

ISSN: 1735-8515 (Online)

Bulletin of the

Iranian Mathematical Society

Vol. 42 (2016), No. 2, pp. 489–497

Title:

Pseudo Ricci symmetric real hypersurfaces of a complex projective space

Author(s):

S. K. Hui and Y. Matsuyama

Published by Iranian Mathematical Society
http://bims.ims.ir

Bull. Iranian Math. Soc.
Vol. 42 (2016), No. 2, pp. 489–497
Online ISSN: 1735-8515

PSEUDO RICCI SYMMETRIC REAL HYPERSURFACES OF A COMPLEX PROJECTIVE SPACE

S. K. HUI* AND Y. MATSUYAMA

(Communicated by Jost-Hinrich Eschenburg)

Abstract. Pseudo Ricci symmetric real hypersurfaces of a complex projective space are classified and it is proved that there are no pseudo Ricci symmetric real hypersurfaces of the complex projective space CP^n for which the vector field ξ from the almost contact metric structure (ϕ, ξ, η, g) is a principal curvature vector field.

Keywords: real hypersurface, complex projective space, pseudo Ricci symmetric.
MSC(2010): Primary: 53C25; Secondary: 53C40.

1. **Introduction**

Let CP^n, $n \geq 2$ be an n-dimensional complex projective space with Fubini-Study metric of constant holomorphic sectional curvature 4 and let M be a real hypersurface of CP^n. Then M has an almost contact metric structure (ϕ, ξ, η, g) induced from the Kähler structure of CP^n. The study of the geometry of real hypersurfaces in the space forms has been an object of significant interest. Important past results have been, for example, the classification of homogeneous real hypersurfaces in CP^n by Takagi [34], the characterization of real hypersurfaces with constant principal curvatures by Kimura [17] and the studies of real hypersurfaces in CP^n with special Ricci tensor by Hamada [9], by Cecil and Ryan [3] and by Ki [14]. Many differential geometers have studied real hypersurfaces of a complex projective space such as Bejancu and Deshmukh [2], Cho and Ki [6], Deshmukh [7], Hamada [10,11], Ikuta [13], Kimura [15,16,18], Kimura and Maeda [19], Maeda [20,21], Maeda [22], Matsuyama [23–25], Okumura [26], Perez et. al [29,30], [31,32], Takagi [35,36], Wang [37] and others.

It is well known that there does not exist a real hypersurface M of CP^n

Article electronically published on April 30, 2016.
Received: 8 May 2014, Accepted: 25 February 2015.
*Corresponding author.

satisfying the condition that the second fundamental tensor A of M is parallel. Again, in [10], Hamada used the condition that the second fundamental tensor A is recurrent, i.e. there exists an 1-form α such that $\nabla A = A \otimes \alpha$. And Hamada [10] proved that there are no real hypersurfaces of a complex projective space with recurrent second fundamental tensor. In this connection, Hui and Matsuyama [12] studied real hypersurfaces of a complex projective space with pseudo parallel second fundamental tensor. Again many differential geometers studied real hypersurfaces of complex projective space satisfying some condition of Ricci tensor. In [14] Ki proved that there are no real hypersurfaces of a complex projective space with parallel Ricci tensor. Again, in [11], Hamada studied the real hypersurfaces of a complex projective space with recurrent Ricci tensor and proved that there are no real hypersurfaces with recurrent Ricci tensor of CP^n under the condition that ξ is a principal curvature vector.

A Riemannian space is said to be Ricci symmetric if its Ricci tensor S of type (0,2) satisfies $\nabla S = 0$, where ∇ denotes the Riemannian connection. During the last five decades, the notion of Ricci symmetry has been weakened by many authors in several ways to a different extent such as Ricci recurrent space [28], Ricci semisymmetric space [33], pseudo Ricci symmetric space by Deszcz [8] and pseudo Ricci symmetric space by Chaki [4].

A non-flat Riemannian space (M^n, g) is said to be pseudo Ricci symmetric [4] if its Ricci tensor S of type (0,2) is not identically zero and satisfies the condition

$$(\nabla_X S)(Y,Z) = 2\alpha(X)S(Y,Z) + \alpha(Y)S(X,Z) + \alpha(Z)S(Y,X), \tag{1.1}$$

for any vector field X, Y, Z, where α is a nowhere vanishing 1-form and ∇ denotes the operator of covariant differentiation with respect to the metric tensor g. The pseudo Ricci symmetric spaces have been also studied by Arslan et. al [1], Chaki and Saha [5], Özen [27] and many others.
The relation (1.1) can be written as

$$(\nabla_X Q)Y = 2\alpha(X)QY + \alpha(Y)QX + S(Y,X)\rho, \tag{1.2}$$

where ρ is the vector field associated to the 1-form α such that $\alpha(X) = g(X,\rho)$ and Q is the Ricci operator, i.e., $g(QX,Y) = S(X,Y)$ for all X, Y.

Motivated by the above studies the present paper deals with the study of pseudo Ricci symmetric real hypersurfaces of a complex projective space. The paper is organized as follows. Section 2 is concerned with some preliminaries. Section 3 is devoted to the study of pseudo Ricci symmetric real hypersurfaces of a pseudo Ricci symmetric complex projective space and it is proved that there are no pseudo Ricci symmetric real hypersurfaces of the complex projective space CP^n for which the vector field ξ from the almost contact metric structure (ϕ, ξ, η, g) is a principal curvature vector field.

2. Preliminaries

Let M be a real hypersurface of the complex projective space CP^n equipped with the Fubini-Study Riemannian metric G. We denote the induced metric on M by g, and let N be a unit normal vector field to M. Then the Kähler structure J on CP^n induces an almost contact metric structure (ϕ, ξ, η, g) on M, i.e., in this setting we have a tensor field ϕ of type (1,1) on M characterized by $g(\phi X, Y) = G(JX, Y)$, and a vector field ξ, an 1-form η on M charaterized by $g(\xi, X) = \eta(X) = G(JX, N)$, and with these data, the following relations hold:

$$\phi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = g(\xi, \xi) = 1 \quad \text{and } \phi\xi = 0. \tag{2.1}$$

The Riemannian connections $\widetilde{\nabla}$ in CP^n and ∇ in M are related by

$$\widetilde{\nabla}_X Y = \nabla_X Y + g(AX, Y)N, \tag{2.2}$$

$$\widetilde{\nabla}_X N = -AX \tag{2.3}$$

for arbitrary vector fields X and Y on M, where A is the second fundamental tensor of M in CP^n. Let TM be the tangent bundle of M. An eigenvector X of the second fundamental tensor A is called a principal curvature vector. Also an eigenvalue λ of A is called a principal curvature.
Now it follows from (2.2) that

$$(\nabla_X \phi)Y = \eta(Y)AX - g(AX, Y)\xi, \tag{2.4}$$

$$\nabla_X \xi = \phi AX. \tag{2.5}$$

Let $\widetilde{R}$ and R be the curvature tensors of CP^n and M respectively. From the expression of the curvature tensor of CP^n, we see that the curvature tensor, Codazzi equation and the Ricci tensor of type (1,1) are given by

$$\begin{aligned} R(X,Y)Z &= g(Y,Z)X - g(X,Z)Y + g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y \\ &- 2g(\phi X, Y)\phi Z + g(AY, Z)AX - g(AX, Z)AY, \end{aligned} \tag{2.6}$$

$$(\nabla_X A)Y - (\nabla_Y A)X = \eta(X)\phi Y - \eta(Y)\phi X - 2g(\phi X, Y)\xi, \tag{2.7}$$

$$QX = (2n+1)X - 3\eta(X)\xi + hAX - A^2 X, \tag{2.8}$$

where $h = traceA$.
Again we have

$$\begin{aligned} (\nabla_X Q)Y &= -3g(\phi AX, Y)\xi - 3\eta(Y)\phi AX + (Xh)AY \\ &+ h(\nabla_X A)Y - A(\nabla_X A)Y - (\nabla_X A)AY. \end{aligned} \tag{2.9}$$

Also we recall the following:

Lemma 2.1. *[22] If ξ is a principal curvature vector, then the corresponding principal curvature a is locally constant.*

Lemma 2.2. *[22] Assume that ξ is a principal curvature vector and the corresponding principal curvature is a. If $AX = \lambda X$ for $X \perp \xi$, then we have $A\phi X = \overline{\lambda}$, where $\overline{\lambda} = \frac{(a\lambda+2)}{(2\lambda-a)}$.*

Theorem 2.3. *[3] Let M be a connected real hypersurface of CP^n, $n \geq 3$, whose Ricci tensor S satisfies $S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y)$ for some smooth functions a and b on M. Then M is locally congruent to one of the following:*
(i) *a geodesic hyper-sphere,*
(ii) *a tube of radius r over a totally geodesic CP^k, $1 \leq k \leq n-2$, where $0 < r < \frac{\pi}{2}$ and $\cot^2 r = \frac{k}{n-k-1}$,*
(iii) *a tube of radius r over a complex quadric Q_{n-1}, where $0 < r < \frac{\pi}{4}$ and $\cot^2 2r = n-2$.*

Theorem 2.4. *[34] Let M be a homogeneous real hypersurface of CP^n. Then M is a tube of radius r over one of the following Kähler submanifolds:*
(A_1) hyperplane CP^{n-1}, where $0 < r < \frac{\pi}{2}$,
(A_2) totally geodesic CP^k, $(1 \leq k \leq n-2)$, where $0 < r < \frac{\pi}{2}$,
(B) complex quadric Q_{n-1}, where $0 < r < \frac{\pi}{4}$,
(C) $CP^1 \times CP^{\frac{n-1}{2}}$, where $0 < r < \frac{\pi}{4}$ and $n(\geq 5)$ is odd,
(D) complex Grassman $cG_{2,5}$, where $0 < r < \frac{\pi}{4}$ and $n = 9$,
(E) Hermitian symmetric space $SO(10)/U(5)$, where $0 < r < \frac{\pi}{4}$ and $n = 15$.

Theorem 2.5. *[16] Let M be a real hypersurface of CP^n. Then M has constant principal curvatures and ξ is a principal curvature vector if and only if M is locally congruent to a homogeneous real hypersurface.*

Theorem 2.6. *[14] There are no real hypersurfaces with parallel Ricci tensor of a complex space form cM^n, $c \neq 0$.*

3. Pseudo Ricci symmetric Real hypersurfaces of a complex projective space CP^n

In this section, we have studied pseudo Ricci symmetric real hypersurfaces of a complex projective space CP^n and prove the following:

Lemma 3.1. *Let M be a connected pseudo Ricci symmetric real hypersurface of a complex projective space CP^n. If all eigenvalues of the Ricci operator Q are constant then the Ricci tensor S of M is parallel.*

Proof. Let $QX = \lambda X$, $QY = \mu Y$ and $QZ = \nu Z$. Then we have

$$g((\nabla_X Q)Y, Z) = (\mu - \nu)g(\nabla_X Y, Z). \tag{3.1}$$

Again we have from (1.2) that

$$g((\nabla_X Q)Y, Z) = 2\alpha(X)\mu g(Y,Z) + \alpha(Y)\nu g(X,Z) + \alpha(Z)\mu g(X,Y).$$

If $\lambda \neq \mu$, $\lambda \neq \nu$ and $\mu \neq \nu$ then $g((\nabla_X Q)Y, Z) = 0$. In the treatment of the case $\mu = \nu$, it follows from (3.1) that $g((\nabla_X Q)Y, Z) = 0$.
Assume that $\mu \neq \nu$, $\lambda = \mu$. Then $g((\nabla_Z Q)X, Y) = 0$.
On the other hand, we have

$$g((\nabla_Z Q)X, Y) = 2\alpha(Z)\mu g(X, Y) + \big[\alpha(X)S(Z, Y) + \alpha(Y)S(X, Z)\big].$$

Thus we obtain $\alpha(Z)\mu g(X, Y) = 0$. Hence $g((\nabla_X Q)Y, Z) = 0$. Consequently, the Ricci tensor S of M is parallel.

From (2.8) and since ξ is principal, the principal curvature vector will also be eigenvectors of S. Thus Ricci tensor of a homogeneous real hypersurface has constant eigenvalues. Again the hypersurface listed in Theorem 2.2 do not have parallel Ricci tensor. Thus from Lemma 3.1 and Theorem 2.3, we obtain

Proposition 3.2. *A homogeneous real hypersurface of CP^n can not be pseudo Ricci symmetric.*

Since all the surfaces in (i) - (iii) of Theorem 2.1 are homogeneous, so by using Theorem 2.1, we have

Corollary 3.3. *A real hypersurface of CP^n, $n \geq 3$ whose Ricci tensor S satisfies $S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y)$ for some smooth functions a and b on M, can not be pseudo Ricci symmetric.*

Now we prove the following:

Theorem 3.4. *There are no pseudo Ricci symmetric real hypersurfaces in the complex projective space CP^n, for which ξ is a principal curvature vector field.*

Proof. Assume that M is a real hypersurface in complex projective space CP^n (with almost contact metric structure (ϕ, ξ, η, g)), which is pseudo Ricci symmetric and for which ξ is a principal curvature vector field. Then by virtue of (2.8) it follows from (1.2) that

$$\begin{aligned} g((\nabla_X Q)Y, Z) \;&=\; 2\alpha(X)\big[(2n+1)g(Y, Z) - 3\eta(Y)\eta(Z) \\ &+\; hg(AY, Z) - g(A^2Y, Z)\big] \\ &+\; \alpha(Y)\big[(2n+1)g(X, Z) - 3\eta(X)\eta(Z) \\ &+\; hg(AX, Z) - g(A^2X, Z)\big] \\ &+\; \alpha(Z)\big[(2n+1)g(X, Y) - 3\eta(X)\eta(Y) \\ &+\; hg(AX, Y) - g(A^2X, Y)\big]. \end{aligned} \tag{3.2}$$

Using (2.9) in (3.2), we get

$$
\begin{aligned}
(3.3) \qquad & 2\alpha(X)\big[(2n+1)g(Y,Z)-3\eta(Y)\eta(Z)+hg(AY,Z)-g(A^2Y,Z)\big]\\
+ \; & \alpha(Y)\big[(2n+1)g(X,Z)-3\eta(X)\eta(Z)+hg(AX,Z)-g(A^2X,Z)\big]\\
+ \; & \alpha(Z)\big[(2n+1)g(X,Y)-3\eta(X)\eta(Y)+hg(AX,Y)-g(A^2X,Y)\big]\\
+ \; & 3\eta(Z)g(\phi AX,Y)+3\eta(Y)g(\phi AX,Z)-(Xh)g(AY,Z)\\
- \; & hg((\nabla_X A)Y,Z)+g(A(\nabla_X A)Y,Z)+g((\nabla_X A)AY,Z)=0
\end{aligned}
$$

for any tangent vectors X, Y, Z.
Putting $Y=\xi$ and $Z=\phi X$ in (3.3), we get

$$
\begin{aligned}
(3.4) \quad & 2\alpha(X)\big[hg(A\xi,\phi X)-g(A^2\xi,\phi X)\big]+\alpha(\xi)\big[hg(AX,\phi X)-g(A^2X,\phi X)\big]\\
+ \; & \alpha(\phi X)\big[2(n-1)\eta(X)+hg(AX,\xi)-g(A^2X,\xi)\big]\\
+ \; & 3g(AX,X)-3\eta(AX)\eta(X)-(Xh)g(A\xi,\phi X)\\
- \; & hg((\nabla_X A)\xi,\phi X)+g(A(\nabla_X A)\xi,\phi X)+g((\nabla_X A)A\xi,\phi X)=0.
\end{aligned}
$$

Let us assume $A\xi=a\xi$. Then by Lemma 2.1, a is constant and hence we get

$$(3.5) \qquad (\nabla_X A)\xi = a\phi AX - A\phi AX.$$

Using (3.5) in (3.4), we obtain

$$
\begin{aligned}
(3.6) \qquad & \alpha(\xi)\big[hg(AX,\phi X)-g(A^2X,\phi X)\big]+2(n-1)\alpha(\phi X)\eta(X)\\
+ \; & 3g(AX,X)-3a\big(\eta(X)\big)^2-hag(\phi AX,\phi X)\\
+ \; & hg(A\phi AX,\phi X)-g(A\phi AX,A\phi X)+a^2g(\phi AX,\phi X)=0
\end{aligned}
$$

for any tangent vector X on M. We choose X as a unit principal curvature vector orthogonal to ξ and by virtue of Lemma 2.2, we have

$$AX=\lambda X \quad \text{and } A\phi X=\overline{\lambda}X,$$

where

$$(3.7) \qquad \overline{\lambda}=\frac{a\lambda+2}{2\lambda-a}.$$

Therefore we obtain

$$(3.8) \qquad \lambda\big[\overline{\lambda}^2-\{h+\alpha(\xi)\}\overline{\lambda}-(a^2-ha+3)\big]=0.$$

Again from (3.7), we get

$$(3.9) \qquad \lambda\overline{\lambda}=\frac{\lambda+\overline{\lambda}}{2}a+1.$$

If $\lambda=\overline{\lambda}$ then (3.9) yields

$$(3.10) \qquad \lambda^2=a\lambda+1.$$

If 0 occurs as a principal curvature (for a principal vector orthogonal to ξ), then it follows from (3.9) that all principal curvatures are constant.
Next assuming that 0 is not a principal curvature (again we consider only

directions orthogonal to ξ), the relation (3.8) shows that there are at most two distinct principal curvatures. If λ and $\overline{\lambda}$ are distinct then we have

$$\lambda + \overline{\lambda} = h + \alpha(\xi) \quad \text{and } \lambda\overline{\lambda} = -(a^2 - ha + 3),$$

which yields

$$-(a^2 - ha + 3) = \frac{\{h + \alpha(\xi)\}a}{2} + 1,$$

i.e.

$$a^2 - \frac{\{h - \alpha(\xi)\}a}{2} + 4 = 0.$$

Thus the coefficients in (3.8) are constants and hence so are λ and $\overline{\lambda}$. The final possibility is that all principal curvatures (with principal vectors orthogonal to ξ) satisfy (3.10) and are again constant.

Hence by Theorem 2.3, it follows that M is locally homogeneous and therefore the Ricci operator of M has constant eigenvalues. Since M is also pseudo Ricci symmetric, so by Lemma 3.1, it follows that the Ricci tensor of M is parallel.

On the other hand, the homogeneous real hypersurfaces of complex projective space CP^n have been classified by Takagi [34] (Theorem 2.2), and it turns out that none of them have parallel Ricci tensor. Therefore we arrive at a contradiction, and thus, no such hypersurface M exists. □

Acknowledgments

This work was done while the first author (S. K. Hui) was a visiting scholar at Chuo University, Japan. The authors wish to express their sincere gratitude to the refree for the valuable suggestions to the improvement of the paper.

References

[1] K. Arslan, R. Ezentas, C. Murathan and C. Özgür, On pseudo Ricci symmetric manifolds, *Balkan J. Geom. and Appl.* **6** (2001), no. 2, 1–5.

[2] A. Bejancu and S. Deshmukh, Real hypersurfaces of CP^n with non-negative Ricci curvature, *Proc. Amer. Math. Soc.* **124** (1996), no. 1, 269–274.

[3] T. E. Cecil and P. J. Ryan, Focal sets and real hypersurfaces in complex projective space, *Trans Amer. Math. Soc.* **269** (1982), no. 2, 481–499.

[4] M. C. Chaki, On pseudo Ricci symmetric manifolds, *Bulgar. J. Phys.* **15** (1988), no. 6, 526–531.

[5] M. C. Chaki and S. K. Saha, On pseudo-projective Ricci symmetric manifolds, *Bulgar. J. Phys.* **21** (1994), no. 1-2, 1–7.

[6] J. T. Cho and U.-Hang Ki, Real hypersurfaces of a complex projective space in terms of the Jacobi operators, *Acta Math. Hungar.* **80** (1998), no. 1-2, 155–167.

[7] S. Deshmukh, Real hypersurfaces of a complex projective space, *Proc. Indian Acad. Math. Sci.* **121** (2011), no. 2, 171–179.

[8] R. Deszcz, On Ricci-pseudosymmetric warped products, *Demonstratio Math.* **22** (1989), no. 4, 1053–1065.

[9] T. Hamada, On real hypersurfaces of a complex projective space with η-recurrent second fundamental tensor, *Nihonkai Math. J.* **6** (1995), no. 2, 153–163.

[10] T. Hamada, On real hypersurfaces of a complex projective space with recurrent second fundamental tensor, *J. Ramanujan Math. Soc.* **11** (1996), no. 2, 103–107.

[11] T. Hamada, On real hypersurfaces of a complex projective space with recurrent Ricci tensor, *Glasg. Math. J.* **41** (1999), no. 3, 297–302.

[12] S. K. Hui and Y. Matsuyama, On real hypersurfaces of a complex projective space with pseudo parallel second fundamental tensor, *Kobe J. Math.*, Accepted.

[13] K. Ikuta, Real hypersurfaces of a complex projective space, *J. Korean Math. Soc.* **36** (1999), no. 4, 725–736.

[14] U.-Hang Ki, Real hypersurfaces with parallel Ricci tensor of a complex space form, *Tsukuba J. Math.* **13** (1989), no. 1, 73–81.

[15] M. Kimura, Real hypersurfaces of a complex projective space, *Bull. Austral. Math. Soc.* **33** (1986) no. 3, 383–387.

[16] M. Kimura, Real hypersurfaces and complex submanifolds in a complex projective space, *Trans. Amer. Math. Soc.* **296** (1986), no. 1, 137–149.

[17] M. Kimura, Sectional curvatures of holomorphic planes on a real hypersurface in CP^n, *Math. Ann.* **27** (1987), no. 3, 487–497.

[18] M. Kimura, Some real hypersurfaces of a complex projective space, *Saitama Math. J.* **5** (1987), 1–5. correction in **10** (1992) 33–34.

[19] M. Kimura and S. Maeda, On real hypersurfaces of a complex projective space, *Math. Z.* **202** (1989), no. 3, 299–311.

[20] S. Maeda, Real hypersurfaces of complex projective space, *Math. Ann.* **263** (1983), no. 4, 473–478.

[21] S. Maeda, Ricci tensors of real hypersurfaces in a complex projective space, *Proc. Amer. Math. Soc.* **122** (1994), no. 4, 1229–1235.

[22] Y. Maeda, On real hypersurfaces of a complex projective space, *J. Math. Soc. Japan* **28** (1976), no. 3, 529–540.

[23] Y. Matsuyama, A characterization of real hypersurfaces in complex projective space, *J. Institute Sci. and Eng., Chuo Univ.* **2** (1996) 11–13.

[24] Y. Matsuyama, A characterization of real hypersurfaces in complex projective space II, *J. Institute Sci. and Eng., Chuo Univ.* **3** (1997) 1–3.

[25] Y. Matsuyama, A characterization of real hypersurfaces in complex projective space III, *Yokohama Math. J.* **46** (1999) 119–126.

[26] M. Okumura, On some real hypersurfaces of a complex projective space, *Trans. Amer. Math. Soc.* **212** (1975) 355–364.

[27] F. Özen, On pseudo M-projective Ricci symmetric manifolds, *Int. J. Pure Appl. Math.* **72** (2011), no. 2, 249–258.

[28] E. M. Patterson, Some theorems on Ricci-recurrent spaces, *J. London Math. Soc.* **27** (1952) 287–295.

[29] J. D. Perez and F. G. Santos, On the Lie derivative of structure Jacobi operator of real hypersurfaces in complex projective space, *Publ. Math. Debrecen* **66** (2005), no. 3-4, 269–282.

[30] J. D. Perez and F. G. Santos, Real hypersurfaces in complex projective space with recurrent structure Jacobi operator, *Differential Geom. Appl.* **26** (2008), no. 2, 218–223.

[31] J. D. Perez, F. G. Santos and Y. J. Suh, Real hypersurfaces in complex projective space whose structure Jacobi operator is Lie ξ-parallel, *Differential Geom. Appl.* **22** (2005), no. 2, 181–188.

[32] J. D. Perez, F. G. Santos and Y. J. Suh, Real hypersurfaces in complex projective space whose structure Jacobi operator is $\mathbb{D}$-parallel, *Bull. Belg. Math. Soc. Simon Stevin* **13** (2006), no. 3, 459–469.

[33] Z. I. Szabó, Structure theorems on Riemannian spaces satisfying $R(X, Y) \cdot R = 0$. I. The local version, *J. Differential Geom.* **17** (1982), no. 4, 531–582.
[34] R. Takagi, On homogeneous real hypersurfaces in a complex projective space, *Osaka J. Math.* **10** (1973) 495–506.
[35] R. Takagi, Real hypersurfaces in a complex projective space with constant principal curvatures, *J. Math. Soc. Japan* **27** (1975) 43–53.
[36] R. Takagi, Real hypersurfaces in complex projective space with constant principal curvatures II, *J. Math. Soc. Japan* **27** (1975), no. 4, 507–516.
[37] Q. M. Wang, Real hypersurfaces with constant principal curvatures in complex projective space I, *Sci. Sinica Ser. A* **26** (1983), no. 10, 1017–1024.

(Shyamal Kumar Hui) Department of Mathematics, Sidho Kanho Birsha University, Purulia-723104, West Bengal, India.
Department of Mathematics, Bankura University, Bankura-722155, West Bengal, India.

E-mail address: shyamal_hui@yahoo.co.in

(Yoshio Matsuyama) Department of Mathematics, Chuo University, Faculty of Science and Engineering, 1-13-27 Kasuga, Bunkyo-ku, Tokyo 112-8551, Japan.

E-mail address: matuyama@math.chuo-u.ac.jp