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ON LIST VERTEX 2-ARBORICITY OF TOROIDAL GRAPHS WITHOUT CYCLES OF SPECIFIC LENGTH

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Abstract. The vertex arboricity $\rho(G)$ of a graph G is the minimum number of subsets into which the vertex set $V(G)$ can be partitioned so that each subset induces an acyclic graph. A graph G is called list vertex k-arborable if for any set $L(v)$ of cardinality at least k at each vertex v of G, one can choose a color for each v from its list $L(v)$ so that the subgraph induced by every color class is a forest. The smallest k for a graph to be list vertex k-arborable is denoted by $\rho_l(G)$. Borodin, Kostochka and Toft (Discrete Math. 214 (2000) 101-112) first introduced the list vertex arboricity of G. In this paper, we prove that $\rho_l(G) \leq 2$ for any toroidal graph without 5-cycles. We also show that $\rho_l(G) \leq 2$ if G contains neither adjacent 3-cycles nor cycles of lengths 6 and 7.
Keywords: Vertex arboricity, toroidal graph, structure, cycle.
MSC(2010): Primary: 05C15; Secondary: 05C70.

1. **Introduction**

All graphs considered in this paper are finite and simple. For a graph G we denote its vertex set, edge set, maximum degree and minimum degree by $V(G)$, $E(G)$, $\Delta(G)$ and $\delta(G)$, respectively.

The *vertex arboricity* of a graph G, denoted by $\rho(G)$, is the smallest number of subsets that the vertices of G can be partitioned into such that each subset induces an acyclic subgraph, i.e. a forest. There is an equivalent definition to the vertex arboricity in terms of the coloring version. An *acyclic* k-coloring of a graph G is a mapping π from $V(G)$ to the set $\{1, 2, \cdots, k\}$ such that each color class induces an acyclic subgraph. The *vertex arboricity* $\rho(G)$ of G is the smallest integer k such that G has an acyclic k-coloring.

Vertex arboricity, also known as point arboricity, was first introduced in 1968 by Chartrand, Kronk and Wall [10] who proved for any graph G with maximum degree Δ that $\rho(G) \leq \lceil \frac{\Delta+1}{2} \rceil$, and that if G is a planar graph, then $\rho(G) \leq 3$.

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Shortly afterward, Chartrand and Kronk [9] showed that their result for planar graphs was sharp and that if G is an outerplanar graph, then $\rho(G) \leq 2$. A result analogous to Brooks's Theorem for vertex arboricity was proved in 1975 by Kronk and Mitchenm [23]; which showed that if G is neither a cycle, nor a complete graph with an odd number of vertices, then $\rho(G) \leq \lceil \frac{\Delta}{2} \rceil$.

The upper bound 3 for $\rho(G)$ on planar graphs has also been studied by Goddard [18], Grunbaum [19] and Poh [25]. Among them, Goddard and Poh, independently, proved a stronger result that the vertex set of any planar graph can be partitioned into three sets such that each set induces a linear forest, that is, a forest satisfies the condition that every component is a path. The path version of vertex arboricity, called linear vertex arboricity, has also been studied extensively in [1, 2, 24].

It was known [17] that determining the vertex arboricity of a graph is NP-hard. Hakimi and Schmeichel [20] showed that determining whether $\rho(G) \leq 2$ is NP-complete for maximal planar graphs. Stein [29], Hakimi and Schmeichel [20] gave a complete characterization of maximal planar graphs with vertex arboricity 2. The reader is referred to [6, 8, 11, 28, 31, 34, 27] for other results about the vertex arboricity of graphs.

A k-cycle is a cycle with k edges. Recently, in 2008, Raspaud and Wang [26] proved that $\rho(G) \leq 2$ if G is a planar graph with no k-cycles for some fixed $k \in \{3, 4, 5, 6\}$ or no triangles at distance less than 2. In 2012, Huang et al. [21] extended the result by showing that $\rho(G) \leq 2$ if G is a planar graph with no 7-cycles, and Chen et al. [12] proved that $\rho(G) \leq 2$ if G is a planar graph with no intersecting triangles.

List-colourings, in which each element is coloured from its own list of colours, were introduced independently by Vizing [30] in 1976 and by Erdös et al. [16] in 1980. In 2000, Borodin, Kostochka and Toft [5] combined and extended the ideas of vertex arboricity and choosability to introduce list vertex arboricity. A graph G is called list vertex k-arborable if for any set $L(v)$ of cardinality at least k at each vertex v of G, one can choose a color for each v from its list $L(v)$ so that the subgraph induced by every color class is a forest. Note that if the list $L(v)$ does not vary from a vertex to another, we have a problem of usual vertex arboricity. The smallest k for a graph to be list vertex k-arborable is denoted by $\rho_l(G)$. Borodin et al. obtained the list vertex arboricity analogue of Brook's Theorem in [5]. It is trivial to see that $\rho(G) \leq \rho_l(G)$. Xue and Wu [33] proved that the list vertex arboricity of bipartite graphs can be arbitrarily large.

In 2008, Borodin and Ivanova [4] proved that every planar graph with no triangles at distance less than 2 is list vertex 2-arborable. In the following year, they proved that $\rho_l(G) \leq 2$ if G is a planar graph without 4-cycles adjacent to 3-cycles in [3]. Zhen and Wu [35] showed that if $\rho(G)$ is close enough to half the number of vertices of G, then $\rho(G) = \rho_l(G)$.

A graph G is k-degenerate if every subgraph of G contains a vertex of degree at most k. A k-degenerate graph admits a linear ordering such that the forward degree of every vertex is at most k. In [26], Raspaud and Wang mentioned that $\rho(G) \leq \lceil \frac{k+1}{2} \rceil$ for any k-degenerate graph G. Recently, Xue and Wu [33] showed that $\rho_l(G) \leq \lceil \frac{k+1}{2} \rceil$ for any k-degenerate graph, and they also showed that $\rho_l(G) \leq 2$ if G is either K_4-minor free, or G is a planar graph without k-cycles for any fixed $k \in \{3, 4, 5, 6\}$.

A torus is a closed surface (compact, connected 2-manifold without boundary) that is a sphere with a unique handle, and a toroidal graph is a graph embeddable in the torus. For a toroidal graph G, we still use G to denote an embedding of G in the torus. Cai, Wang and Zhu [7] discussed the structural properties of toroidal graphs without short cycles and gave some results on choosability.

The investigation of vertex arboricity on higher surfaces were actually done by Kronk, and Cook. In 1969, Kronk [22] proved that every connected graph of genus n has vertex arboricity $\rho(G) \leq \lfloor \frac{9+\sqrt{1+48n}}{4} \rfloor$. In 1974, Cook [15] obtained an analogue of the result of Kronk and gave some upper bound of graphs on genus. By their results, $\rho(G) \leq 4$ for any toroidal graph G; and $\rho(G) \leq 2$ for any toroidal graph G with girth at least 5. In 1975, Kronk and Michem [23] improved the latter result by showing that $\rho(G) \leq 2$ if G is a toroidal graph with no triangles.

In this paper, we consider the list vertex arboricity of toroidal graphs without cycles of specific length. More precisely, we give the following theorems.

Theorem 1.1. *Let G be a toroidal graph. Then $\rho_l(G) \leq 2$ if G contains no 5-cycles.*

Theorem 1.1 implies some of the following results. Every planar (toroidal) graph without 5-cycles is 4-choosable in [32, 7], and every planar graph without 5-cycles is vertex 2-arborable in [26].

Note that toroidal graphs without 6-cycles and 7-cycles are not 4-choosable, so neither of them are list vertex 2-arborable. The 4-choosability of a planar (toroidal) graphs without adjacent triangles is still open.

In this paper, we also consider the list vertex arboricity of the graphs containing neither adjacent triangles nor cycles of lengthgs 6 and 7, and give the following result.

Theorem 1.2. *Let G be a toroidal graph. Then $\rho_l(G) \leq 2$ if G contains no adjacent 3-cycles and no cycles of lengths 6 and 7.*

In [26], Raspaud and Wang asked the following question:

Question 1.3. [26] What is the maximum integer μ for all $k \in \{3, \cdots, \mu\}$, a planar graph G with no k-cycles has $\rho(G) \leq 2$?

Recently, Choi and Zhang [14] showed Theorem 1.4, which says that forbidding 4-cycles in toroidal graphs is sufficient to guarantee vertex arboricity at most 2.

Theorem 1.4. [14] *If G is a toroidal graph with no 4-cycles, then $\rho(G) \le 2$.*

Since the complete graph on 5 vertices is a toroidal graph with no cycles of length at least 6 and has vertex arboricity at least 3, we completely answer the Question 1.3 for toroidal graphs.

2. Notations

We use $b(f)$ to denote the boundary walk of a face f and write $f = [v_1v_2v_3 \cdots v_n]$ if $v_1, v_2, v_3, \cdots, v_n$ are the vertices of $b(f)$ in a cyclic order. We use $N_G(v)$ and $d_G(v)$ to denote the set and number of vertices adjacent to a vertex v, respectively. A face f is incident with all vertices and edges on $b(f)$. The *degree* of a face f of G, denoted also by $d_G(f)$, is the length of its boundary walk, where cut edges are counted twice. When no confusion may occur, we write $N(v)$, $d(v)$, $d(f)$ instead of $N_G(v)$, $d_G(v)$ and $d_G(f)$. A face f of G is called a simple face if $b(f)$ forms a cycle. Obviously, when $\delta(G) \ge 2$ for $k \le 5$, or G is 2-connected, each k-face is a simple face. A vertex (face) of degree k is called a k-vertex (k-face). If $r \le k$ or $1 \le k \le r$, then a k-vertex (k-face) is called an r^+- or r^--vertex (r^+- or r^--face), respectively.

For a vertex $v \in V(G)$, let $n_i(v)$ denote the number of i-vertices adjacent to v, and $m_j(v)$ the number of j-faces incident with v. For a face $f \in F(G)$, let $n_i(f)$ denote the number of i-vertices incident with f, and $m_j(f)$ the number of j-faces adjacent to f. Two faces are adjacent if they share at least one common edge.

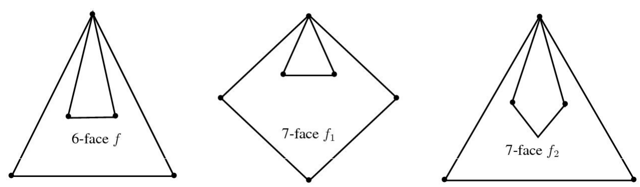
3. Structural properties

Let G be a toroidal graph without 6-cycles, 7-cycles and without adjacent 3-cycles. Obviously every subgraph of G also carries such properties. In this section, We investigate the structural properties of G which will be used to establish upper bounds of list vertex arboricity of such toroidal graphs.

Lemma 3.1. *Let G be a toroidal graph without 6-cycles and 7-cycles and without adjacent 3-cycles. Then G is 3-degenerate.*

Proof. Assume to the contrary that the lemma is false. Let G be a connected counterexample which is embedded on the torus. Thus $\delta(G) \ge 4$ and without 6-cycles, 7-cycles, and without adjacent triangles.

A face f is *frugal* if some vertex t is incident to it twice. Obviously, any 6-face or 7-face in G must be a *frugal* face for G contains no 6-cycles and 7-cycles. A list of faces of a vertex v is *consecutive* if it is a sublist of the list of faces incident to v in cyclic order. Two adjacent faces are *normally adjacent* if they have only two vertices in common (clearly, only one common edge on their facial walk), or are *abnormally adjacent* (that is, they have at least three vertices in common). □



First we prove some structural results needed for the proof of Lemma 3.1.

Claim 3.2. G contains no 3-face adjacent to a 5-face.

Proof. Let f be a 3-face with $b(f) = [v_1v_2v_3]$, and let f_1 be a 5-face with $b(f_1) = [v_1v_2u_1u_2u_3]$. Then $v_3v_2u_1u_2u_3v_1v_3$ forms a 6-cycle, unless $v_3 \in \{u_1, u_2, u_3\}$. But, if $v_3 = u_1$, then $d(v_2) = 2$, which is a contradiction to the minimum degree of G. If $v_3 = u_2$, it appears adjacent 3-cycles. By symmetry, $v_3 \neq u_3$. □

Claim 3.3. A 3-face must be normally adjacent to a 4-face.

Proof. Let f be a 3-face with $b(f) = [v_1v_2v_3]$, and let f_1 be a 4-face with $b(f_1) = [v_1v_2u_1u_2]$. A *abnormally adjacent* of f and f_1 will contradict to the fact G contains no adjacent 3-cycles. □

Claim 3.4. G contains no two adjacent 4-faces $f_1 = v_1v_2v_3v_4$ and $f_2 = v_1v_2u_1u_2$.

Proof. Obvious two faces f_1 and f_2 must be *abnormally adjacent*, or a 6-cycle appears. By the symmetry, we consider $v_3 = u_1$, or $v_3 = u_2$. If $v_3 = v_1$, then $d(v_2) = 2$ which contradicts the fact $d_G(v) \geq 4$ for any $v \in V(G)$. So $v_3 = u_2$, we have two adjacent 3-cycles $v_1v_2v_3(= u_2), v_2u_1u_2(= v_3)$. □

Proposition 3.5. *There is no 4^+-vertex v with $d(f_1), d(f_2), d(f_3) \in \{3, 4\}$ where f_1, f_2, f_3 are three consecutive faces incident with v.*

Proof. Let v be a 4^+-vertex with three consecutive faces f_1, f_2, f_3 incident with it and all the degrees of faces in f_1, f_2, f_3 are in $\{3, 4\}$. For G contains no adjacent 3-cycles. We just consider the following four cases $343, 434, 443, 444$ of all the degree combinations of f_1, f_2, f_3. Moreover, by Claim 3.4, we just consider two cases 343 and 434.

Case 1. $d(f_1 = [vv_1v_2]) = 3, d(f_2 = [vv_2xv_3]) = 4, d(f_3 = [vv_3v_4]) = 3$.

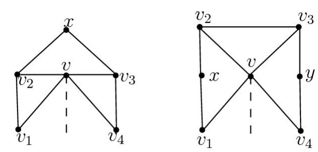
By Claim 3.3, $v_1, v_4 \notin \{v_2, v_3, x\}$. If $v_1 \neq v_4$, then $vv_1v_2xv_3v_4v$ is a 6-cycle. Yet, if $v_1 = v_4$, two adjacent triangles vv_1v_2 and vv_4v_3 will appear.

Case 2. $d(f_1 = [vv_1xv_2]) = 4, d(f_2 = [vv_2v_3]) = 3, d(f_3 = [vv_3yv_4]) = 4$.

By Claim 3.3, $x, y, v_1, v_4 \notin \{v_2, v_3, v\}$. If $v_1, x \notin \{y, v_4\}$, then $vv_1xv_2v_3yv_4v$ is a 7-cycle. By symmetry, we consider the following two subcases.

Subcase 1. $x = y$. Then $v_2v_3y(= x)$ and vv_2v_3 are adjacent 3-cycles.

Subcase 2. $x = v_4$. Then f_2, f_3 and $vv_1x(= v_4)$ is a case in Case 1.



Case 1: 343 Case 2: 434

□

Claim 3.6. G contains no 3-face adjacent to a 6-face.

Proof. By the properties of G, a 6-face must be *frugal* 6-face as depicted in Figure 1. So is it. □

Claim 3.7. A 3-face is adjacent to at most one *frugal* 7-face.

Proof. By the properties of G, a 7-face must be *frugal* 7-face as depicted in Figure 1. So we get it by the fact that G contains no adjacent 3-cycles and G contains no three consecutive faces with the face f_1, f_2, f_3 and the degree of these consecutive faces are 4, 3, 4 respectively, which is excluded by Proposition 3.5.

□

The Euler's formula $|V| + |F| - |E| = 0$ can be rewritten in the following form:

$$\sum_{v\in V(G)}\{d_G(v)-4\}+\sum_{f\in F(G)}\{d_G(f)-4\}=0. \tag{3.1}$$

Let ω be a weight on $V(G)\cup F(G)$ by defining $\omega(v)=d_G(v)-4$ if $v\in V(G)$, and $\omega(f)=d_G(f)-4$ if $f\in F(G)$. Then the total sum of the weights is zero. To prove Lemma 3.1, we will introduce some rules to transfer weights between the elements of $V(G)\cup F(G)$ so that the total sum of the weights is kept constant while the transferring is in progress. However, once the transferring is finished, we can show that the resulting weight ω^* satisfies $\sum_{x\in V(G)\cup F(G)}\omega^*(v)>0$. This contradiction to (3.1) will complete the proof.

Our transferring rules are as follows:

(R1) Each 5^+-vertex transfers $\frac{w(v)}{\lfloor\frac{2}{3}d(v)\rfloor}$ to each incident 4^--face.

(R2) Each 5^+-face transfers $\frac{w(f)}{d(f)}$ to each adjacent 4^--face.

The following simple facts will be used frequently in our discussions.

Fact 1. $\frac{w(v)}{\frac{2}{3}d(v)}\geq\frac{1}{2}$ while $d(v)\geq 6$, and $\frac{w(f)}{d(f)\geq\frac{1}{2}}$ while $d(f)\geq 8$.

Fact 2. $|m_3(v)|\leq\lfloor\frac{1}{2}d(v)\rfloor$.

Fact 3. $|m_{4^-}(v)|\leq\lfloor\frac{2}{3}d(v)\rfloor$.

Fact 4. $w^*(x)\geq 0$ while $d(x)\geq 4$ for all $x\in V(G)\cup F(G)$.

Fact 5. Let f be a 3-face, then $m_4(f)\leq 1$. Let f_1 be a 4-face, then $m_3(f_1)\leq 1$.

Fact 2 is true by the properties of G. Fact 3 is true by Proposition 1. Fact 1 can be got by direct calculation. Fact 4 can be easily seen by the discharging rules **(R1)** and **(R2)**. As to Fact 5, for the 4-face $f_1=[v_1v_2v_3v_4]$, we have f_1 can be adjacent to two 3-faces on the two opposite edges v_1v_2, v_3v_4 or v_1v_4, v_2v_3 by Proposition 3.5, but in this case, either a 6-cycle or two adjacent 3-cycles will appear. This completes the proof of Fact 5.

Let f be a face of G. Now we consider the case while $d(f)=3$.

By Fact 5, we have $m_4(f)\leq 1$. Moreover $m_3(f)=0$ because G contains no adjacent 3-cycles.

If $m_4(f)=0$, we have $m_5(f)=m_6(f)=0$ by Claim 1 and Claim 4 and $m_7(f)\leq 1$ by Claim 5. If $m_7(f)=0$, then $w^*(f)\geq w(f)+3\cdot\frac{1}{2}=\frac{1}{2}>0$ by **Fact 1**. While $m_7(f)=1$, then $w^*(f)\geq w(f)+2\cdot\frac{1}{2}+\frac{3}{7}=\frac{3}{7}>0$ by **Fact 1** and **(R2)**.

If $m_4(f)=1$, then the other two faces adjacent to f must be 8^+-face. In this case, if f is incident with at least one 5^+-vertex, a 5-vertex transfers $\frac{5-4}{\lfloor\frac{2\cdot 5}{3}\rfloor}$ to f, then $w^*(f)\geq w(f)+2\cdot\frac{1}{2}+\frac{1}{3}=\frac{1}{3}>0$ by **(R1)** and **(R2)**. While $n_{5^+}(f)=0$. $w^*(f)\geq w(f)+2\cdot\frac{1}{2}=0$ by **(R2)**.

Now, we get that $\omega^*(x)\geq 0$ for each $x\in V(G)\cup F(G)$. It follows that

$$0 \leq \sum_{x \in V(G) \cup F(G)} \omega^*(x) = \sum_{x \in V(G) \cup F(G)} \omega(x) = 0.$$

If $\sum_{x \in V(G) \cup F(G)} \omega^*(x) > 0$, we are done. Assume that $\sum_{x \in V(G) \cup F(G)} \omega^*(x) = 0$.

Claim 3.8. G contains no 4-face.

Proof. Assume f' is a 4-face in $F(G)$, it must be adjacent to at least three 5^+-faces by **Fact 5** and **Claim** 3.4. But there will have $w^*(f') \geq w(f') + 3 \cdot \frac{1}{5} > 0$.

By Claim 3.8, G contains no 4-face, so for any 3-face, $m_4(f) = 0$, then we conclude that G must contain no 3-face by the discussion above. Subsequently, G contain no 5^+-faces since any 5^+-face sends no charge to the adjacent faces at this case by **(R2)**. Such a toroidal graph does not exist. □

4. Proof of the theorems

Let G be a counterexample to Theorem 1.1 or Theorem 1.2 with the fewest number of vertices. It is easy to see that G is 2-connected, and we have:

Claim 4.1. $\delta(G) \geq 4$.

Proof. If $\delta(G) \leq 3$, then there exists a vertex v in G such that $d(v) \leq 3$. Let $G' = G - v$. Then G' admits an acyclic coloring ϕ. Since $d(v) \leq 3$, there always exists a color which lies in $L(v)$ which is assigned to at most one neighbor of v under the map ϕ, then we can extend ϕ to an acyclic L-coloring of G by assigning this color to v with $\phi(v) \in L(v)$. □

We will use the following two results for the proof of Theorem 1.1.

Lemma 4.2. [7] *Let G be a toroidal graph without 5-cycles. Then $\delta \leq 4$; and $\delta(G) = 4$ if and only if G is 4-regular.*

Theorem 4.3. [5, 33] *Let G be a connected graph. If G is neither a cycle nor a complete graph of odd order, then $\rho_l(G) \leq \lceil \frac{\Delta(G)}{2} \rceil$.*

By Claim 4.1, $\delta(G) \geq 4$. By Lemma 4.2, G is a 4-regular graph. Obviously it is not a complete graph of odd order because G contains no 5-cycles. This fact completes the proof of Theorem 1.1 by Theorem 4.3.

By Lemma 1 and the following theorem posed in [33], Theorem 1.2 is trivial.

Theorem 4.4. [33] *For any graph G, $\rho_l(G) \leq \lceil \frac{deg(G)+1}{2} \rceil$ where $deg(G)$ is the minimum number k for which G is k-degenerate.*

5. **Further considerations**

For all planar graphs, it was known that $\rho(G) \leq 2$ if a planar graph contains no cycle for any fixed $k \in \{3,4,5,6,7\}$. A toroidal graph wtihout 6-cycles and 7-cycles may not be 4-choosable, of course, not be list vertex 2-arborable. In fact, for $n = 4,5,6$, K_n is a toroidal graph with $\chi_l(K_n) = n$ but without $(n+1)$-cycle. Moreover, it will be noted that excluding cycles of length eight does not forbid the complete graph K_7 and thus the values of k between 3 and 7 are of the most interest.

In [13], Choi proved every toroidal graph containing neither K_5^- nor 6-cycles is 4-choosable. And this result is sharp in the sense that forbidding only one of a K_5^- or 6-cycles in a toroidal graph does not guarantee the 4-choosability. Moreover, a graph containg neither K_5 nor 6-cycles can not assure its list vertex arboricity at most two. (See Theorem 4.1 in [13]). In [33], Xue and Wu proved that for any positive integer n, there is a bipartite graph G such that $\rho_l(G) \geq n$. Here we pose the following conjecture.

Conjecture 5.1. Let G be a toroidal graph without adjacent triangles. Then $\rho_l(G) \leq 2$.

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