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GROWTH OF MEROMORPHIC SOLUTIONS FOR COMPLEX DIFFERENCE EQUATIONS OF MALMQUIST TYPE

Y. Y. ZHANG, Z. S. GAO* AND J. L. ZHANG

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Abstract. In this paper, we give some necessary conditions for a complex difference equation of Malmquist type

$$\sum_{j=1}^{n} f(z+c_j) = \frac{P(f(z))}{Q(f(z))},$$

where $n(\in \mathbb{N}) \geq 2$, and $P(f(z))$ and $Q(f(z))$ are relatively prime polynomials in $f(z)$ with small functions as coefficients, admitting a meromorphic function of finite order. Moreover, the properties of finite order transcendental meromorphic solutions for complex difference equation $\prod_{j=1}^{n} f(z+c_j) = P(f(z))/Q(f(z))$ are also investigated.
Keywords: Complex difference, meromorphic, Malmquist type.
MSC(2010): Primary: 30D35; Secondary: 39A10.

1. Introduction

In the whole paper, a meromorphic function always means meromorphic in the whole complex plane. We assume that the reader is familiar with the basic notions of Nevanlinna's value distribution theory of meromorphic functions, see, e.g., [4,10]. Let $f(z)$ be a meromorphic function, we use $\sigma(f)$, $\lambda(f)$ and $\lambda(1/f)$ to denote the order of growth, the exponent of convergence of the zeros and the exponent of convergence of the poles of $f(z)$, respectively. In addition, we denote by $S(r,f)$ any quantity that satisfies the condition $S(r,f) = o(T(r,f))$ as $r \to \infty$ outside of a possible exceptional set of finite logarithmic measure, and a meromorphic function $a(z)$ ($\not\equiv \infty$) is called a small function with respect to $f(z)$ provided that $T(r,a(z)) = S(r,f)$.

As we all know, the celebrated Malmquist theorem shows that a complex differential equation $f'(z) = R(z,f(z))$, where $R(z,f(z))$ is rational in both

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arguments, and which admits a transcendental meromorphic solution $f(z)$ in the complex plane, reduces into a Riccati differential equation

$$f'(z) = a(z) + b(z)f(z) + c(z)f(z)^2 \tag{1.1}$$

with rational coefficients, and all meromorphic solutions of (1.1) have finite order. For more details concerning equation (1.1), as well as for generalizations of the Malmquist theorem, see, e.g., [10]. Recently, as the research on the difference analogues of Nevanlinna theory is becoming active, many authors (see, e.g., [1–3, 5, 6, 8, 11]) started to consider the growth of order and existence of meromorphic functions of complex difference equations of Malmquist type. By using the difference analogue of the lemma on the logarithmic derivative, Chiang and Feng [3] gave the necessary conditions for a complex difference equation of Malmquist type admitting a meromorphic function of finite order by proving the following Theorem 1.1.

Theorem 1.1. *(see [3]) Let $c_1, \cdots, c_n$ be nonzero complex constants. If the difference equation*

$$\sum_{j=1}^{n} f(z+c_j) = \frac{P(f(z))}{Q(f(z))} = \frac{a_p f(z)^p + a_{p-1} f(z)^{p-1} + \cdots + a_1 f(z) + a_0}{b_q f(z)^q + b_{q-1} f(z)^{q-1} + \cdots + b_1 f(z) + b_0} \tag{1.2}$$

with rational coefficients $a_i(z)$, $b_j(z)$ admits a finite order transcendental meromorphic solution, then we have $d = \max\{p, q\} \le n$.

Remark 1.2. From the following Theorem 1.3, we know that if (1.2) admits a finite order transcendental meromorphic solution, we also have $p \le q + 1$.

The difference equations of Malmquist type may have rational solutions. For example, $f(z) = -1/z$ satisfies the following equation

$$f(z+1) + f(z-1) = \frac{(z+2)f(z) + 1}{1 - f(z)^2}.$$

In this paper, we study the growth of transcendental meromorphic solutions for complex difference equations of Malmquist type. We first prove the following Theorem 1.3.

Theorem 1.3. *Suppose that $c_1, c_2, \cdots, c_n$ are distinct, nonzero constants, and that $f(z)$ is a transcendental meromorphic solution of complex difference equation of Malmquist type*

$$\sum_{j=1}^{n} f(z+c_j) = \frac{P(f(z))}{Q(f(z))} = \frac{a_p f(z)^p + a_{p-1} f(z)^{p-1} + \cdots + a_1 f(z) + a_0}{b_q f(z)^q + b_{q-1} f(z)^{q-1} + \cdots + b_1 f(z) + b_0}, \tag{1.3}$$

where $n(\in \mathbb{N}) \ge 2$, and $P(f(z))$ and $Q(f(z))$ are relatively prime polynomials in $f(z)$ with coefficients a_s $(s = 0, \cdots, p)$ and b_t $(t = 0, \cdots, q)$ such that $a_p b_q \not\equiv 0$ and satisfy $T(r, a_s) = S(r, f)$ and $T(r, b_t) = S(r, f)$. Then

(1) if $p > q + 1$, we have $\sigma(f) = \infty$;

(2) if $f(z)$ is an entire function of finite order, we have $p = 1$ and $q = 0$.

It is natural to ask what happens if we ignore the condition in Theorem 1.3 that all the coefficients a_s $(s = 0, \cdots, p)$ and b_t $(t = 0, \cdots, q)$ are all small functions of $f(z)$. We consider this case and obtain the following Theorem 1.4.

Theorem 1.4. *Suppose that c_1, c_2, $\cdots$, c_n are distinct, nonzero constants and that $P(f(z))$ and $Q(f(z))$ are relatively prime polynomials in $f(z)$ with coefficients a_s $(s = 0, \cdots, p)$ and b_t $(t = 0, \cdots, q)$ such that $a_p b_q \not\equiv 0$, and there is a dominant coefficient a_l or b_m satisfying*

$$\sigma_0 = \sigma(a_l) > \max\{\sigma(a_s), \sigma(b_t) : 0 \le s \le p, 0 \le t \le q, s \neq l\}$$

or

$$\sigma_0 = \sigma(b_m) > \max\{\sigma(a_s), \sigma(b_t) : 0 \le s \le p, 0 \le t \le q, t \neq m\},$$

and $\sigma_0 < \infty$. If $p > q+1$ and $f(z)$ is a finite order transcendental meromorphic solution of (1.3), then $\sigma(f) = \sigma_0$.

In the rest of this paper, we continue to investigate the transcendental meromorphic solutions of complex difference equations of the following type

$$(1.4) \quad \prod_{j=1}^{n} f(z + c_j) = \frac{P(f(z))}{Q(f(z))} = \frac{a_p f(z)^p + a_{p-1} f(z)^{p-1} + \cdots + a_1 f(z) + a_0}{b_q f(z)^q + b_{q-1} f(z)^{q-1} + \cdots + b_1 f(z) + b_0}.$$

Chiang and Feng [3] proved that if (1.4) with rational coefficients admitting a meromorphic function of finite order, then $d = \max\{p, q\} \le n$. We consider the finite order transcendental meromorphic solution of (1.4) and prove the following Theorem 1.5 which is similar to Theorem 1.1 in [8].

Theorem 1.5. *Let c_1, c_2, $\cdots$, c_n be distinct, nonzero constants. Suppose that $f(z)$ is a finite order transcendental meromorphic solution of complex difference equation (1.4), where $n (\in \mathbb{N}) \ge 2$, and $P(f(z))$ and $Q(f(z))$ are relatively prime polynomials in $f(z)$ with coefficients a_s $(s = 0, \cdots, p)$ and b_t $(t = 0, \cdots, q)$ such that $a_0 a_p b_q \not\equiv 0$ and satisfy $T(r, a_s) = S(r, f)$ and $T(r, b_t) = S(r, f)$. If $q \ge 1$, then we have $\lambda(f) = \lambda(1/f) = \sigma(f)$.*

2. Some lemmas

Lemma 2.1. *(see [3]) Let c_1, c_2 be two complex numbers such that $c_1 \neq c_2$ and let $f(z)$ be a meromorphic function with finite order. Let σ be the order of $f(z)$, then for each $\varepsilon > 0$, we have*

$$m\left(r, \frac{f(z + c_1)}{f(z + c_2)}\right) = O(r^{\sigma - 1 + \varepsilon}).$$

Lemma 2.2. *(see [9]) Let $f(z)$ be a meromorphic function of finite order of a difference equation of the form*

$$U(z, f)P(z, f) = Q(z, f),$$

where $U(z,f)$, $P(z,f)$ and $Q(z,f)$ are difference polynomials with all the coefficients $a_\lambda(z)$ being small functions as understood in the usual Nevanlinna's theory, i.e., $T(r,a_\lambda(z)) = O(r^{\sigma-1+\varepsilon}) + S(r,f)$. The maximum total degree $\deg_f U(r,f) = n$ in $f(z)$ and its shifts, and $\deg_f Q(r,f) \le n$. Moreover, we assume that $U(r,f)$ contains just one term of maximal total degree in $f(z)$ and its shifts. Then for each $\varepsilon > 0$,

$$m(r, P(z,f)) = O(r^{\sigma-1+\varepsilon}) + S(r,f),$$

possibly outside of an exceptional set of finite logarithmic measure.

In what follows, we say that $f(z)$ is a meromorphic function with more than $S(r,f)$ poles of a certain type in the sense that the integrated counting function of these poles is not of type $S(r,f)$, and we use ∞^k (0^l) to denote a pole (zero) of $f(z)$ with multiplicity k (l).

Lemma 2.3. *(see [6]) Suppose that $f(z)$ is a meromorphic solution of (1.3) with more than $S(r,f)$ poles (counting multiplicities). Let z_j denote the zeros and poles of the coefficients a_i which are small meromorphic functions with respect to $f(z)$. Let m_j be the maximum order of the zeros and poles of the functions a_i at z_j. Then for any $\varepsilon > 0$, there are at most $S(r,f)$ points z_j such that*

$$f(z_j) = \infty^{k_j},$$

where $m_j \ge \varepsilon k_j$.

Lemma 2.4. *(see [10]) Let f(z) be a meromorphic function, then for all irreducible rational function in $f(z)$*

$$R(f(z)) = \frac{a_p f(z)^p + a_{p-1} f(z)^{p-1} + \cdots + a_1 f(z) + a_0}{b_q f(z)^q + b_{q-1} f(z)^{q-1} + \cdots + b_1 f(z) + b_0},$$

with meromorphic coefficients a_s $(s = 0, \cdots, p)$ and b_t $(t = 0, \cdots, q)$ which are small functions of $f(z)$ and $d = \max\{p,q\}$, the characteristic function of $R(f(z))$ satisfies

$$T(r, R(f(z))) = dT(r,f) + S(r,f).$$

Lemma 2.5. *(see [3]) Let $f(z)$ be a meromorphic function with order $\sigma = \sigma(f)$, $\sigma < \infty$, and let c be a fixed nonzero complex number, then for each $\varepsilon > 0$, we have*

$$T(r, f(z+c)) = T(r,f) + O(r^{\sigma-1+\varepsilon}) + O(\log r).$$

Lemma 2.6. *(see [3]) Let $f(z)$ be a meromorphic function with order $\sigma = \sigma(f)$, $\sigma < \infty$, and let c be a fixed nonzero complex number, then for each $\varepsilon > 0$, we have*

$$N(r, f(z+c)) = N(r,f) + O(r^{\sigma-1+\varepsilon}) + O(\log r).$$

Lemma 2.7. *(see [7]) Let $f(z)$ be a nonconstant finite order meromorphic solutions of $P(z,f)=0$, where $P(z,f)$ is a polynomial in $f(z)$. If $P(z,a)\not\equiv 0$ for a meromorphic function $a(z)$ satisfying $T(r,a)=S(r,f)$, then we have*

$$m\left(r,\frac{1}{f-a}\right)=S(r,f).$$

3. **Proof of Theorem 1.3**

Proof. (1) We use a similar argument as that in the proof of Proposition 5.4 in [9]. Suppose on the contrary that $f(z)$ is a transcendental meromorphic solution of (1.3) of finite order σ. For simplicity, let

$$F_L(z):=\sum_{j=1}^{n} f(z+c_j)$$

and

$$F_R(z):=\frac{a_pf(z)^p+a_{p-1}f(z)^{p-1}+\cdots+a_1f(z)+a_0}{b_qf(z)^q+b_{q-1}f(z)^{q-1}+\cdots+b_1f(z)+b_0}.$$

Since $p>q+1$, then by Lemma 2.2, we conclude that $m(r,f)=S(r,f)$. Therefore, f has more than $S(r,f)$ poles, counting multiplicity. Denoting points in the pole sequence by z_i. We may invoke the notation introduced in Lemma 2.3 to denote $f(z_i)=\infty^{k_i}$. By Lemma 2.3, f has more than $S(r,f)$ poles so that we have $m_i<\varepsilon k_i$ at z_i. Here m_i refers to the coefficients a_p, $\cdots$, a_0 and b_q, $\cdots$, b_0 of (1.3). Denote the sequence of such poles by $z_{1,i}$, and take this sequence as our starting point. Supposing, as we may, that $\varepsilon<1/4$, we see that

$$F_R(z_{1,i})=\infty^{k'_{2,i}},\quad k'_{2,i}\geq(p-\varepsilon)k_{1,i}-(q+\varepsilon)k_{1,i}\geq(2-2\varepsilon)k_{1,i}.$$

Comparing this with F_L, we conclude that at least one of the points $z_{1,i}+c_1$, $\cdots$, $z_{1,i}+c_n$ is a pole of f of multiplicity $k_{2,i}\geq k'_{2,i}$. We first apply Lemma 2.3 to obtain that there are more than $S(r,f)$ such points $z_{2,i}$ with $f(z_{2,i})=\infty^{k'_{2,i}}$ and $m_{2,i}<\varepsilon k_{2,i}$. We then pick only one of these points, denoting it by $z_{2,i}$. Continuing to the next phase, we observe that $F_R(z_{2,i})=\infty^{k'_{3,i}}$, and we fix, for each permitted $z_{2,i}$, a pole $z_{3,i}$ of the next phase so that $f(z_{3,i})=\infty^{k_{3,i}}$, where

$$k_{3,i}\geq k'_{3,i}\geq(2-2\varepsilon)k_{2,i}\geq(2-2\varepsilon)^2k_{1,i}.$$

Then by induction, we may finally choose a sequence z_m of poles of $f(z)$ which satisfy the conditions $f(z_m)=\infty^{k_m}$ and $k_m\geq(2-2\varepsilon)^{m-1}k_1\geq(2-2\varepsilon)^{m-1}$. We now estimate the counting function $N(r,f)$. Let $C=\max(|c_1|,\cdots,|c_n|)$ and denote $r_m=|z_1|+(m-1)C$, then it is geometrically obvious that

$$z_m\in B(z_1,(m-1)C)\subset B(0,|z_1|+(m-1)C)=B(0,r_m).$$

For m large enough, we have $r_m \leq 2(m-1)C$, which suggests that

$$n(r_m, f) \geq (2-2\varepsilon)^{m-1} \geq (3/2)^{m-1}.$$

Hence,

$$N(2r_m, f) \geq (\log 2)(3/2)^{m-1} \geq (\log 2)(3/2)^{r_m/3C}.$$

This means that $f(z)$ is of infinite order, which obviously contradicts to our assumption that $f(z)$ is of finite order, and hence $\sigma(f) = \infty$.

(2) Suppose that $f(z)$ is an entire function of finite order. If $q \geq p \geq 1$, then we deduce from (1.3) that

$$\begin{aligned} & a_p f(z)^p + a_{p-1} f(z)^{p-1} + \cdots + a_1 f(z) + a_0 \\ = & \sum_{j=1}^{n} f(z+c_j)(b_q f(z)^q + b_{q-1} f(z)^{q-1} + \cdots + b_1 f(z) + b_0). \end{aligned}$$

By Lemma 2.2 and the above equation, we have

$$m\left(r, \sum_{j=1}^{n} f(z+c_j)\right) = S(r,f). \tag{3.1}$$

By Lemma 2.4 and (1.3), we get

$$T\left(r, \sum_{j=1}^{n} f(z+c_j)\right) = qT(r, f(z)) + S(r,f). \tag{3.2}$$

Equations (3.1) and (3.2) and Lemma 2.6 imply that

$$nN(r,f) \geq N\left(r, \sum_{j=1}^{n} f(z+c_j)\right) + S(r,f) = qT(r, f(z)) + S(r,f) \tag{3.3}$$

which is a contradiction to our assumption that $f(z)$ is an entire function. Therefore, $q < p$. If $p > q+1$, we know from the first part that $f(z)$ is a meromorphic function of infinite order. So $p = q+1$. By Lemma 2.4 and (1.3), we get

$$T\left(r, \sum_{j=1}^{n} f(z+c_j)\right) = pT(r, f(z)) + S(r,f). \tag{3.4}$$

If $p \geq 2$, we deduce from Lemma 2.1 and (1.3) that

$$\begin{aligned} & m\left(r, \sum_{j=1}^{n} f(z+c_j)\right) = m\left(r, \sum_{j=1}^{n} \frac{f(z+c_j)}{f(z)} f(z)\right) \\ \leq & m\left(r, \sum_{j=1}^{n} \frac{f(z+c_j)}{f(z)}\right) + m(r, f(z)) \leq T(r,f) + S(r,f). \end{aligned} \tag{3.5}$$

It follows from (3.4) and (3.5) and Lemma 2.6 that

$$nN(r,f)\geq N\left(r,\sum_{j=1}^{n}f(z+c_j)\right)+S(r,f)\geq (p-1)T(r,f(z))+S(r,f),$$

which is also a contradiction to that $f(z)$ is an entire function. Thus we get $p=1$ and $q=0$. □

4. Proof of Theorem 1.4

Proof. Without loss of generality, we may suppose that

$$\sigma_0=\sigma(a_p)>\max\{\sigma(a_s),\sigma(b_t):0\leq s\leq p-1,0\leq t\leq q\}.$$

We claim that $\sigma(a_p)\leq\sigma(f)$. Otherwise, we may suppose that $\sigma(a_p)>\sigma(f)$, and we have

$$T(r,a_s)=S(r,a_p),\quad s=0,1,\cdots,p-1,$$
$$T(r,b_t)=S(r,a_p),\quad t=0,1,\cdots,q,$$
$$T(r,f^i)=iT(r,f)=S(r,a_p),\quad i=1,\cdots,\max\{p,q\}.$$

By Lemma 2.4 and the above equations, we have

$$T\left(r,\frac{a_pf(z)^p+a_{p-1}f(z)^{p-1}+\cdots+a_1f(z)+a_0}{b_qf(z)^q+b_{q-1}f(z)^{q-1}+\cdots+b_1f(z)+b_0}\right)=T(r,a_p)+S(r,a_p). \tag{4.1}$$

On the other hand, by Lemma 2.5, we obtain

$$T\left(r,\sum_{j=1}^{n}f(z+c_j)\right)\leq\sum_{j=1}^{n}T(r,f(z+c_j))+\log n\leq nT(r,f(z))+S(r,f). \tag{4.2}$$

From (1.3), (4.1) and (4.2), we obtain

$$T(r,a_p)+S(r,a_p)\leq nT(r,f(z))+S(r,f)=S(r,a_p)+S(r,f),$$

which is impossible. So we get $\sigma(a_p)\leq\sigma(f)$. However, if $\sigma(f)>\sigma_0$ when $p>q+1$, then all the coefficients are small with respect to $f(z)$ and from Theorem 1.3 (1), we know that $\sigma(f)=\infty$, a contradiction. Thus we have $\sigma(f)=\sigma_0$. □

5. Proof of Theorem 1.5

Proof. Suppose that $f(z)$ is a finite order transcendental meromorphic solution of (1.4). First, we prove that $\lambda(f)=\sigma(f)$. We deduce from (1.4) that

$$P(z,f(z))=\sum_{j=1}^{n}f(z+\eta_j)(b_qf(z)^q+b_{q-1}f(z)^{q-1}+\cdots+b_1f(z)+b_0)$$
$$-a_pf(z)^p+a_{p-1}f(z)^{p-1}+\cdots+a_1f(z)+a_0.$$

We notice that

$$P(z,0)=-a_0\not\equiv 0.$$

It follows from Lemma 2.7 that

$$m\left(r, \frac{1}{f(z)}\right) = S(r, f)$$

for all r possibly outside of an exceptional set of finite logarithmic measure. Therefore,

$$N\left(r, \frac{1}{f(z)}\right) = T(r, f) + S(r, f)$$

for all r possibly outside of an exceptional set of finite logarithmic measure. Thus we have $\lambda(f) = \sigma(f)$.

Second, we prove that $\lambda(1/f) = \sigma(f)$. Set

$$H(z, f(z)) = b_q f(z)^q + b_{q-1} f(z)^{q-1} + \cdots + b_1 f(z) + b_0,$$

and rewrite (1.4) into the following form

$$\prod_{j=1}^{n} f(z + c_j) H(z, f(z)) = a_p f(z)^p + a_{p-1} f(z)^{p-1} + \cdots + a_1 f(z) + a_0.$$

Note that $p \leq n$. Then by Lemma 2.2 and the above equation, we have

$$m(r, H(z, f(z))) = S(r, f). \tag{5.1}$$

By Lemma 2.4 and (1.4), we have

$$T(r, H(z, f(z))) = qT(r, f(z)) + S(r, f). \tag{5.2}$$

It follows from (5.1) and (5.2) and Lemma 2.6 that

$$qT(r, f(z)) + S(r, f) = N(r, H(z, f(z))) + S(r, f) \leq qN(r, f) + S(r, f).$$

Thus we have $\lambda(1/f) = \sigma(f)$ and this completes the proof. □

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