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**Mathematical modeling, analysis and simulation of Ebola epidemics**

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## MATHEMATICAL MODELING, ANALYSIS AND SIMULATION OF EBOLA EPIDEMICS

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(Communicated by Fatemeh Helen Ghane)

**ABSTRACT.** Mathematical models are the most important tools in epidemiology to understand previous outbreaks of diseases and to better understand the dynamics of how infections spread through populations. Many existing models closely approximate historical disease patterns. This article investigates the mathematical model of the deadly disease with severe and uncontrollable bleeding, Ebola which is currently becoming the headache of the whole world though effort to control is undergoing. In this paper a new mathematical model of the Ebola epidemic is built. Besides, the basic reproduction number is calculated and the stability of both disease free and endemic equilibrium is proved. Finally, numerical simulations are executed to further consolidate the analysis of the deadly disease Ebola.

**Keywords:** Basic reproduction number, global stability, equilibrium, epidemic model.

**MSC(2010):** Primary: 37N25; Secondary: 92D30.

### 1. Introduction

Ebola is a disease of humans and other primates caused by an Ebola virus. Symptoms start two days to three weeks after contacting the virus with a fever, sore throat, muscle pain and headaches [2, 7, 9–11]. Typically, vomiting, diarrhea and rash flow, along with decreased functioning of the liver and kidneys. Around this time, the affected people may begin to bleed within the body and externally. The virus may be acquired upon contact with blood or bodily fluids of an infected animal. Spreading through the air has not been documented in the natural environment. Fruit bats are believed to be a carrier and may spread the virus without being affected [4, 5, 8, 12, 13, 15, 17]. Once human infection occurs, the disease may spread between people, as well. Male survivors may be able to transmit the disease via semen for nearly two months. To make

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the diagnosis, typically other diseases with similar symptoms such as malaria, cholera and other viral hemorrhagic fevers are first excluded. To confirm the diagnosis, blood samples are tested for viral antibodies, viral RNA, or the virus itself. What makes the disease the worst of all is, no specific treatment for it is yet available. Efforts to help those who are infected are supportive and include giving either oral rehydration therapy (slightly sweet and salty water to drink) or intravenous. The disease has a high risk of death, killing between 50% and 90% of those infected with the virus [3, 14]. The disease typically occurs in tropical regions of sub-Saharan Africa. The countries affected have some with the world's lowest literacy rates and Public-health campaigns started too late as well as didn't reach enough people besides the fragile health system. These things create the difficulty to control the disease. It has affected Guinea, Sierra Leone, Liberia and Nigeria [1, 18]. It is now becoming the worst disease with the highest fatality rate and is a big headache for the world. It seeks an urgent solution. According to world health organization there are more than 9000 deaths till now. Efforts are under way to develop a vaccine; however, none yet exists.

## 2. Mathematical model

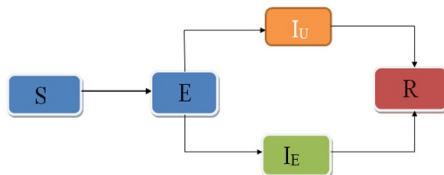
A compartmental model with a closed population was used to describe the natural history and epidemiology of Ebola. Briefly, the population is divided into five compartments: Susceptible individuals (**S**) may become Exposed (**E**) after contact with an Ebola infected individual. As one of the basic and the most important reason for high Ebola spread is the lack of awareness or education for raising healthy literacy and others, we divided the Ebola infected population into, uneducated infected individuals (**I<sub>U</sub>**) and educated infected individuals (**I<sub>E</sub>**) class after the disease incubation period, thereafter capable of infecting others including nurses, doctors etc. at hospitals and with a chance of infecting others before being removed from the model (**R**), or they may recover, at which point they are similarly removed.

The susceptible population is increased by the recruitment of individuals into the population at the rate  $\lambda$  and may acquire infection after contact with infected uneducated individual at the rate  $\beta_1$  and infected educated individual at the rate  $\beta_2$ . The susceptible individuals are further decreased by the natural death at the rate  $\mu$ .

The population of exposed individuals is generated by the infection of susceptible individuals at the rates  $\beta_1$  and  $\beta_2$ . This population is further decreased by development of Ebola disease symptoms at the rate  $\alpha$  and natural death at a rate  $\mu$ .

The population of uneducated infected individuals is generated at the rate  $\rho$ . It is decreased by death due to Ebola at the rate  $\delta_1$  and natural death at the rate  $\mu$ .

FIGURE 1. Compartmental flow of a mathematical model for Ebola epidemics.



The population of educated infected individuals is generated at the rate  $(\alpha - \rho)$  and decreased by death due to Ebola at the rate  $\delta_2$  and the natural death at the rate  $\mu$ . We assumed the death rate of educated infected individuals  $\delta_2$  is less than that of uneducated infected individuals  $\delta_1$ .

Finally, the Ebola infected individuals are recovered /removed at a rate  $\gamma$  and decreased by the natural death at the rate  $\mu$ . The system of ordinary differential equations describing this model is given below.

$$(2.1) \quad \frac{dS}{dt} = \lambda - \frac{\beta_1 S(I_U)}{N} - \frac{\beta_2 S(I_E)}{N} - \mu S$$

$$(2.2) \quad \frac{dE}{dt} = \frac{\beta_1 S(I_U)}{N} + \frac{\beta_2 S(I_E)}{N} - (\alpha + \mu)E$$

$$(2.3) \quad \frac{dI_U}{dt} = \rho E - (\mu + \delta_1)I_U$$

$$(2.4) \quad \frac{dI_E}{dt} = (\alpha - \rho)E - (\mu + \delta_2)I_E$$

$$(2.5) \quad \frac{dR}{dt} = \gamma(I_U + I_E) - \mu R$$

### 3. Model parameters

TABLE 1. Model parameters for simulation (Source: World Health Organization, WHO, Ebola 2014)

Parameter	Average Value
Total number of population (N)	5000
Recruitment rate ( $\lambda$ )	1100
Death rate due to nature and other ( $\mu$ )	0.045
Rate of infection, uneducated ( $\rho$ )	0.036
Transmission (contact) rate, uneducated ( $\beta_1$ )	0.867
Transmission (contact) rate, educated ( $\beta_2$ )	0.133
Death rate due to Ebola, uneducated ( $\delta_1$ )	0.25
Death rate due to Ebola, educated ( $\delta_2$ )	0.15
Rate of removal ( $\gamma$ )	0.31
Rate of infection ( $\alpha$ )	0.052

#### 4. Basic properties

Since the model monitors changes in the human population, all the variables and parameters are assumed to be positive for all  $t \geq 0$ .

The model is therefore analyzed in a suitable feasible region:

$$D = \{S(t), E(t), I_U(t), I_E(t), R(t) \in R_+^5\}$$

with initial conditions  $S(0) \geq 0, E(0) \geq 0, I_U(0) \geq 0, I_E(0) \geq 0$  and  $R(0) \geq 0$  is positively invariant for the system (2.1) to (2.5).

#### 5. Positivity of the solution

For the above system it is necessary to prove that all the state variables are non-negative so that the solutions of the system with positive initial conditions remain positive for all  $t > 0$ . We thus state the following lemma.

**Lemma 5.1.** *If  $S(0) \geq 0, E(0) \geq 0, I_U(0) \geq 0, I_E(0) \geq 0$  and  $R(0) \geq 0$  then the solutions  $S(t), E(t), I_U(t), I_E(t)$  and  $R(t)$  are all positive for all  $t > 0$ .*

*Proof.* To get a contradiction, assume that there exists positive reals  $t_1, t_2, t_3, t_4$  and  $t_5$  for which the following hold:

- (1)  $S(t_1) = 0, S'(t_1) < 0$ , and for all  $0 \leq t \leq t_1$  one has that  $E(t) \geq 0, I_U(t) \geq 0, I_E(t) \geq 0$  and  $R(t) \geq 0$ ;
- (2)  $E(t_2) = 0, E'(t_2) < 0$ , and for all  $0 \leq t \leq t_2$  we have that  $S(t) \geq 0, I_U(t) \geq 0, I_E(t) \geq 0$  and  $R(t) \geq 0$ ;
- (3)  $I_U(t_3) = 0, I_U'(t_3) < 0$ , and for all  $0 \leq t \leq t_3$  one has that  $S(t) \geq 0, E(t) \geq 0, I_E(t) \geq 0$  and  $R(t) \geq 0$ ;
- (4)  $I_E(t_4) = 0, I_E'(t_4) < 0$ , and for all  $0 \leq t \leq t_4$  we have that  $S(t) \geq 0, E(t) \geq 0, I_U(t) \geq 0$  and  $R(t) \geq 0$ ;
- (5) Finally,  $R(t_5) = 0, R'(t_5) < 0$ , and for all  $0 \leq t \leq t_5$  one has that  $S(t) \geq 0, E(t) \geq 0, I_U(t) \geq 0$  and  $I_E(t) \geq 0$ .

The first case contradicts the assumption  $S'(t) = \lambda > 0$  meaning that  $S(t) \geq 0, t \geq 0$ . The second case contradicts the fact  $E'(t_2) = \frac{\beta_1 S I_U + \beta_2 S I_E}{N} \geq 0$  that is  $E(t) \geq 0$ , for all  $t > 0$ . By the analogous arguments, it can be shown that  $I_U(t) \geq 0, I_E(t) \geq 0, R(t) \geq 0$ , for all  $t \geq 0$ . Thus, the solutions of  $S(t), E(t), I_U(t), I_E(t)$  and  $R(t)$  remain positive for all  $t > 0$ . □

#### 6. Analysis of the model

**6.1. Existence of the disease free equilibrium state,  $E_0$ .** At the disease free equilibrium state we have absence of infection. Thus, all the Ebola infected classes will be zero and the entire population will comprise of only Ebola free, susceptible individuals. A disease free equilibrium state of the model above is unique and exists at the point:  $E_0 = (S^*, E^*, I_U^*, I_E^*, R^*) = (\frac{\lambda}{\mu}, 0, 0, 0, 0)$

**6.2. The basic reproduction number.** The basic reproduction number,  $R_0$  of the system (2.1) to (2.5) can be obtained by using the next generation matrix method formulated in [6, 16].

As our population is closed, let  $X = (E, I_U, I_E)^T$  then  $\frac{dX}{dt} = f(x) - v(x)$  where:

$$(6.1) \quad f(x) = \begin{pmatrix} \beta_1(I_U) + \beta_2(I_E) \\ \rho E \\ 0 \end{pmatrix},$$

and

$$(6.2) \quad v(x) = \begin{pmatrix} (\alpha + \mu)E \\ (\mu + \delta_1)I_U \\ (\mu + \delta_2)I_E - (\alpha - \rho)E \end{pmatrix}.$$

The Jacobian matrices of  $f(x)$  and  $v(x)$  evaluated at the disease free equilibrium,  $E_0$  are:

$$(6.3) \quad Df(E_0) = F = \begin{pmatrix} 0 & \beta_1 & \beta_2 \\ \rho & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

and

$$(6.4) \quad Dv(E_0) = V = \begin{pmatrix} \alpha + \mu & 0 & 0 \\ 0 & \mu + \delta_1 & 0 \\ \rho - \alpha & 0 & \mu + \delta_2 \end{pmatrix}.$$

The model reproduction number, denoted by  $R_0$  is thus given by:

$$R_0 = \frac{1}{2} \left[ \frac{\beta_2(\alpha - \rho)}{(\mu + \delta_2)(\alpha + \mu)} + \sqrt{\left[ \frac{\beta_2(\alpha - \rho)}{(\mu + \delta_2)(\alpha + \mu)} \right]^2 + \frac{4\beta_1\rho}{(\mu + \delta_1)(\alpha + \mu)}} \right]$$

### 6.3. Local stability of the disease free equilibrium, $E_0$ .

**Theorem 6.1.** *The disease free equilibrium  $E_0$  is locally asymptotically stable for  $R_0 < 1$  and unstable otherwise.*

*Proof.* To prove the local stability of the disease free equilibrium, we used the jacobian stability method. If the eigenvalues of (F-V) have negative real parts then the disease free equilibrium is locally stable. Using F and V from equations (6.3) and (6.4):

$$(6.5) \quad F - V = \begin{pmatrix} -\alpha - \mu & \beta_1 & \beta_2 \\ \rho & -\delta_1 - \mu & 0 \\ \alpha - \rho & 0 & -\mu - \delta_2 \end{pmatrix}.$$

Using characteristic equation  $|(F - V) - \lambda I| = 0$ , the following equation is obtained.

$\lambda^3 + [(\alpha + 3\mu + \delta_1 + \delta_2) + (\mu + \delta_2)(\alpha + 2\mu + \delta_1)](\lambda^2) + [(\mu + \delta_1)(\alpha + \mu) - \beta_1\rho - \beta_2(\alpha - \rho)](\lambda) + (\mu + \delta_2)[(\mu + \delta_1)(\alpha + \mu) - \beta_1\rho] + \beta_2(\rho - \alpha)(\mu + \delta_1) = 0$ , where  $\lambda$  is the eigenvalue in this case. As all the coefficients are positive for all  $R_0 < 1$  then all the eigenvalues are negative. Besides, the product of the coefficient of  $\lambda^2$  and the coefficient of  $\lambda$  is greater than the constant term for  $R_0 < 1$ . Therefore, for  $R_0 < 1$  the disease free equilibrium is locally asymptotically stable.  $\square$

**6.4. Global stability of the disease free equilibrium.**

**Theorem 6.2.** *For system (2.1) to (2.5), the disease free equilibrium is globally asymptotically stable if  $R_0 < 1$*

*Proof.* To prove comparison theorem was used. The rate of change of the variables  $(E, I_U, I_E, R)$  of the above system can be re-written as:

$$\begin{pmatrix} \frac{dE}{dt} \\ \frac{dI_U}{dt} \\ \frac{dI_E}{dt} \\ \frac{dR}{dt} \end{pmatrix} = (F' - V') \begin{pmatrix} E \\ I_U \\ I_E \\ R \end{pmatrix} - (1 - \frac{\mu S}{\lambda}) \begin{pmatrix} 0 & \frac{\beta_1 S_0}{N} & \frac{\beta_2 S_0}{N} & 0 \\ P & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E \\ I_U \\ I_E \\ R \end{pmatrix}$$

where  $(F')$  and  $(V')$  are jacobian matrices (of order 4) evaluated at the disease free equilibrium and  $S_0 = \frac{\lambda}{\mu}$ . Clearly,

$$(6.6) \quad \begin{pmatrix} \frac{dE}{dt} \\ \frac{dI_U}{dt} \\ \frac{dI_E}{dt} \\ \frac{dR}{dt} \end{pmatrix} \leq (F' - V') \begin{pmatrix} E \\ I_U \\ I_E \\ R \end{pmatrix}$$

Since, the eigenvalues of the matrix  $(F' - V')$  have negative real parts (this comes from the local stability results in [16, Lemma 1]) then the system (2.1) to (2.5) is stable whenever  $R_0 < 1$ . So  $(E, I_U, I_E, R) \rightarrow (0, 0, 0, 0)$  and  $S \rightarrow \frac{\lambda}{\mu}$  as  $t \rightarrow \infty$ . By the comparison theorem [6, 14]  $(S, E, I_U, I_E, R) \rightarrow E_0$  as  $t \rightarrow \infty$ . Therefore,  $E_0$  is globally asymptotically stable.  $\square$

**6.5. Endemic equilibrium.**

**6.5.1. Existance of the endemic equilibrium.** If  $R_0 > 1$ , the system (2.1) to (2.5) has a unique endemic equilibrium:  $E^*(S^*, E^*, I_U^*, I_E^*, R^*)$  where:

$$(6.7) \quad S^* = \frac{N^2(\mu + \delta_1)(\mu + \delta_2)(\alpha + \mu)}{N\beta_1\rho(\mu + \delta_2) + N\beta_2(\alpha - \rho)(\mu + \delta_1)}$$

$$(6.8) \quad E^* = \frac{\lambda - \mu}{\alpha + \mu} S^*$$

$$(6.9) \quad I_U^* = \frac{\rho(\alpha - \mu)}{(\mu + \beta_1)(\mu + \alpha)} S^*$$

$$(6.10) \quad I_E^* = \frac{\alpha(\alpha - \rho)}{\rho(\mu + \delta_2)} I_U^*$$

$$(6.11) \quad R^* = \frac{\gamma(I_U^* + I_E^*)}{\mu}$$

**6.5.2. Local stability of endemic equilibrium.** By evaluating the Jacobian matrices at the endemic equilibrium it can be easily shown (similar to the method for local stability of disease free equilibrium) that the characteristics roots of the matrix have negative real parts. Therefore, we say the endemic equilibrium is locally asymptotically stable.

**6.5.3. Global stability of endemic equilibrium.** By making change of variables and using Lyapunov method, define the function:

$V = X^{*2} + Y^{*2} + Z^{*2} + W^{*2} + A^{*2}$  where,

$$(6.12) \quad X^* = S - \frac{\lambda N}{\beta_1 I_U + \beta_2 I_E + \mu N}$$

$$(6.13) \quad Y^* = E - \frac{\beta_1 S I_U \beta_2 S I_E}{N(\mu + \alpha)}$$

$$(6.14) \quad Z^* = I_U - \frac{\rho E}{\mu + \delta_1}$$

$$(6.15) \quad W^* = I_E - \frac{(\alpha - \rho)E}{\mu + \delta_2}$$

$$(6.16) \quad A^* = R - \frac{\gamma(I_U + I_E)}{\mu}$$

Clearly,  $V(0, 0, 0, 0, 0) = (0, 0, 0, 0, 0)$  and  $V(X, Y, Z, W, A) > 0$  for all  $(X, Y, Z, W, A)$  in the region. That is,  $V$  is positive definite. Then, the partial derivative of  $V$  about the system gives:

$$V' = 2\left[S - \frac{\lambda N}{\beta_1 I_U + \beta_2 I_E + \mu N}\right]\left(\frac{dS}{dt}\right) + 2\left[E - \frac{\beta_1 S I_U \beta_2 S I_E}{N(\mu + \alpha)}\right]\left(\frac{dE}{dt}\right) + 2\left[I_U - \frac{\rho E}{\mu + \delta_1}\right]\left(\frac{dI_U}{dt}\right) + 2\left[I_E - \frac{(\alpha - \rho)E}{\mu + \delta_2}\right]\left(\frac{dI_E}{dt}\right) + 2\left[R - \frac{\gamma(I_U + I_E)}{\mu}\right]\left(\frac{dR}{dt}\right)$$



where,

$$\frac{dX^*}{dt} = \frac{dS}{dt}, \frac{dY^*}{dt} = \frac{dE}{dt}, \frac{dZ^*}{dt} = \frac{dI_U}{dt}, \frac{dW^*}{dt} = \frac{dI_E}{dt}, \frac{dA^*}{dt} = \frac{dR}{dt}$$

then,

$$V' = -2 \left[ \left( S - \frac{\lambda N}{\beta_1 I_U + \beta_2 I_E + \mu N} \right)^2 \right] \left( \frac{\beta_1 I_U + \beta_2 I_E + N\mu}{N} \right) - 2 \left[ \left( E - \frac{\beta_1 S I_U \beta_2 S I_E}{N(\mu + \alpha)} \right)^2 \right] (\alpha + \mu) - 2 \left[ \left( I_U - \frac{PE}{\mu + \delta_1} \right)^2 \right] (\mu + \delta_1) - 2 \left[ \left( I_E - \frac{(\alpha - P)E}{\mu + \delta_1} \right)^2 \right] (\mu + \delta_2) - 2 \left[ \left( R - \frac{\gamma(I_U + I_E)}{\mu} \right)^2 \right] (\mu) \leq 0.$$

Therefore, as  $V'$  is negative definite, our system is globally asymptotically stable.

### 7. Numerical simulation

To illustrate the analytical results obtained above, we give some simulations using the parameters values of Table 1 in Section 3 above. The results are given below. Figure 2 shows when  $R_0 = 0.695 < 1$ , the disease free equilibrium is globally asymptotically stable. This means the disease dies out. Figure 3 shows when  $R_0 = 1.627 > 1$ , the endemic equilibrium is globally asymptotically stable. This means the disease persists in the population.

### 8. Conclusion

In overall, the dynamical behavior of the formulated Ebola epidemic model is investigated and the basic reproduction number, which plays a vital role in controlling the spread of Ebola is calculated. Our new model has the detail about all compartments and we found it works for the current Ebola outbreak very well. The parameter values used are all the latest values. When the reproductive number,  $R_0 < 1$  the disease equilibrium is globally asymptotically stable. The disease free equilibrium is globally asymptotically stable, which implies the disease will die out. From our study we observed that when contact rate of susceptible is decreasing so does the number of the infected population. The biological implication of this is that, the contact rate is playing a very important role in controlling the spread of Ebola. Hence, isolation of the Ebola patient and providing great awareness (education) are highly the crucial tools to fight Ebola. When  $R_0 > 1$  the Endemic equilibrium is globally asymptotically stable, that implies Ebola will sustain and lead to epidemic eventually. Therefore, starting from personal hygiene isolating the patient, putting on protective gloves, disposal of wastes safely and safe burial practices are among important things to be taken into consideration. Besides, in order to prevent epidemics, through the analysis of the model the government must

FIGURE 2. When  $(\beta_1 = 0.333, \beta_2 = 0.11, \rho = 0.035, R_0 = 0.695 < 1)$ , the disease free equilibrium  $E_0$  is globally asymptotically stable.

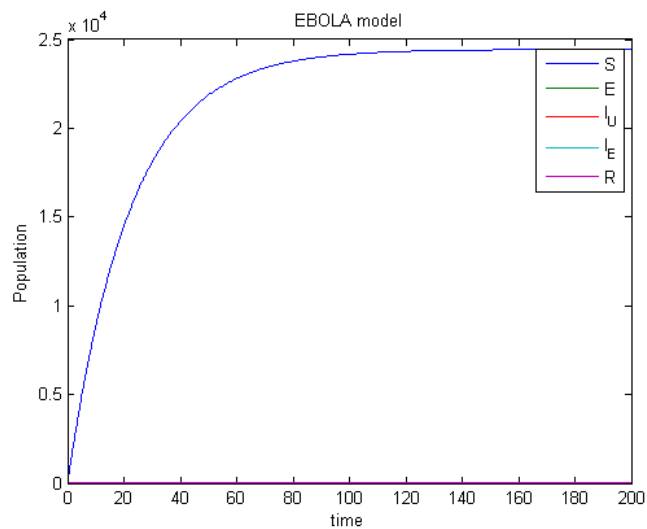
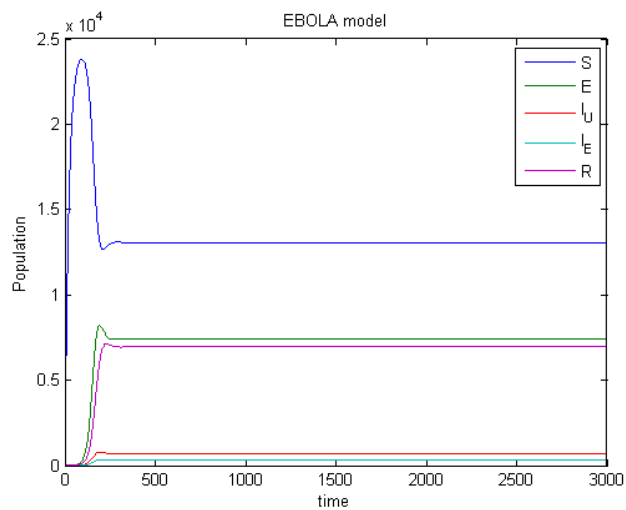


FIGURE 3. When  $(\beta_1 = 0.867, \beta_2 = 0.133, \rho = 0.036, R_0 = 1.6275 > 1)$ , the endemic equilibrium  $E^*$  is globally asymptotically stable.



strictly manage the policy on creating the greatest awareness (education) on Ebola and carry it out. This in turn helps for health campaigning and raising health literacy which as seen from our study helps to control the disease. We finally strongly believe that our study will play its own role in the current effort of controlling the Ebola outbreak in West Africa.

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