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STRONGLY $k-$SPACES

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Abstract. In this paper, we introduce the notion of strongly $k-$spaces (with the weak (=finest) pre-topology generated by their strongly compact subsets). We characterize the strongly $k-$spaces and investigate the relationships between preclosedness, locally strongly compactness, pre-first countableness and being strongly $k-$space.
Keywords: Strongly compact sets, preopen sets, $k-$spaces.
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1. Introduction

In [1], Arens introduced a category of Hausdorff spaces called $k-$spaces with the property that each subset which intersects every compact set in a closed set is itself a closed set. The concept of a $k-$space or a compactly generated space is widely encountered in the literature, [9,10,20,22,23]. The Hausdorff property is imposed to guarantee that compact subsets are closed, [20]. However, the definition of a $k-$space disagreeing the requirement of Hausdorff property has been preferred in [14, 17, 21].

In this paper, we give the necessary and sufficient conditions for a space X to be a strongly $k-$space which need not be pre-Hausdorff. Our fundamental aim is to define a strongly $k-$space generated by its strongly compact subsets. The strong version of compactness was introduced in [2] in terms of preopen sets defined by [11]. A space X is called strongly compact if every preopen cover of X has a finite subcover, [2, 12]. It is proved in [7] that a space X is strongly compact if and only if each infinite subset of X has a pre-limit point. Besides, in [16, Proposition (3.6)], it is observed that the pre-irresolute image of a strongly compact space is strongly compact.

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2. **Preliminaries**

Let (X,τ) be a topological space and A be a subset of the topological space X. The closure of A and the interior of A are denoted by $\mathrm{cl}\,(A)$ and $\mathrm{int}\,(A)$, respectively. Also, the power set of X is denoted by $\exp(X)$. Let us recall the basic notions which will be needed in the next section.

A subset S of (X,τ) is called preopen if $S\subseteq \mathrm{int}\,(\mathrm{cl}\,(S))$ and the family of preopen sets is denoted by $p\tau$ and called the pre-topology on X, [11]. A subset F of (X,τ) is called preclosed if its complement is a preopen or equivalently $\mathrm{cl}\,(\mathrm{int}\,(F))\subseteq F$, [13]. For example, a dense subset of (X,τ) is a preopen set. In [11], it is proved that an arbitrary union of preopen sets is preopen.

For $A\subset X$, it is denoted by $\mathrm{cl}_{p\tau}A$ (preclosure of A) the intersection of all preclosed sets containing A, i.e., the smallest preclosed set containing A and by $\mathrm{int}_{p\tau}A$ (preinterior of A) the union of preopen sets contained in A, i.e., the biggest preopen set contained in A. Thus, $\mathrm{int}_{p\tau}A=\bigcup\{S:S\subset A \text{ and } S\in p\tau\}$ and $\mathrm{cl}_{p\tau}A=\bigcap\{F:A\subset F \text{ and } X-F\in p\tau\}$, [11].

Let x be a point of the space X and $U\subset X$, U is called a pre-neighborhood of x in X if there exists $S\in p\tau$ such that $x\in S\subset U$, [19].

A class B_x of preopen sets containing x is called a pre-local base at x, if there exists $S_x\in \mathrm{B}_x$ with $x\in S_x\subseteq S$, for each preopen set S containing x, [3].

A pre-topological space $(X,p\tau)$ is said to be a pre-first countable space if there exists a countable pre-local base at every $x\in X$, [3].

Let A be a subset of a topological space X. A point $x\in X$ is called a pre-limit point of A if every preopen set $S\subset X$ containing x contains a point of A other than x, [8].

Let $(X,p\tau)$ and $(Y,p\tau')$ be two pre-topological spaces, then a function $f:(X,p\tau)\to(Y,p\tau')$ is said to be pre-irresolute iff $f^{-1}(S)\in p\tau$ for all $S\in p\tau'$, [5]. A function $f:(X,p\tau)\to(Y,p\tau')$ is called precontinuous if the preimage of every open subset of Y is a preopen subset of X, [15].

A sequence $\{x_n\}_{n\in N}$ is said to be pre-converges to a point x of X if $\{x_n\}_{n\in N}$ is eventually in every preopen set containing x, [18].

A space X is called strongly compact if every preopen cover of X has a finite subcover, [12]. A topological space X is strongly compact if and only if it is compact and every infinite subset of X has nonempty interior, [6]. Let f be a pre-irresolute surjection from X onto Y. If X is strongly compact, then Y is strongly compact, [12].

A space X is said to be locally strongly compact at $x\in X$ iff x has a neighborhood which is strongly compact in X. Also, X is called locally strongly compact if for each $x\in X$ there exists a preopen neighborhood U of x with strongly compact $\mathrm{cl}_{p\tau}U$.

3. **Strongly $k-$spaces**

In this section, our aim is to define and study strongly $k-$spaces by using the concepts of preopen sets and strongly compact sets.

Definition 3.1. A pre-topological space is a strongly $k-$space (strongly compactly generated space) if the following holds:

A subset $A \subseteq X$ is preopen whenever $A \cap K$ is preopen in K for every strongly compact subset $K \subseteq X$.

Equivalently, $A \subseteq X$ is preclosed if and only if $A \cap K$ is preclosed in K for every strongly compact subset $K \subseteq X$.

Example 3.2. Any strongly compact space is a strongly $k-$space.

Example 3.3. Let $X = \{a, b, c, d\}$ and

$$\tau = \{\varnothing, X, \{a\}, \{d\}, \{a, b\}, \{a, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}\}.$$

Then (X, τ) is a topological space such that $p\tau = \tau$. Every subset $K \subset X$ is strongly compact (K being a finite set). It is easy to verify that X is a strongly $k-$space since A is preopen in X when $A \cap K$ is preopen in K for each subspace K of X. On the other hand, it can be seen that X is a strongly $k-$space because it is strongly compact.

According to a theorem of Cohen [4], we give the relation between locally strongly compact spaces and strongly $k-$spaces in the following theorem.

Theorem 3.4. *A space is a strongly $k-$space iff it is a quotient space of a locally strongly compact space.*

Proof. Let $(X, p\tau)$ be a strongly $k-$space and $\wp$ be the family of all strongly compact subsets of X. The disjoint union of all $K \in \wp$ with the relative topology inherited from the space X is a locally strongly compact space. Let us denote it by $Y = \bigcup\limits_{K \in \wp} K$. Then an identity mapping $f : Y \to X$ can be defined such that $K \in \wp$ corresponds to the strongly compact subset K of X. Let $p\tau_f$ be a generalized quotient pre-topology on X. We need to verify that the quotient pre-topology generated by this mapping coincides with the original pre-topology on X. It is clear that, $p\tau \subset p\tau_f$. Then we have to prove that $p\tau_f \subset p\tau$ to show the space X is coincident with the quotient space of space Y. Hence, let $A \in p\tau_f$. Since X is a strongly $k-$space, if $A \cap K$ is preopen in K then $A \in p\tau$. Let us show $A \cap K$ is preopen in K for all strongly compact $K \subseteq X$. Considering $A \in p\tau_f$, $f^{-1}(A)$ is preopen subset in Y. The intersection of a strongly compact set and a preopen set is preopen in its induced pre-topology, that is, $f^{-1}(A) \cap K$ is a preopen subset in K. Besides $f^{-1}(A) \cap K$ is a subset of $K \subset Y$, it is a subset of $K \subset X$ such that $f^{-1}(A) \cap K = A \cap K$. Hence $A \cap K$ is preopen in K, namely, $A \in p\tau$.

Conversely, let Y be a locally strongly compact space and $f : Y \to X$ be the generalized quotient mapping. Since $f : Y \to X$ is a surjective function the quotient pre-topology on X is the collection of subsets of X that have preopen inverse images under f. In other words, the quotient pre-topology is the finest pre-topology on X for which f is a pre-irresolute function. Let A be a non-empty subset of X such that $K \cap A$ is preopen in K for every strongly compact $K \subseteq X$. Then we need to verify that A is preopen in X to show X is a strongly $k-$space. Since Y is locally strongly compact, for each $y \in Y$ there exists a preopen neighborhood U of y with strongly compact $\mathrm{cl}_{p\tau}U$. By the fact that f is pre-irresolute, $f(\mathrm{cl}_{p\tau}U)$ is strongly compact. According to the assumption that intersection of A and each compact set of X is relatively preopen, the subset $A \cap f(\mathrm{cl}_{p\tau}U)$ is preopen in $f(\mathrm{cl}_{p\tau}U)$. The set $f^{-1}(A) \cap \mathrm{cl}_{p\tau}U$ is preopen in $\mathrm{cl}_{p\tau}U$, since f is pre-irresolute. Moreover, $f^{-1}(A) \cap U$ is preopen in U. On the other hand, $Y = \bigcup_{y \in U} U$ and the subsets U are preopen sets. Thus $f^{-1}(A)$ is preopen in Y. Consequently, A is preopen in X and this completes the proof. □

To express the relation between strongly $k-$space and preclosedness, we need to give the following lemmas.

Lemma 3.5. *Let $F \subseteq A$ providing that $(X, p\tau)$ is a pre-topological space and $A \subset X$. The set F is preclosed in the subspace A iff there is a preclosed set H in the space X such that $F = A \cap H$.*

Proof. Let F be a preclosed subset in the subspace A. Then $A \setminus F$ is a preopen subset in the subspace A. Thus, there is a preopen subset S in X such that $A \setminus F = A \cap S$. That is, $F = A \cap (X \setminus S)$. Then, the set $H = X \setminus S$ is preclosed in X and $F = A \cap H$.
Conversely, let H be a preclosed set in X such that $F = A \cap H$. Then, $A \setminus F = A \cap (X \setminus H)$. The set $A \setminus F$ is preopen in A since $X \setminus H$ is preopen in X. Thus, F is preclosed in the subspace A. □

Lemma 3.6. *Let $(X, p\tau)$ be a pre-topological space, the subset A be preclosed in X and $F \subseteq A$. The set F is preclosed in the subspace A iff the set F is preclosed in the subspace X.*

Proof. Since F is preclosed in the subspace A, from Lemma 3.5, there is a preclosed set H in the space X such that $F = A \cap H$. Since the subsets A and H are preclosed, it can be written such that $A \supset \mathrm{cl}(\mathrm{int}(A))$ and $H \supset \mathrm{cl}(\mathrm{int}(H))$. Then $A \cap H \supset \mathrm{cl}(\mathrm{int}(A)) \cap \mathrm{cl}(\mathrm{int}(H)) \supset \mathrm{cl}(\mathrm{int}(A \cap H))$, that is, $A \cap H$ is preclosed. Converse of the assertion is obvious. □

Theorem 3.7. *A preclosed (preopen) subspace of a strongly $k-$space is a strongly $k-$space.*

Proof. Let F be a preclosed subspace of a strongly $k-$space X and the intersection of $H \subset F$ and every strongly compact set $L \subset F$ be preclosed in L. We need to show that H is preclosed in F. Suppose that K denotes any strongly compact subset in X. The intersection of the preclosed subset F and the strongly compact subset K is strongly compact in X. Moreover, $F \cap K$ is a strongly compact subset of F with respect to subspace pre-topology. If we denote $F \cap K = L$, then $H \cap L$ is preclosed in F. By the fact that F is preclosed, it is seen from the Lemma 3.6 that $H \cap L$ is preclosed in X. Under the assumption that X is a strongly $k-$space, H is preclosed in X. This gives us H is preclosed in F, too. □

The proof for a preopen subspace is identical.

Theorem 3.8. *A space is a strongly $k-$space if and only if each point has a pre-neighborhood whose interior of closure is a strongly $k-$space.*

Proof. Suppose that every point x in X has a pre-neighborhood whose interior of closure is a strongly k-space. To prove that X is a strongly k-space, we have to indicate that A is a preopen set in K whenever $A \cap K$ is preopen for every strongly compact set K. Suppose $x \in A$. By hypothesis, any pre-neighborhood of $x \in X$ is U such that $\mathrm{int}\,(\mathrm{cl}\,(U))$ is a strongly $k-$space. For every strongly compact $L \subset \mathrm{int}\,(\mathrm{cl}\,(U))$, $(A \cap \mathrm{int}\,(\mathrm{cl}\,(U))) \cap L = A \cap L$ is a preopen set. Therefore, $A \cap \mathrm{int}\,(\mathrm{cl}\,(U))$ is preopen in $\mathrm{int}\,(\mathrm{cl}\,(U))$. Namely, $A \cap \mathrm{int}\,(\mathrm{cl}\,(U)) \subset \mathrm{int}\,(\mathrm{cl}\,(A \cap \mathrm{int}\,(\mathrm{cl}\,(U)))) \subset \mathrm{int}\,(\mathrm{cl}\,(A)) \cap \mathrm{int}\,(\mathrm{cl}\,(U))$. A is a preopen due to $x \in \mathrm{int}\,(\mathrm{cl}\,(A))$ when $x \in A \cap \mathrm{int}\,(\mathrm{cl}\,(U))$, that is, $A \subset \mathrm{int}\,(\mathrm{cl}\,(A))$.
Conversely, let X be a strongly $k-$space and U be any pre-neighborhood of any point x in X. Obviously, $\mathrm{int}\,(\mathrm{cl}\,(U))$ is preopen. Then the proof is obvious from Theorem 3.7. □

Theorem 3.9. *A space is a strongly $k-$space if there exists a strongly compact set K such that $x \in \mathrm{int}\,(\mathrm{cl}\,(A \cap K))$ for each subset A and $x \in A$.*

Proof. Suppose that the intersection of A and every strongly compact set of X is preopen. Let K be any strongly compact set such that $x \in \mathrm{int}\,(\mathrm{cl}\,(A \cap K))$ for $x \in A$. By hypothesis, $A \cap K \subset \mathrm{int}\,(\mathrm{cl}\,(A \cap K)) \subset \mathrm{int}\,(\mathrm{cl}\,(A)) \cap \mathrm{int}\,(\mathrm{cl}\,(K))$, then $x \in \mathrm{int}\,(\mathrm{cl}\,(A))$. Thus A is preopen, that is, X is a strongly $k-$space. □

Theorem 3.10. *A locally strongly compact space is a strongly $k-$space.*

Proof. Let $A \subset X$. Suppose that the intersection of A and every strongly compact set of X is preopen. For any point x in A, there is a strongly compact pre-neighborhood K of x since X is locally strongly compact. $A \cap K \subset \mathrm{int}\,(\mathrm{cl}\,(A \cap K))$ because $A \cap K$ is preopen. Thus there is a pre-neighborhood K such that $x \in \mathrm{int}\,(\mathrm{cl}\,(A \cap K))$. Then the proof is obvious from Theorem 3.9. □

Example 3.11. The collection $\zeta = \{(-\infty,\ a) : a \in \mathbb{R}\} \cup \{\mathbb{R},\ \varnothing\}$ establishes a topology (left-ray topology) and so a pre-topology on $\mathbb{R}$. Since the sets of the form $(-\infty,\ a]$ are compact and every infinite subsets of them has nonempty preinterior, these sets are strongly compact. Then $(\mathbb{R}, \zeta)$ is a locally strongly compact space because there exists a strongly compact neighborhood $(-\infty,\ x]$ for each point $x \in \mathbb{R}$. By virtue of Theorem 3.10, $(\mathbb{R}, \zeta)$ is a strongly $k-$space, too. The set of rational numbers $\mathbb{Q}$ is a preopen but not open subset of $\mathbb{R}$. By considering Theorem 3.7, $\mathbb{Q}$ is a strongly $k-$space. But $\mathbb{Q}$ is not compact and so not strongly compact.

Now, to express the relation between being pre-first countable and being strongly $k-$space we need to give the following lemmas:

Lemma 3.12. *Let pre-topological space $(X, p\tau)$ be a pre-first countable space. For a point $x \in X$ and a subset $A \subset X$, $x \in \mathrm{cl_{p\tau}A}$ iff there is a sequence $\{x_n\}_{n\in\mathbb{N}}$ which pre-converges to x in A.*

Proof. Let $x \in \mathrm{cl_{p\tau}A}$ and X be a pre-first countable space. Then the point $x \in X$ has a countable pre-neighborhood basis $\{U_n\}_{n\in\mathbb{N}}$ which reduce one within the other without loss of generality. For all $n \in \mathbb{N}$, $U_n \cap A \neq \emptyset$ because $x \in \mathrm{cl_{p\tau}A}$. If it is chosen $x_n \in U_n \cap A$, it is constituted a sequence $\{x_n\}_{n\in N}$ in A. It is clear that the sequence $\{x_n\}_{n\in N}$ that is constituted in this way pre-converges to the point x.
Conversely, let it be given a sequence $\{x_n\}_{n\in N}$ in A such that $< x_n >\to x$. By the definition of pre-convergence, for every pre-neighborhood U of x there is at least one $n_0 \in \mathbb{N}$ such that $x_n \in U$ for every $n \geq n_0$. Thus $x_n \in U \cap A$. Namely $x \in \mathrm{cl_{p\tau}A}$. □

Lemma 3.13. *Let $(X, p\tau)$ be a pre-topological space. If $\{x_n\}_{n\in\mathbb{N}}$ is a sequence which consists of elements of X and pre-converges to point x in X, $K = \{x\} \cup \{x_n : n \in \mathbb{N}\}$ is a strongly compact set.*

Proof. Let the family $\Omega = \{S_i : i \in I\}$ be a preopen cover of K. Since $x \in K \subseteq \bigcup\limits_{i\in I} S_i$, there exists at least one $i_0 \in I$ such that $x \in S_{i_0}$. On the other hand, considering the sequence $\{x_n\}_{n\in\mathbb{N}}$ which pre-converges to the point x, there is an $n_0 \in \mathbb{N}$ such that $x_n \in U_{i_0}$ for every $n \geq n_0$. Let us choose a $S_n \in \Omega$ such that $x_n \in S_n$ for each $n < n_0$. In this case, $x_n \in \bigcup\limits_{i=1}^{n_0-1} S_i$ for $n < n_0$. Then $K \subseteq S_{i_0} \cup \left(\bigcup\limits_{i=1}^{n_0-1} S_i\right)$. In other words, the family $\{S_{i_0}\} \cup \{S_i : i = 1, 2, \ldots, n_0 - 1\}$ is a finite preopen subcover of the preopen cover $\Omega = \{S_i : i \in I\}$. Thus $K = \{x\} \cup \{x_n : n \in \mathbb{N}\}$ is a strongly compact set. □

Theorem 3.14. *A pre-first countable space is a strongly $k-$space.*

Proof. Let $x \in \mathrm{cl}_{\mathrm{p}\tau}\mathrm{A}$. By Lemma 3.12, there is a sequence $\{x_n\}_{n\in\mathbb{N}}$ which pre-converges to x in A. If it is chosen $K = \{x\} \bigcup \{x_n : n \in \mathbb{N}\}$, it was proven that K is strongly compact in Lemma 3.13. Since K is strongly compact for every $x \in X$, X is locally strongly compact. The result follows from Theorem 3.10. □

Definition 3.15. A subspace A of a space X has property (*) if whenever $S \subset A$ and $S \cap K$ is preopen (or preclosed) in $A \cap K$ for each strongly compact set K in X, then S is preopen (or preclosed) in A.

Theorem 3.16. *A subspace A of a space X is a strongly $k-$space iff A has property (*) and $A \cap K$ is a strongly $k-$space for each strongly compact set K in X.*

Proof. Let a subspace A of X be a strongly $k-$space. Suppose $S \subset A$ meets each strongly compact set $K \subset X$ in a preopen set in $A \cap K$. Then, $S \cap K$ is a preopen in $A \cap K$. By the hypothesis, S is a preopen in A. Also, $A \cap K$ is preopen subset of A for every strongly compact set K. By Theorem 3.7, $A \cap K$ is strongly $k-$space.
Conversely, let $S \cap K \subset A \cap K$ be preopen for $S \subset A$ and every strongly compact set $K \subset A$. Assume that $A \cap L$ is a strongly $k-$space for every strongly compact $L \subset X$, and $M \subset A \cap L$ is the strongly compact set. Then $M \cap L$ is a strongly compact subset. By hypothesis, $S \cap (M \cap L)$ is preopen in A for the strongly compact set $M \cap L$. Therefore, $S \cap L$ is preopen in $A \cap L$. Hence, S is preopen in A. This verifies that A is a strongly $k-$space.

□

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