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THE RAMSEY NUMBERS OF LARGE TREES VERSUS WHEELS

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Abstract. For two given graphs G_1 and G_2, the Ramsey number $R(G_1, G_2)$ is the smallest integer n such that for any graph G of order n, either G contains G_1 or the complement of G contains G_2. Let T_n denote a tree of order n and W_m a wheel of order $m+1$. To the best of our knowledge, only $R(T_n, W_m)$ with small wheels are known. In this paper, we show that $R(T_n, W_m) = 3n - 2$ for odd m with $n > 756m^{10}$.
Keywords: Ramsey number, tree, wheel.
MSC(2010): Primary: 05C55; Secondary: 05C15.

All graphs considered in this paper are finite simple graphs without loops. For two given graphs G_1 and G_2, the *Ramsey number* $R(G_1, G_2)$ is the smallest integer n such that for any graph G of order n, either G contains G_1 or $\overline{G}$ contains G_2, where $\overline{G}$ is the complement of G. Let $|G|$ be the number of vertices of G. The *neighborhood* $N(v)$ of a vertex v is the set of vertices adjacent to v in G and $N[v] = N(v) \cup \{v\}$. The *minimum degree* of G is denoted by $\delta(G)$. We use T_n to denote a tree of order n. We use C_m and mK_n to denote a cycle of order m and the disjoint union of m copies of K_n, respectively. A *Wheel* $W_m = K_1 + C_m$ is a graph of $m+1$ vertices, where K_1 is called the hub of the wheel.

Ramsey number involving trees or wheels have been studied in several research, for a survey see [8]. Some Ramsey values $R(T_n, W_m)$ for small wheels W_5, W_6, W_7, W_9 have been shown in [2, 5–7, 9]. To the best of our knowledge, there is no other known tree-wheel Ramsey values. In this paper, we evaluate the Ramsey numbers of $R(T_n, W_m)$ for large trees and wheels. The main result of this paper is the following theorem.

Theorem 0.1. *$R(T_n, W_m) = 3n - 2$ for odd m with $n > 756m^{10}$.*

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In [3], Burr et al. considered the Ramsey number involving a tree versus a cycle and established the following result.

Lemma 0.2. *[3]* $R(T_n, C_m) = 2n - 1$ *if* m *is odd and* $n > 756m^{10}$.

Proof of Theorem. Let G be a graph with $|G| = 3n - 2$ such that m is odd and $n > 756m^{10}$. If there is a vertex $v \in V(G)$ such that $|N[v]| \leq n - 1$, we have $|G - N[v]| \geq 2n - 1 \geq R(T_n, C_m)$ and hence $\overline{G} - N[v]$ contains a C_m. Therefore, $\overline{G}$ contains a $W_m = \{v\} + C_m$. Otherwise, for any vertex v, $|N[v]| \geq n$, which shows that $\delta(G) \geq n - 1$. So G contains every tree with n vertices(See Ex. 4.1.9 [1]). Hence, we have $R(T_n, W_m) \leq 3n - 2$. The lower bound is due to $3K_{n-1}$. □

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