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ON ANNIHILATOR IDEALS IN SKEW POLYNOMIAL RINGS

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Abstract. This article examines annihilators in the skew polynomial ring $R[x;\alpha,\delta]$. A ring is strongly right AB if every non-zero right annihilator is bounded. In this paper, we introduce and investigate a class of McCoy rings which satisfies Property (A) and the proposed conditions by P.P. Nielsen [J. Algebra 298 (2006) 134-141]. We assume that R is an (α,δ)-compatible ring, and prove that, if R is nil-reversible then the skew polynomial ring $R[x;\alpha,\delta]$ is strongly right AB. It is also shown that, every right duo ring with an automorphism α is skew McCoy. Moreover, if R is strongly right AB and skew McCoy, then $R[x;\alpha]$ and $R[x;\delta]$ have right Property (A).
Keywords: McCoy ring, strongly right AB ring, nil-reversible ring, CN ring, rings with Property (A).
MSC(2010): Primary: 16D25; Secondary: 16D70, 16S34.

1. **Introduction**

Throughout this article, all rings are associative with identity. Let α be a ring endomorphism and δ an α-derivation of R, that is, δ is an additive map such that $\delta(ab) = \delta(a)b+\alpha(a)\delta(b)$, for all $a, b \in R$. We denote $R[x;\alpha,\delta]$ the Ore extension whose elements are the polynomials over R, the addition is defined as usual and the multiplication satifies in the relation $xa = \alpha(a)x + \delta(a)$, for any $a \in R$.

According to N. Jacobson [27], a right ideal of R is *bounded* if it contains a non-zero ideal of R. From E.H. Feller [15], a ring R is right (*left*) *duo* if every right (left) ideal is an ideal, and C. Faith [12] said a ring would be called *strongly right bounded* if every non-zero right ideal is bounded. The class of strongly bounded rings has been observed by many authors (e.g. [6,27,48,49]).

Due to H. Bell [5], a ring R is said to have *the insertion of factors property* (simply, *IFP*) if $ab = 0$ implies $aRb = 0$ for $a, b \in R$. Note that a ring R has

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IFP if and only if any right (or left) annihilator is an ideal. Rings with IFP are also called *semi-commutative*, see [41]. Right (resp. left) duo rings are both strongly right (resp. left) bounded and semi-commutative.

In [26], S.U. Hwang, N.K. Kim and Y. Lee introduced a condition that is a generalization of strongly bounded rings and semi-commutative rings, calling a ring *strongly right* AB if every non-zero right annihilator is bounded.

There is another important ring theoretic condition common in the literature related to the zero divisor and annihilator conditions. P.P. Neilsen in [44], calls a ring R right *McCoy* (resp. left McCoy), if for each pair of non-zero polynomials $f(x), g(x) \in R[x]$ with $f(x)g(x) = 0$, there exists a non-zero element $r \in R$ (resp. $c \in R$) with $f(x)r = 0$ (resp. $cg(x) = 0$). According to G.F. Birkenmeier [6] a ring R is called 2-*primal*, if the prime radical of R coincides with the set of nilpotent elements in R. Another property between commutativity and 2-primality is what P.M. Cohn in [9] calls a *reversible* ring. A ring R is reversible if for each $a, b \in R$, $ab = 0$ implies that $ba = 0$. We direct the reader to the excellent papers [1, 37, 38] for a nice introduction to some standard zero-divisor conditions.

P.P. Neilsen [44] raised a question: is there a natural class of McCoy rings, which includes all reversible rings and all rings R such that $R[x]$ is semi-commutative? We use this to define a new class of rings strengthening the condition for reversible rings. This property between "reversible" and "McCoy" is what we call nil-reversible rings. We say a ring R is *nil-reversible*, if $ab = 0 \Leftrightarrow ba = 0$, where $b \in nil(R)$.

An important theorem in commutative ring theory, related to zero-divisor conditions, is that if I is an ideal in a Noetherian ring and if I consists entirely of zero divisors, then the annihilator of I is nonzero. This result fails for some non-Noetherian rings, even if the ideal I is finitely generated. J.A. Huckaba and J.M. Keller [24], say that a commutative ring R has *Property* (A) if every finitely generated ideal of R consisting entirely of zero divisors has nonzero annihilator. Many authors have studied commutative rings with Property (A) ([3,18,23,24,36,46], etc.), and have obtained several results which are useful in studying commutative rings with zero-divisors. C.Y. Hong, N.K. Kim, Y. Lee and S.J. Ryu [22] extended Property (A) to noncommutative rings, and study such rings and several extensions with Property (A).

The recent surge of interest in a quantum groups and quantized algebras has brought renewed interest in general skew polynomial rings, due the fact that many of these quantized algebras and their representations can be expressed in terms of iterated skew polynomial rings. This development calls for a thorough study of skew polynomial rings $R[x; \alpha, \delta]$.

In section 2 we assume that R is an (α,δ)-compatible ring, and prove that, if R is nil-reversible then $R[x; \alpha, \delta]$ is strongly right AB. It is also shown that, every right duo ring with an automorphism α is skew McCoy; and whenever

$R[x;\alpha]$ is strongly right AB, then R is skew McCoy. Also if R is strongly right AB and skew Armendariz, then $R[x;\alpha,\delta]$ is strongly right AB. Whenever $R[x;\alpha,\delta]$ is strongly right AB and $r_{R[x;\alpha,\delta]}(Y) \neq 0$, then $r_R(Y) \neq 0$, for any $Y \subseteq R[x;\alpha,\delta]$. We then conclude that, nil-reversible rings is a larger class than the class asked by P.P. Nielsen [44], and satisfies the conditions. Indeed, nil-reversible rings is a natural class of McCoy rings which includes reversible rings, and all rings R such that $R[x]$ is strongly right (or left) AB (and hence all rings R such that $R[x]$ is semi-commutative). In section 3, it is shown whenever $R[x;\alpha]$ is strongly right AB, then $R[x;\alpha]$ has right Property (A). Moreover, if R is strongly right AB and skew McCoy, then the skew polynomial rings $R[x;\alpha]$ and $R[x;\delta]$ have right Property (A).

For any non-empty subset X of R, annihilators will be denoted by $r_R(X)$ and $l_R(X)$. We write $Z_l(R)$, $Z_r(R)$ for the set of all left zero-divisors of R and the set of all right zero-divisors of R.

2. Rings whose right annihilators are bounded

The notion of bounding a one-sided ideal by a two-sided ideal goes back at least to N. Jacobson [27]. He said that a right ideal of R is *bounded* if it contains a non-zero ideal of R. This concept has been extended in several ways. From C. Faith [12], a ring R is called *strongly right* (resp. *left*) *bounded* if every non-zero right (resp. left) ideal of R contains a non-zero ideal. A ring is called *strongly bounded* if it is both strongly right and strongly left bounded. Right (resp. left) duo rings are strongly right (resp. left) bounded and semi-commutative. G.F. Birkenmeier and R.P. Tucci [6, Proposition 6] showed that a ring R is right duo if and only if R/I is strongly right bounded for all ideals I of R.

A ring R is called *right* (resp. *left*) *AB* if every essential right (resp. left) annihilator of R is bounded.

Definition 2.1 ([26]). A ring R is called *strongly right* (resp. *left*) *AB* if every non-zero right (resp. left) annihilator of R is bounded; R is called *strongly AB* if R is strongly right and strongly left AB.

Obviously strongly right bounded rings and semi-commutative rings are both strongly right AB, but the converse statements are not necessarily true in either case as it is shown by the authors in [26, Example 2.3].

Definition 2.2. We say a ring R is *nil-reversible*, if for every $a \in R, b \in nil(R)$, $ab = 0 \Leftrightarrow ba = 0$.

Proposition 2.3. *Nil-reversible rings are* 2*-primal.*

Proof. Let R be a nil-reversible ring and $a \in nil(R)$. Then we get $a^k = 0$, for some positive integer k. So we have $a^{k-1}Ra = 0$ and hence $a^{k-2}Ra \subseteq nil(R)$.

This yields $a^{k-2}RaRa = 0$, as R is nil-reversible. We also have $a^{k-3}RaRa \subseteq nil(R)$, so $a^{k-3}RaRaRa = 0$. Continuing in this way we obtain $(aR)^k = 0$. This shows that R is a 2-primal ring. $\square$

Example 2.4. Let $A = F\langle x, y\rangle$ where x and y are noncommuting indeterminates and let F be a field. Let I be the two-sided ideal $AxyA + Ay^2xA + Ayx^2A$. Note that every element of $R = A/I$ can be written uniquely in the form $a + \sum_{i=1}^{n} a_i x^i + \sum_{j=1}^{m} b_j y^j + cyx$, where $a, a_i, b_j, c \in F$, for some integers m, n. It is not hard to see that $nil(R) = Fyx$ and for every $r = s + \sum_{i=1}^{n} r_i x^i + \sum_{j=1}^{m} t_j y^j + dyx \in R$, where $s, r_i, t_i, d \in F$ and m, n are integers, we have $nil(R)r = snil(R) = nil(R)s = rnil(R)$. This implies that R is a CN-ring and so a nil-reversible ring. As $xy = 0$ but $yx \neq 0$, then R is not reversible ring.

According to J. Krempa [32], an endomorphism α of a ring R is said to be *rigid* if $a\alpha(a) = 0$ implies $a = 0$, for $a \in R$. A ring R is said to be α-*rigid* if there exists a rigid endomorphism α of R. Every domain D with a monomorphism α is an α-rigid ring.

In [17], the second author and E. Hashemi introduced (α, δ)-compatible rings and studied their properties. A ring R is α-*compatible* if for each $a, b \in R$, we have $ab = 0$ if and only if $a\alpha(b) = 0$. In this case, clearly the endomorphism α is injective. Moreover, if δ is an α-derivation, R is said to be δ-*compatible* if for each $a, b \in R$, $ab = 0 \Rightarrow a\delta(b) = 0$. A ring R is (α, δ)-*compatible* if it is α-compatible and δ-compatible. Also by [17, Lemma 2.2], a ring R is α-rigid if and only if R is (α, δ)-compatible and reduced (i.e., have no nonzero nilpotent elements).

Lemma 2.5 ([17, Lemma 2.1])**.** *Let R be an (α, δ)-compatible ring. Then the following statements hold:*

(1) *If $ab = 0$, then $a\alpha^n(b) = \alpha^n(a)b = 0$ for all positive integers n.*

(2) *If $\alpha^k(a)b = 0$ for some positive integer k, then $ab = 0$.*

(3) *If $ab = 0$, then $\alpha^n(a)\delta^m(b) = 0 = \delta^m(a)\alpha^n(b)$ for all positive integers m, n.*

According to T.Y. Lam, A. Leroy and J. Matczuk [34], for any ring R, with an automorphism α and an α-derivation δ, and for integers i, j with $0 \leq i \leq j$, $f_i^j \in End(R, +)$ denotes the map which is the sum of all possible words in α, δ built with i letters α and $j - i$ letters δ. It is easy to prove, by induction, that for $a, b \in R$, we have $f_l^n(ab) = \sum_{i=1}^{n} f_i^n(a) f_l^i(b)$. For instance, $f_0^0 = 1, f_j^j = \alpha^j, f_0^j = \delta^j$. For any $f(x) \in R[x; \alpha, \delta]$, we denote by C_f the set of all coefficients of $f(x)$.

Lemma 2.6 ([45, Lemma 2.4])**.** *Let α be an endomorphism and δ an α-derivation of R and assume R is an (α, δ)-compatible ring. Then $ab \in nil(R)$ implies $af_i^j(b) \in nil(R)$ for all $0 \leq i \leq j$ and $a, b \in R$.*

Lemma 2.7 ([45, Lemma 2.6])**.** *Let R be an (α,δ)-compatible 2-primal ring and $f(x) = a_0 + a_1x + \cdots + a_nx_n \in R[x;\alpha,\delta]$. Then $f(x) \in nil(R[x;\alpha,\delta])$, if and only if $a_i \in nil(R)$ for all $0 \leq i \leq n$.*

Lemma 2.8. *Let R be an (α,δ)-compatible ring and suppose for some $X \subseteq S = R[x;\alpha,\delta]$ that there is $0 \neq c \in R$ with $XRc = 0$. Then we have $XSc = 0$.*

Proof. As a special case of [45, Corollary 2.1], if $abc = 0$ in R then for $i \leq j$ and $k \leq l$, $af_j^j i(b)f_k^l(c) = 0$. This shows that for any $f(x) \in X$ and $g(x) \in S$, all the coefficients of $f(x)g(x)c$ are zero. □

Theorem 2.9. *Let R be an (α,δ)-compatible ring. If R is nil-reversible, then $S = R[x;\alpha,\delta]$ is a strongly AB ring.*

Proof. We prove the right case, the left case is similar. Suppose $X \subseteq S$ and $r_S(X) \neq 0$. Let $Xg(x) = 0$, for some $g(x) = b_0 + b_1x + \cdots + b_nx^n \in S$, with minimal degree such that $b_n \neq 0$.

Case1: $g(x) \in nil(S)$. We show that $Xb_j = 0$, for every $0 \leq j \leq n$. Assume, on the contrary, that $Xb_k \neq 0$ for some $0 \leq k \leq n$. Then there exists $f(x) = a_0 + a_1x + \cdots a_mx^m \in X$ such that $f(x)b_k \neq 0$. On the other hand we have $f(x)g(x) = 0$. Then $a_m\alpha^m(b_n) = 0$. So $a_mb_n = 0$, by (α,δ)-compatibility of R. Since R is nil-reversible and using Lemma 2.7, $nil(S) = nil(R)[x;\alpha,\delta]$, we have $b_na_m = 0$. Now take $g_1(x) = g(x)a_m$. But $deg(g_1(x)) < deg(g(x))$ and $Xg_1(x) = 0$, which contradicts our assumption that $g(x)$ has minimal degree such that $f(x)g(x) = 0$, thus $g_1(x) = 0$. We have

$$0 = g(x)a_m = \sum_{i=0}^{n} b_if_0^i(a_m) + \sum_{i=1}^{n} b_if_1^i(a_m)x + \cdots + b_n\alpha^n(a_m)x^n.$$

So we obtain $b_n\alpha^n(a_m) = 0$, and by Lemma 2.5, we have $b_na_m = 0$. So $b_nf_i^j(a_m) = 0, 0 \leq i \leq j$, since R is (α,δ)-compatible. Also we get $b_{n-1}\alpha^{n-1}(a_m)$ $+ b_nf_{n-1}^n(a_m) = 0$ and so $b_{n-1}\alpha^{n-1}(a_m) = 0$. Continuing this procedure yields that $b_ja_m = 0, 0 \leq j \leq n$. Since R is nil-reversible $a_mb_j = 0$, so $a_mg(x) = 0$. From $f(x)g(x) = 0$ we get $(a_0 + \cdots a_{m-1}x^{m-1})(b_0 + b_1x + \cdots + b_nx^n) = 0$. Continuing in this way we can show that $a_ig(x) = 0$ for each $0 \leq i \leq m$, which contradicts with our assumption that $f(x)b_k \neq 0$. Thus $Xb_j = 0$, $0 \leq j \leq n$, and this implies $XRb_j = 0$, as $nil(S) = nil(R)[x;\alpha,\delta]$ by Lemma 2.7 and R is (α,δ)-compatible and nil-reversible ring. We conclude that $XSb_j = 0$, and so S is strongly right AB.

Case 2: $g(x) \notin nil(S)$. Then we have two cases:
(i): $g(x)C_X \neq 0$. In this case there exists $a \in C_X$ such that $g(x)a \neq 0$. Then there exists $h(x) = c_0 + c_1x + \cdots + c_kx^k \in X$ with $a \in C_h$. From $Xg(x) = 0$, we get $h(x)g(x) = 0$. Since $nil(R)$ is an ideal of R, by Proposition 2.3, and R is (α,δ)-compatible, it is easy to see that $c_ib_j \in nil(R), 0 \leq i \leq k, 0 \leq j \leq n$. Hence $b_ja \in nil(R)$ and that $b_jf_i^j(a) \in nil(R)$ therefore $g(x)a \in nil(S)$ since

$Xg(x) = 0$ and we reduce to the previous case.

(ii): $g(x)C_X = 0$. When $XRb_j = 0$ for some $0 \leq j \leq n$, there is nothing to prove. Now assume that $XRb_j \neq 0$ for all $0 \leq j \leq n$. Then there exists $f(x) = a_0 + a_1x + \cdots + a_mx^m \in X$, such that $f(x)Rb_j \neq 0$. So we have $a_kx^krC_g \neq 0$ for some $r \in R$ and so we get $0 \leq k \leq m$. By (α, δ)-compatibility of R, we have $a_krC_g \neq 0$. On the other hand, we get $C_ga_k = 0$, because $g(x)C_X = 0$ and R is (α, δ)-compatible. So we have $a_kf_i^j(rC_g) \in nil(R)$, by Lemma 2.6. It follows that $a_kx^krC_gC_X = 0$, since $g(x)C_X = 0$. Hence, by nil-reversibility, we have $C_XRa_krC_g = 0$. By Lemma 2.8, we get $XSa_krC_g = 0$ and we are done. $\square$

The result applies to polynomial rings R[x] where R is nil-reversible.

By M.P. Darzin [11] a ring R is a *CN-ring* whenever every nilpotent element of R is central. D. Khurana et al. [29], introduced the notion of *unit-central* rings (i.e., every invertible element of it lies in center), and show that each unit-central ring is a CN-ring. It is clear that CN-rings and reversible rings are nil-reversible.

Corollary 2.10. *If R is an (α, δ)-compatible CN-ring, then $R[x; \alpha, \delta]$ is strongly AB.*

In [16], the second author, M. Habibi and A. Alhevaz produced several classes of (α, δ)-compatible reversible rings.

Example 2.11. Consider the following ring of matrices over a reduced ring R:

$$S = \left\{ \begin{pmatrix} a & 0 & 0 & c \\ 0 & a & 0 & 0 \\ 0 & 0 & a & d \\ 0 & 0 & 0 & a \end{pmatrix} |a, c, d \in R \right\}.$$

Let k be a central invertible element of R. So $\alpha : S \to S$ is an automorphisms of S, where $\alpha \left(\begin{pmatrix} a & 0 & 0 & c \\ 0 & a & 0 & 0 \\ 0 & 0 & a & d \\ 0 & 0 & 0 & a \end{pmatrix} \right) = \begin{pmatrix} a & 0 & 0 & kc \\ 0 & a & 0 & 0 \\ 0 & 0 & a & d \\ 0 & 0 & 0 & a \end{pmatrix}$. We show that S is a nil-reversible α-compatible ring.

If

$$A = \begin{pmatrix} a & 0 & 0 & c \\ 0 & a & 0 & 0 \\ 0 & 0 & a & d \\ 0 & 0 & 0 & a \end{pmatrix}, B = \begin{pmatrix} 0 & 0 & 0 & e \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

with $AB = 0$, then $ae = af = 0$, so $ea = fa = 0$, since R is reduced. Therefore $BA = 0$, and so S is nil-reversible.

If in S, for

$$C=\begin{pmatrix} a & 0 & 0 & c\\ 0 & a & 0 & 0\\ 0 & 0 & a & d\\ 0 & 0 & 0 & a\end{pmatrix}, D=\begin{pmatrix} b & 0 & 0 & e\\ 0 & b & 0 & 0\\ 0 & 0 & b & f\\ 0 & 0 & 0 & b\end{pmatrix},$$

$CD=0$, then since R is reduced, $ab=ae=cb=af=db=0$. So $C\alpha(D)=0$. The converse is clear since k is invertible. Therefore S is an α-compatible and nil-reversible ring.

A ring R is said to be *Armendariz* if for polynomials $f(x)=a_0+a_1x+\cdots+a_nx^n$ and $g(x)=b_0+b_1x+\cdots+b_mx^m$ in $R[x]$, $f(x)g(x)=0$ implies $a_ib_j=0$ for each $0\le i\le n$, $0\le j\le m$. This definition was given by M.B. Rege and S. Chhawchharia in [47] using the name Armendariz since E. Armendariz had proved in [2, Lemma 1] that reduced rings satisfied this condition. Also, by D.D. Anderson, V. Camillo [1, Theorem 4], a ring R is Armendariz if and only if so is $R[x]$.

According to A. Moussavi and E. Hashemi [40, Definition 1], a ring R with an endomorphism α and an α-derivation δ is (α,δ)-*skew Armendariz*, if for polynomials $f(x)=a_0+a_1x+\cdots+a_nx^n$ and $g(x)=b_0+b_1x+\cdots+b_mx^m$ in $R[x;\alpha,\delta]$, $f(x)g(x)=0$ implies $a_ix^ib_j=0$ for each $0\le i\le n$, $0\le j\le m$.

Note that an (α,δ)-compatible ring R is (α,δ)-skew Armendariz if and only if for each pair of non-zero polynomials $f(x)=\sum_{i=0}^m a_ix^i, g(x)=\sum_{j=0}^n b_jx^j\in R[x;\alpha,\delta]$ with $f(x)g(x)=0$ then $a_ib_j=0$, for each i,j.

Definition 2.12 ([16, p. 2])**.** A ring R is called (α,δ)-*skew McCoy* (or skew McCoy for short) if for each pair of non-zero polynomials $f(x)=\sum_{i=0}^m a_ix^i, g(x)=\sum_{j=0}^n b_jx^j\in R[x;\alpha,\delta]$ with $f(x)g(x)=0$ then there exists a non-zero element $r\in R$ with $f(x)r=0$.

Theorem 2.13. *Let R be an (α,δ)-compatible, where α is an automorphism of R. If $S=R[x;\alpha,\delta]$ is strongly right AB and $r_S(Y)\neq 0$, then $r_R(Y)\neq 0$, for any $Y\subseteq S$. In particular, R is an (α,δ)-skew McCoy ring.*

Proof. Suppose $Y\neq 0$, and $r_S(Y)\neq 0$. Then $Yh(x)=0$, for $0\neq h(x)=c_0+c_1x+\cdots+c_tx^t\in S$. Here we can set $c_t\neq 0$. If $t=0$, then we are done, and so assume $t\ge 1$. There exists an ideal $0\neq L\subseteq r_S(Y)$ such that $YL=0$, as S is strongly right AB. The rest of the proof is the asme as that of [21, Theorem 1]. □

Corollary 2.14. *The class of McCoy rings includes nil-reversible rings and all rings R such that $R[x]$ is strongly right AB.*

Therefore, we conclude that, nil-reversible rings is a larger class of rings which satisfy the conditions asked by P.P. Nielsen [44, p. 136]. Indeed, nil-reversible rings is a natural class of McCoy rings which includes reversible rings,

CN rings, all rings R such that $R[x]$ is strongly right (or left) AB (and hence all rings R such that $R[x]$ is semi-commutative).

Lemma 2.15. *Let R be a semi-commutative ring with a compatible automorphism α. If $f(x)=\sum_{i=0}^{m} a_i x^i, g(x)=\sum_{j=0}^{n} b_j x^j \in R[x;\alpha]$ with $f(x)g(x)=0$, then $a_0^{n+1}g(x)=0$.*

Proof. We adopt the proof of [7, Lemma 5.4]. Clearly $a_0b_0=0$. Assume by induction that $a_0^{l+1}b_l=0$ for all $l<j$. Since α is a compatible, then we get $\alpha^t(a_0^{l+1})b_l=a_0^{l+1}\alpha^s(b_l)=0$, for integers $s,t,\ l<j$. We can rewrite $f(x)=c_0+xc_1+\cdots+x^m c_m$ for some $c_i\in R$, $1\le i\le m$. The degree j part of the equation $f(x)g(x)=0$ yields $\sum_{i=0}^{j}\alpha^i(c_ib_{j-i})=0$. Multiplying on the left by a_0^j, we have

$$0=\sum_{i=0}^{j} a_0^j\alpha^i(c_ib_{j-i})=a_0^{j+1}b_j.$$

□

Theorem 2.16. *Every right duo ring with a compatible automorphism α, is α-skew McCoy.*

Proof. The proof is the same as that of [7, Theorem 8.2]. □

Let $\mathfrak{C}$ denote the class of rings R which have the property that $R[x]$ is semi-commutative, and let $\mathfrak{D}$ denote the class of rings R which have the property that $R[x]$ is strongly AB. The following diagram shows all implications among these properties (with no other implications holding, except by transitivity):

$$\begin{tikzcd}
 & \textit{right duo} \arrow[r] & \textit{right McCoy} & \\
\textit{commutative} \arrow[ur] \arrow[r] \arrow[dr] \arrow[dd, dashed] & \textit{reversible} \arrow[r] & \textit{nil-reversible} \arrow[r] \arrow[dd, dashed] & \textit{McCoy} \arrow[ul] \\
 & \textit{unit-central} \arrow[r] & \textit{CN} \arrow[u] & \\
\mathfrak{C} \arrow[rr] & & \mathfrak{D} \arrow[uur, dashed] &
\end{tikzcd}$$

We notice that $R[x]$ need not be strongly right AB when R is a strongly right AB (or semi-commutative) ring, as we see in the following:

Example 2.17 ([44, p. 138])**.** Let $k=\mathbb{F}_2\langle a_0,a_1,a_2,a_3,b_0,b_1\rangle$ be the free associative algebra (with 1) over $\mathbb{F}_2$ generated by six indeterminates (as labeled above). Let I be the ideal generated by the following relations:

$$\langle a_0b_0,\ a_0b_1+a_1b_0,\ a_1b_1+a_2b_0,\ a_2b_1+a_3b_0,\ a_3b_1,\ a_0a_j,\ a_3a_j,\ a_1a_j+a_2a_j,\\ b_sb_t, b_sa_j\rangle,$$

$0 \le i,j \le 3; 0 \le s,t \le 1$. Let $R = k/I$. Think of $\{a_0,a_1,a_2,a_3,b_0,b_1\}$ as elements of R satisfying the relations in I, suppressing the bar notation. Put $F(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ and $G(x) = b_0 + b_1x$. The first row of relations in I guarantees that $F(x)G(x) = 0$ in $R[x]$. It is shown in [44, p. 138] that $F(x), G(x) \ne 0$ in $R[x]$. Further, Nielsen demonstrated that R is semi-commutative and so it is a strongly right AB ring. Also he proved that R is left McCoy but not right McCoy. But by Theorem 2.13 we conclude that $R[x]$ is not strongly right AB.

Theorem 2.18. *Let R be an (α,δ)-compatible ring. If R is (α,δ)-skew Armendariz and strongly right AB, then $R[x;\alpha,\delta]$ is strongly right AB.*

Proof. We adopt the proof of [26, Proposition 4.6]. Assume R is strongly right AB and $X \subseteq R[x;\alpha,\delta]$ with $r_{R[x;\alpha,\delta]}(X) \ne 0$ and let C be the set of all coefficients of polynomials in X. Take non-zero $f(x) = a_0 + a_1x + \cdots + a_nx^n \in r_{R[x;\alpha,\delta]}(X)$. Then for any $g(x) = b_0 + b_1x + \cdots + b_mx^m \in X, g(x)f(x) = 0$. Since R is (α,δ)-skew Armendariz and (α,δ)-compatible, $b_ia_j = 0$, for all i,j. Thus $a_j \in r_R(C), 0 \le j \le n$, entailing $r_R(C) \ne 0$. Since R is strongly right AB, there exists a non-zero ideal I of R such that $r_R(C) \supseteq I$. So $CRt = 0$, for each $t \in I$. By (α,δ)-compatibility of R, $XR[x;\alpha,\delta]t = 0$. Therefore $R[x;\alpha,\delta]$ is strongly right AB. □

The following generalizes [26, Proposition 4.6] from the $R[x]$ case.

Proposition 2.19. *Let α be an automorphism and R an (α,δ)-compatible ring. If $R[x;\alpha,\delta]$ is strongly right AB, then R is skew McCoy and strongly right AB.*

Proof. We adopt the proof of [26, Proposition 4.5]. Suppose that $S = R[x;\alpha,\delta]$ is strongly right AB. Let $X \subseteq R$ with $r_R(X) \ne 0$. Note that $r_R(X) = r_S(X) \cap R$. Since $r_R(X) \ne 0$, we get $r_S(X) \ne 0$. But S is strongly right AB, so there is a non-zero ideal L of S such that $r_S(X) \supseteq L$. For every $h(x) = c_0 + c_1x + \cdots + c_tx^t \in L$, $Sh(x)S \subseteq L$. So $XRh(x) \subseteq XSh(x) = 0$. This implies that $XRc_k = 0, 0 \le k \le t$. So $r_R(X) \supseteq Rc_kR$. Therefore R is strongly AB. □

According to [50, Definition 2.1], a ring R is called *left power-serieswise McCoy* if whenever two power-series $f(x) = \sum_0^\infty a_ix^i, g(x) = \sum_0^\infty b_jx^j \in R[[x]]$ satisfy $f(x)g(x) = 0$, then there exists $0 \ne r \in R$ such that $rg(x) = 0$. Power-serieswise McCoy rings are McCoy.

Proposition 2.20. *Let R be a right power-serieswise McCoy ring and $Z_l(R[x])$ be a countable set. If R is strongly right AB then $R[x]$ is strongly right AB.*

Proof. Assume $X \subseteq R[x]$ and $r_{R[x]}(X) \neq 0$. So there exists a countable set I such that $X = \bigcup_{i\in I}\{f_i\} \subseteq R[x]$, with $f_i = a_{i0} + a_{i1}x + \cdots + a_{in_i}x^{n_i} \in R[x]$. Then there exists $0 \neq g(x) \in R[x]$ such that $Xg(x) = 0$. Then we have $F(x)g(x) = 0$ where $F(x) = f_1 + f_2x^{n_1+1} + ... + f_{n_k}x^{n_1+\cdots+n_{k-1}+1} + ...$. Since R is power-serieswise right McCoy, there exists $0 \neq c \in R$ such that $F(x)c = 0$. So $f_ic = 0$ and hence $a_{ij}c = 0$. Since R is strongly right AB, there exists an ideal J such that $a_{ij}J = 0$ for every i, j. For every $0 \neq d \in J$, we have $a_{ij}Rd = 0$ for any $i, j \in I$. So $f_iRd = 0$ for every $i \in I$. Since $X = \bigcup_{i\in I}\{f_i\}$, $XRd = 0$. By [20, Lemma 2.1], $XR[x]d = 0$. So $R[x]$ is strongly right AB. □

Definition 2.21 ([8, Definition 2])**.** A ring R is said to have *the right finite intersection property* (simply, *right FIP*) if, for any subset X of R, there exists a finite subset X_0 of X such that $r_R(X) = r_R(X_0)$.

Proposition 2.22. *Let α be an automorphism and R be an α-compatible right duo ring. If $R[x;\alpha]$ has right FIP, then $R[x;\alpha]$ is strongly right AB.*

Proof. Assume that $X \subseteq R[x;\alpha]$ and $r_{R[x;\alpha]}(X) \neq 0$. Then there exists a finite subset X_0 of X such that $r_{R[x;\alpha]}(X) = r_{R[x;\alpha]}(X_0)$, as $R[x;\alpha]$ has right FIP. Assume that $X_0 = \{f_1, f_2, \ldots, f_k\}$, where $f_i = a_{i0}+a_{i1}x+\cdots+a_{in_i}x^{n_i}$, $1 \leq i \leq k$ with positive integers n_i. Take $F(x) = f_1 + x^{n_1+1}f_2 + \cdots + x^{n_1+\cdots+n_{k-1}+1}f_k$. Then for some $0 \neq g(x) \in R[x;\alpha]$ we have $F(x)g(x) = 0$. By 2.16 right duo rings are α-skew McCoy, so there exists $0 \neq c \in R$ such that $a_{ij}x^jc = 0$, $1 \leq i \leq k, 0 \leq j \leq n_i$. Since R is strongly right AB, there exist an ideal J such that $a_{ij}J = 0$. For $0 \neq d \in J$, we have $a_{ij}Rd = 0$, $1 \leq i \leq k, 0 \leq j \leq n_i$. Thus we have $X_0R[x;\alpha]d = 0$, by α-compatibility of R. So $0 \neq R[x;\alpha]dR[x;\alpha] \subseteq r_{R[x;\alpha]}(X)$ and hence $R[x;\alpha]$ is strongly right AB. □

From the preceding results, it is natural to raise the following:

Question. *If R is a right duo ring, then does the polynomial ring $R[x]$ is strongly right AB?*

C. Faith [14, Abstract] called a ring R *right zip* provided that if the right annihilator $r_R(X)$ of a subset X of R is zero, then there exists a finite subset $Y \subseteq X$ such that $r_R(Y) = 0$. The concept of zip rings was initiated by J.M. Zelmanowitz [51] and appeared in various papers. Zelmanowitz stated that any ring satisfying the descending chain condition on right annihilators is a right zip ring, but the converse does not hold. Extensions of zip rings were studied by several authors. J.A. Beachy and W.D. Blair [4, Proposition 1.9] showed that if R is a commutative zip ring, then the polynomial ring $R[x]$ over R is a zip ring. By W. Cortes [10, Theorem 2.9], if R is a (α, δ)-compatible and (α, δ)-skew Armendariz ring, where α is an endomorphism of R and δ is an α-derivation, then R is left zip if and only if $R[x;\alpha,\delta]$ is left zip.

Proposition 2.23. *Let R be an α-compatible and α-skew McCoy ring. If R is strongly right AB and right zip, then $R[x;\alpha]$ is strongly right AB.*

Proof. Let $S = R[x;\alpha]$, $r_S(X) \neq 0$, where $X \subseteq S$. Assume, on the contrary, that $r_S(XRS) = 0$. Since RS is right zip, there exists a finite subset $X_0 = \{f_1h_1, f_2h_2, \ldots, f_kh_k\} \subseteq XRS$, where $f_i = a_{i0} + a_{i1}x + \cdots + a_{in_i}x^{n_i} \in X$, $h_i \in S$, $1 \leq i \leq k$, such that $r_S(X_0) = 0$. Since $0 \neq r_S(X) \subseteq r_S(\{f_1, \ldots, f_k\})$. So $\{f_1, \ldots, f_k\}g(x) = 0$, for some $0 \neq g(x) \in S$. Put $F(x) = f_1 + f_2x^{n_1+1} + \cdots + f_kx^{n_1+\cdots+n_{k-1}+1}$. Since $f_ig(x) = 0$ for every $1 \leq i \leq k$, $F(x)g(x) = 0$. Since R is α-skew McCoy, there exists $0 \neq c \in R$ such that $F(x)c = 0$. So $f_i\alpha^i(c) = 0$. Therefore $a_{ij}c = 0$, $1 \leq i \leq k, 0 \leq j \leq n_i$, as R is α-compatible. Since R is strongly right AB, there exists an ideal J such that $a_{ij}J = 0$ for every i, j. For every $0 \neq d \in J$, $a_{ij}Rd = 0$, $1 \leq i \leq k, 0 \leq j \leq n_i$. Since R is α-compatible, $f_iSd = 0$. That is contradiction since we assume $r_{R[x;\alpha]}(X_0) = 0$. □

Proposition 2.24. *Let R be a δ-compatible and δ-skew McCoy ring. If R is strongly right AB and right zip, then $R[x;\delta]$ is strongly right AB.*

Proof. The proof begins as in that of Proposition 2.23 except that here $F(x) = f_1 + x^{n_1+1}f_2 + \cdots + x^{n_1+\cdots+n_{k-1}+1}f_k$. Since R is δ-skew McCoy, there exists $0 \neq c \in R$ such that $F(x)c = 0$. So $f_kc = 0$, hence $a_{kn_k}c = 0$. Since R is δ-compatible, $a_{kn_k}x^{n_k}c = a_{kn_k}cx^{n_k} + n_ka_{kn_k}\delta(c)x^{n_k-1} + \cdots + a_{kn_k}\delta^{n_k}(c) = 0$. This implies $f_kc = (a_{k0} + \cdots + a_{kn_{k-1}}x^{n_k-1})c = 0$. By similar argument, we have $a_{kj}x^{kj}c = 0, 0 \leq j \leq n_k$. Therefore $F(x)c = f_1 + x^{n_1+1}f_2 + \cdots + x^{n_1+\cdots+n_{k-2}+1}f_{k-1} = 0$. We finally obtain that $f_ic = 0$. Therefore $a_{ij}c = 0$, $1 \leq i \leq k, 0 \leq j \leq n_i$, as R is δ-compatible. Since R is strongly right AB, there exists an ideal J such that $a_{ij}J = 0$ for every i, j. For every $0 \neq d \in J$, $a_{ij}Rd = 0$, $1 \leq i \leq k, 0 \leq j \leq n_i$. Since R is δ-compatible, $f_iR[x;\delta]d = 0$. That is contradiction since we assume $r_{R[x;\alpha]}(X_0) = 0$. □

Let R be a ring and σ denotes an endomorphism of R with $\sigma(1) = 1$. We denote the identity matrix and unit matrices in the full matrix ring $M_n(R)$, by I_n and E_{ij}, respectively. In [35], T.K. Lee, Y. Zhou introduced a subring of the *skew triangular matrix* ring as a set of all triangular matrices $T_n(R)$, with addition pointwise and a new multiplication subject to the condition $E_{ij}r = \sigma^{j-i}(r)E_{ij}$. So $(a_{ij})(b_{ij}) = (c_{ij})$, where $c_{ij} = a_{ii}b_{ij} + a_{i,i+1}\sigma(b_{i+1,j}) + \cdots + a_{ij}\sigma^{j-i}(b_{jj})$, for each $i \leq j$ and denoted it, by $T_n(R,\sigma)$.

The subring of the skew triangular matrices with constant diagonals is denoted by $T(R,n,\sigma)$. We can denote $A = (a_{ij}) \in T(R,n,\sigma)$ by $(a_{11}, \ldots, a_{1n})$.

Then

$$T(R,n,\sigma)=\left\{\begin{pmatrix} a_1 & a_2 & \cdots & a_n\\ 0 & a_1 & \ddots & \vdots\\ \vdots & \vdots & \ddots & a_2\\ 0 & 0 & \cdots & a_1\end{pmatrix} \mid a_i\in R,\ 1\le i\le n.\right\}$$

is a ring with addition pointwise and multiplication given by:

$$\begin{pmatrix} a_1 & a_2 & \cdots & a_n\\ 0 & a_1 & \ddots & \vdots\\ \vdots & \vdots & \ddots & a_2\\ 0 & 0 & \cdots & a_1\end{pmatrix}\begin{pmatrix} b_1 & b_2 & \cdots & b_n\\ 0 & b_1 & \ddots & \vdots\\ \vdots & \vdots & \ddots & b_2\\ 0 & 0 & \cdots & b_1\end{pmatrix}=$$

$$\begin{pmatrix} a_1b_1 & a_1b_2+a_2\sigma(b_1) & \cdots & a_1b_n+a_2\sigma(b_{n-1})+\cdots+a_n\sigma^{n-1}(b_1)\\ 0 & a_1b_1 & \ddots & \vdots\\ \vdots & & \ddots & a_1b_2+a_2\sigma(b_1)\\ 0 & 0 & \cdots & a_1b_1\end{pmatrix}.$$

In the special case, when $\sigma=id_R$, we use $T(R,n)$ instead of $T(R,n,\sigma)$. On the other hand, there is a ring isomorphism $\varphi:R[x;\sigma]/(x^n)\to T(R,n,\sigma)$, given by $\varphi(\sum_{i=0}^{n-1}a_ix^i)=(a_0,a_1,\dots,a_{n-1})$. So $T(R,n,\sigma)\cong R[x;\sigma]/(x^n)$, where (x^n) is the ideal generated by x^n.

Let α and σ be endomorphisms of R and δ is an α-derivation, with $\alpha\sigma=\sigma\alpha$ and $\delta\sigma=\sigma\delta$. The endomorphism α of R is extended to the endomorphism $\bar{\alpha}:T(R,n,\sigma)\to T(R,n,\sigma)$ defined by $\bar{\alpha}((a_{ij}))=(\alpha(a_{ij}))$ and the α-derivation δ of R is also extended to $\bar{\delta}:T(R,n,\sigma)\to T(R,n,\sigma)$ defined by $\bar{\delta}((a_{ij}))=(\delta(a_{ij}))$.

Theorem 2.25. *Let R be a ring. Assume α and σ are rigid endomorphisms and δ an α-derivation of R such that $\alpha\sigma=\sigma\alpha$ and $\delta\sigma=\sigma\delta$. Then $T(R,n,\sigma)$ is an $(\overline{\alpha},\overline{\delta})$-compatible, nil-reversible and $(\bar{\alpha},\bar{\delta})$-skew Armendariz ring.*

Proof. Let $A=(a_0,\cdots,a_{n-1})\in T(n,R,\sigma)$ and $B=(0,b_1,\cdots,b_{n-1})$ be an element of $nil(T(n,R,\sigma))$ such that $AB=0$. Thus $a_ib_j=0$, for each $0\le i,j\le n-1$, by [16, Theorem 2.2] and so $b_ja_i=0$, since R is reduced. Hence $BA=0$, and so $T(R,n,\sigma)$ is nil-reversible. By [16, Theorem 2.3], $T(R,n,\sigma)$ is $(\overline{\alpha},\overline{\delta})$- compatible and by [16, Theorem 2.8] it is $(\bar{\alpha},\bar{\delta})$-skew Armendariz. □

Proposition 2.26. *Let R be a ring and Δ be a multiplicatively closed subset of R consisting of central regular elements. Then R is nil-reversible if and only if $\Delta^{-1}R$ is nil-reversible.*

Proof. Let $\alpha\beta = 0$ with $\alpha = u^{-1}a, \beta = v^{-1}b, u, v \in \Delta$ and $a \in R, b \in nil(R)$. Since Δ is contained in the center of R, we have $0 = \alpha\beta = u^{-1}av^{-1}b = (u^{-1}v^{-1})ab = (uv)^{-1}ab$ and $ab = 0$. But R is nil-reversible by supposition, so $ba = 0$ and we have $\beta\alpha = v^{-1}bu^{-1}a = (vu)^{-1}ba = 0$; hence $\Delta^{-1}R$ is nil-reversible. □

Lemma 2.27. *For a ring R, $R[x;\alpha]$ is nil-reversible if and only if $R[x, x^{-1};\alpha]$ is nil-reversible.*

Proof. Let $\Delta = \{1, x, x^2, \cdots\}$. Then Δ is a multiplicatively closed subset of central regular elements in $R[x;\alpha]$. Since $R[x, x^{-1};\alpha] = \Delta^{-1}R[x;\alpha]$, it follows that $R[x, x^{-1};\alpha]$ is nil-reversible by Proposition 2.26. □

Proposition 2.28. *Let R be an α-compatible ring. If R is α-skew Armendariz, then the following statements are equivalent:*
(1) *R is nil-reversible.*
(2) *$R[x;\alpha]$ is nil-reversible.*
(3) *$R[x, x^{-1};\alpha]$ is nil-reversible.*

Proof. By Lemma 2.27 and the fact that the class of nil-reversible rings is closed under subring, it suffices to prove (1) $\Rightarrow$ (2). Let $f(x) = \sum_{i=0}^{n} a_i x^i \in R[x;\alpha]$ and $g(x) = \sum_{j=0}^{m} b_j x^j \in nil(R[x;\alpha])$. By Lemma 2.7 and Proposition 2.3, each $b_j \in nil(R), 0 \le j \le m$. Since R is skew Armendariz and α-compatible, $a_i b_j = 0$ for every $0 \le i \le n, 0 \le j \le m$. Therefore $b_j a_i = 0$, since R is nil-reversible. Consequently we have $g(x)f(x) = 0$, since R is α-compatible. So $R[x;\alpha]$ is nil-reversible. □

The endomorphism α on R can be extended to the skew polynomial ring $R[x;\alpha]$ by $\bar{\alpha}(\sum_{i=0}^{n} a_i x^i) = \sum_{i=0}^{n} \alpha(a_i)x^i$; and the derivation δ of R is also extended to the differential polynomial ring $R[x;\delta]$ by $\bar{\delta}(\sum_{i=0}^{n} a_i x^i)=\sum_{i=0}^{n} \delta(a_i)x^i$ by $\delta(f) = fx - xf$.

Theorem 2.29. *Let R be a α-skew Armendariz ring. Then R is nil-reversible α-compatible if and only if $R[x;\alpha]$ is nil-reversible $\bar{\alpha}$-compatible.*

Proof. Let R be nil-reversible. Let $f(x) = \sum_{i=0}^{n} a_i x^i \in R[x;\alpha]$ and $g(x) = \sum_{j=0}^{m} b_j x^j \in nil(R[x;\alpha])$ with $f(x)g(x) = 0$. As R is 2-primal by Lemma 2.3, $b_j \in nil(R)$ for each $0 \le j \le m$, by Lemma 2.7. Since R is α-skew Armendariz and α-compatible, $a_i b_j = 0$ for every $0 \le i \le n, 0 \le j \le m$. Therefore $b_j \alpha^i(a_i) = 0$, since R is nil-reversible and α-compatible. Consequently we have $g(x)f(x) = 0$. So $R[x;\alpha]$ is nil-reversible. We have also $a_i \alpha^i(b_j) = 0$ if and only if $f(x)\bar{\alpha}(g(x)) = 0$, so $R[x;\alpha]$ is $\bar{\alpha}$-compatible.. □

Theorem 2.30. *Let R be a δ-skew Armendariz ring with a derivation δ. Then R is nil-reversible δ-compatible if and only if $R[x;\delta]$ is nil-reversible $\bar{\delta}$-compatible.*

Proof. Let R be nil-reversible. Let $f(x)=\sum_{i=0}^{n}a_ix^i\in R[x;\delta]$ and $g(x)=\sum_{j=0}^{m}b_jx^j\in nil(R[x;\delta])$ with $f(x)g(x)=0$. By Lemma 2.7, each $b_j\in nil(R), 0\leq j\leq m$, as R is 2-primal by Lemma 2.3. Since R is δ-skew Armendariz, $a_i\delta^l(b_j)=0$ for every $0\leq i\leq n, 0\leq j\leq m, l\geq 0$. Therefore $b_ja_i=0$ and so $b_j\delta^l(a_i)=0, l\geq 0$, since R is nil-reversible and δ-compatible. Consequently we have $g(x)f(x)=0$. So $R[x;\delta]$ is nil-reversible. We have also $a_i\delta^l(b_j)=0, l\geq 0$, so $f(x)\bar{\delta}(g(x))=0$.

□

For any ring R, the triangular matrix ring $T_n(R)$ is not nil-reversible (and hence not reversible). Consider $e_{13}\in nil(T_n(R))$. Then we have $e_{33}e_{13}=0$ but $e_{13}e_{33}\neq 0$.

The following example [25, Example 2] shows that if R is a nil-reversible ring, then $R[x]$ need not be nil-reversible.

Example 2.31 ([25, Example 2])**.** Let $\mathbb{Z}_2$ be the field of integers modulo 2 and assume that $A=\mathbb{Z}_2[a_0,a_1,a_2,b_0,b_1,b_2,c]$ is the free algebra of polynomials with zero constant terms in noncommuting indeterminates a_0,a_1,a_2,b_0,b_1,b_2, c over $\mathbb{Z}_2$. Note that A is a ring without identity and consider an ideal of the ring $\mathbb{Z}_2+A$, say I, generated by

$$a_0b_0, a_0b_1+a_1b_0, a_0b_2+a_1b_1+a_2b_0, a_1b_2+a_2b_1, a_2b_2, a_0rb_0, a_2rb_2,$$
$$b_0a_0, b_0a_1+b_1a_0, b_0a_2+b_1a_1+b_2a_0, b_1a_2+b_2a_1, b_2a_2, b_0ra_0, b_2ra_2,$$
$$(a_0+a_1+a_2)r(b_0+b_1+b_2); (b_0+b_1+b_2)r(a_0+a_1+a_2), \textit{ and } r_1r_2r_3r_4,$$

where $r,r_1,r_2,r_3,r_4\in A$. Then clearly $A^4\in I$. Next let $R=(\mathbb{Z}_2+A)/I$ and consider $R[x]\cong(\mathbb{Z}_2+A)[x]/I[x]$. Then R is reversible [31, Example 2.1] and so it is nil-reversible. Now consider $f(x)=a_0+a_1x+a_2x^2$, $g(x)=b_0+b_1x+b_2x^2$. Obviously $C_f, C_g\in nil(R)$, so $f,g\in nil(R[x])$. We have $f(x)g(x)=0$. But $a_0cb_1+a_1cb_0\notin I$, so $(a_0+a_1x+a_2x^2)c(b_0+b_1x+b_2x^2)\notin I[x]$. Thus $R[x]$ is not nil-reversible.

Example 2.32. Let α and σ be rigid endomorphisms a ring R with $\alpha\sigma=\sigma\alpha$. Then $T(R,n,\sigma)$ is $\bar{\alpha}$-compatible, nil-reversible and skew Armendariz, by Theorem 2.25, and that $T(R,n,\sigma)[x;\bar{\alpha}]$ is an $\bar{\alpha}$-compatible and nil-reversible ring, by Theorem 2.30.

3. Rings with property (A)

J.A. Huckaba and J.M. Keller [24] introduced the following: a commutative ring R has Property (A) if every finitely generated ideal of R consisting entirely of zero-divisors has a nonzero annihilator. Property (A) was originally studied by Y. Quentel [46]. Y. Quentel used the term Condition (C) for Property (A). The class of commutative rings with Property (A) is quite large. For example, Noetherian rings [28, p. 56], rings whose prime ideals are maximal [18], the polynomial ring $R[x]$ and rings whose classical ring of quotients are von Neumann regular [18], are examples of rings with Property (A). Using Property (A), G. Hinkle, J.A. Huckaba [19] extend the concept Kronecker function rings from integral domains to rings with zero divisors. Many authors have studied commutative rings with Property (A), and have obtained several results which are useful studying commutative rings with zero-divisors. C.Y. Hong, N.K. Kim, Y. Lee and S.J. Ryu [22] extended the notion of Property (A) to noncommutative rings:

Definition 3.1 ([22, Definition 1.1])**.** A ring R has *right* (*left*) *Property* (A) if for every finitely generated two-sided ideal $I \subseteq Z_l(R)$ (resp. $Z_r(R)$), there exists nonzero $a \in R$ (resp. $b \in R$) such that $Ia = 0$ (resp. $bI = 0$). A ring R is said to have *Property* (A) if R has the right and left Property (A).

Definition 3.2 ([42, Definition 2.1])**.** A ring R with a monomorphism α, is called α*-weakly rigid* if for each $a, b \in R, aRb = 0$ if and only if $a\alpha(Rb) = 0$.

A ring R with a derivation δ is called δ*-weakly rigid* if for each $a, b \in R, aRb = 0$ implies $a\delta(b) = 0$.

Lemma 3.3 ([42, Lemma 3.1])**.** *Let R be an α-weakly rigid ring. Then for each $a, b \in R$ and positive integers $i, j, aRb = 0$ if and only if $\alpha^i(a)R\alpha^j(b) = 0$.*

Lemma 3.4 ([42, Lemma 3.2])**.** *Let R be a δ-weakly rigid ring. Then for each $a, b \in R$ and positive integers $i, j, aRb = 0$ implies $aR\delta^j(b) = 0$.*

Let R be an α-rigid ring with ACC on right annihilators. So R is (α, δ)-compatible and reduced. The subring of the triangular matrices with constant main diagonal is denoted by $S(R, n)$. By [42, Theorem 2.9], $S(R, n)$ is $(\overline{\alpha}, \overline{\delta})$-weakly rigid and by [26, Theorem 2.2], $S(R, n)$ is strongly right AB. According to [22, Corollary 1.7, Theorem 2.1], $S(R, n)$ has right Property (A).

We also note that the endomorphism α on R can be extended to the skew polynomial ring $R[x; \alpha]$ by $\alpha(\sum_{i=0}^{n} a_i x^i) = \sum_{i=0}^{n} \alpha(a_i)x^i$.

Theorem 3.5. *Let R be an α-weakly rigid ring and $f(x) \in S = R[x; \alpha]$. If $r_S(f(x)S) \neq 0$ then $r_S(f(x)S) \cap R \neq 0$.*

Proof. We apply the method of Hirano in the proof of [20, Theorem 2.2], see also [30]. Let $f(x) = a_0 + a_1x + \cdots + a_mx^m \in R[x; \alpha]$. If $deg(f(x)) = 0$ or $f(x) = 0$, then the assertion is clear. Let $deg(f(x)) = m > 0$. Assume, to the contrary, that $r_{R[x;\alpha]}(f(x)R[x; \alpha]) \cap R = 0$ and let $g(x) = b_0 + b_1x + \cdots + b_nx^n$ be a polynomial of minimal degree in $r_{R[x;\alpha]}(f(x)R[x; \alpha])$, such that $b_n \neq 0$. Since $f(x)R[x; \alpha]g(x) = 0$, we have $f(x)Rg(x) = 0$. Therefore $a_mR\alpha^m(b_n) = 0$. Since R is α-weakly rigid, we have $a_mR\alpha^k(b_n) = 0$ for all $k \geq 0$. This implies that

$$a_mSg(x) = a_mS(b_{n-1}x^{n-1} + \cdots + b_0),$$

and

$$0 = f(x)Sg(x) \supseteq f(x)S(a_m Sg(x)) = f(x)S(a_m S(b_{n-1}x^{n-1} + \cdots + b_0)).$$

So $a_m R(b_{n-1}x^{n-1} + \cdots + b_0) \subseteq r_S(f(x)S)$. Since $g(x)$ is of minimal degree in $r_S(f(x)S)$, forces $a_m R(b_{n-1}x^{n-1} + \cdots + b_0) = 0$. We moreover obtain $a_m R b_j = 0$ for all $0 \leq j \leq n$. Since R is α-weakly rigid, we have $a_m R\alpha^k(b_j) = 0, k \geq 0$. Therefore $(a_{m-1}x^{m-1} + \cdots + a_0)Sg(x) = 0$. So $(a_{m-1}x^{m-1} + \cdots + a_0)Rg(x) = 0$. Hence $a_{m-1}Rb_n = 0$, since R is α-weakly rigid. By repeat the same computation, we obtain

$$a_{m-1}x^{m-1}Sg(x) = a_{m-1}x^{m-1}S(b_{n-1}x^{n-1} + \cdots + b_0) \subset$$
$$a_{m-1}S(b_{n-1}x^{n-1} + \cdots + b_0) = 0.$$

So $a_{m-1}x^{m-1}Rg(x) = 0$, and so $a_{m-1}R\alpha^k(b_j) = 0$ for each $0 \leq j \leq n, k \geq 0$, since R is α-weakly rigid. By repeat the same computation we obtain that $a_i x^i Rg(x) = 0$. Since R is α-weakly rigid $a_i R\alpha^k(b_j) = 0$ for all i, j and $k \geq 0$. So $a_i x^i R x^k b_j = 0$ for all i, j. This implies $a_i x^i S b_j = 0$ for all j, and so $f(x)Sb_j = 0$ proving our claim. □

Proposition 3.6. *Let R be an α-weakly rigid ring. Then $S = R[x; \alpha]$ has right Property (A) if and only if whenever $f(x)S \subseteq Z_l(S)$, $r_S(f(x)S) \neq 0$.*

Proof. We adopt the proof of [22, Lemma 2.8]. Let $I = \sum_{i=1}^{k} R[x;\alpha]f_i(x) R[x;\alpha] \subseteq Z_l(R[x;\alpha])$, where $f_i(x) = a_{i0} + a_{i1}x + \cdots + a_{in_i}x^{n_i}$. Put $g(x) = f_1 + x^{n_1+1}f_2 + \cdots + x^{n_1+\cdots+n_{k-1}+1}f_{n_k} \in I$. Thus $g(x)R[x;\alpha] \subseteq I$. By hypothesis, $r_{R[x;\alpha]}(g(x)R[x;\alpha]) = r_{R[x;\alpha]}(R[x;\alpha]g(x)R[x;\alpha]) \neq 0$. So $r_{R[x;\alpha]}(R[x;\alpha]g(x) R[x;\alpha]) \cap R \neq 0$, by Theorem 3.5. Thus for some nonzero $r \in R$, $(R[x;\alpha]g(x) R[x;\alpha])r = 0$. Since $Rg(x)R \subseteq R[x;\alpha]g(x)R[x;\alpha]$ and R is α-weakly rigid, we have $Ra_{ij}Rr = 0$. Thus $Rx^k a_{ij} x^j R x^t r = 0$, since R is α-weakly rigid. So $Ir = (\sum_{i=1}^{k} R[x;\alpha]f_i(x)R[x;\alpha])r = 0$. Therefore $R[x;\alpha]$ has right Property (A). The converse is clear. □

Theorem 3.7. *Let R be an α-compatible ring. If R is strongly right AB and α-skew McCoy, then $R[x;\alpha]$ has right Property (A).*

Proof. Let $X = f(x)R[x;\alpha] \subseteq Z_l([x;\alpha])$, where $f(x) = a_0 + a_1x + \cdots + a_nx^n$. By hypothesis, there exists $g(x) \in R[x;\alpha]$ such that $f(x)g(x) = 0$. Since R is α-compatible and α-skew McCoy, there exists $0 \neq c \in R$ such that $a_ic = 0$, for each i. Since R is strongly right AB, there exists an ideal J such that $a_iJ = 0$ for each i. So for every $0 \neq d \in J$, $a_iRd = 0$, for each i. Since R is α-compatible, we have $fR[x;\alpha]d = 0$. This implies that $R[x;\alpha]$ has right Property (A), by Proposition 3.6. □

Corollary 3.8. *For an α-compatible ring R with any of the conditions:*

(i) *right duo;*
(ii) *CN-ring;*

$R[x;\alpha]$ has right Property (A).

This generalizes [31, Proposition 2.10] and [31, Corollary 2.11].

Theorem 3.9. *Let R an α-compatible ring for an automorphism α of R. If $R[x;\alpha]$ is strongly right AB, then $R[x;\alpha]$ has right Property (A).*

Proof. Let $X = f(x)R[x;\alpha] \subseteq Z_l([x;\alpha])$, where $f(x) = a_0 + a_1x + \cdots + a_nx^n$. By hypothesis, there exists $g(x) \in R[x;\alpha]$ such that $f(x)g(x) = 0$. Since R is α-compatible and α-skew McCoy, there exists $0 \neq c \in R$ such that $a_ic = 0$, for each i. Since $R[x;\alpha]$ is strongly right AB, by 2.18, R is strongly right AB, so there exists an ideal J such that $a_iJ = 0$. So for every $0 \neq d \in J$, $a_iRd = 0$, for each i. Since R is α-compatible, we have $fR[x;\alpha]d = 0$. This implies that $R[x;\alpha]$ has right Property (A), by Proposition 3.6. □

Corollary 3.10. *If $R[x]$ is a strongly right AB, then $R[x]$ has right Property (A).*

Theorem 3.11. *Let R be a δ-weakly rigid ring and $f(x) \in S = R[x;\delta]$. If $r_S(f(x)S) \neq 0$ then $r_S(f(x)S) \cap R \neq 0$.*

Proof. Let $f(x) = a_0 + a_1x + \cdots + a_mx^m$. If $deg(f(x)) = 0$ or $f(x) = 0$, then the assertion is clear. Let $deg(f(x)) = m > 0$. Assume, to the contrary, that $r_{R[x;\delta]}(f(x)R[x;\delta]) \cap R = 0$ and let $g(x) = b_0 + b_1x + \cdots + b_nx^n \in r_{R[x;\delta]}(f(x)R[x;\delta])$ of minimal degree, with $b_n \neq 0$. From $f(x)R[x;\delta]g(x) = 0$ we get $f(x)Rg(x) = 0$. Therefore $a_mRb_n = 0$. Since R is δ-weakly rigid, $a_m\delta^k(Rb_n) = 0$ for all $k \geq 0$. So $a_mx^mRb_n = a_mRb_nx^m + ma_m\delta(Rb_n)x^{m-1} + \cdots + a_m\delta^m(Rb_n) = 0$. This implies

$$0 = f(x)Sg(x) \supseteq f(x)S(a_mSg(x)) = f(x)S(a_mS(b_{n-1}x^{n-1} + \cdots + b_0)).$$

So $a_mR(b_{n-1}x^{n-1} + \cdots + b_0) \subseteq r_S(f(x)S)$. Since $g(x)$ is of minimal degree in $r_S(f(x)S)$, $a_mR(b_{n-1}x^{n-1} + \cdots + b_0) = 0$. We moreover obtain $a_mRb_j = 0$ for all $0 \leq j \leq n$. Since R is δ-weakly rigid, we have $a_m\delta^k(Rb_j) = 0, k \geq 0$. By similar argument $a_mx^mRb_j = 0$. Therefore $(a_{m-1}x^{m-1} + \cdots + a_0)Sg(x) = 0$. Thus $a_{m-1}Rb_n = 0$, so $a_{m-1}\delta^k(Rb_n) = 0$ for all $k \geq 0$. So $a_{m-1}x^{m-1}Rb_n = a_{m-1}Rb_nx^{m-1} + (m-1)a_{m-1}\delta(Rb_n)x^{m-2} + \cdots + a_{m-1}\delta^{m-1}(Rb_n) = 0$. By repeat the same computation, we obtain $a_iRb_j = 0$, for each i, j. Since R is δ-weakly rigid, we have $a_iR\delta^k(b_j) = 0$ for all i, j and $k \geq 0$. So $a_ix^iRx^kb_j = 0, k \geq 0$. Therefore $f(x)Sb_j = 0$ for all j, proving our claim. □

Proposition 3.12. *Let R be a δ-weakly rigid ring. Then $S = R[x;\delta]$ has right Property (A) if and only if whenever $f(x)S \subseteq Z_l(S)$, $r_S(f(x)S) \neq 0$.*

Proof. Let $I = \sum_{i=1}^{k} R[x;\delta]f_i(x)R[x;\delta]$ be a subset of $Z_l(R[x;\delta])$, where $f_i(x) = a_{i0} + a_{i1}x + \cdots + a_{in_i}x^{n_i}$. Put $g(x) = f_1 + f_2x^{n_1+1} + \cdots + f_kx^{n_1+\cdots+n_{k-1}+1} \in I$. Thus $g(x)R[x;\delta] \subseteq I$. We have $r_{R[x;\delta]}(g(x)R[x;\delta]) = r_{R[x;\delta]}(R[x;\delta]g(x)R[x;\delta])$

$\neq 0$. So $r_{R[x;\delta]}(R[x;\delta]g(x)R[x;\delta]) \cap R \neq 0$, by Theorem 3.11. Hence for some nonzero $c \in R$, $(R[x;\delta]g(x)R[x;\delta])c = 0$. So $Rg(x)Rc = 0$ and $Rf_{n_k}Rc = 0$. Hence $Ra_{kn_k}Rc = 0$ and so $Ra_{kn_k}R\delta^l(c) = 0$, for each $l \geq 0$, since R is δ-weakly rigid. So $Ra_{kn_k}x^{n_k}Rc = Ra_{kn_k}Rcx^{n_k} + n_kRa_{kn_k}\delta(Rc)x^{n_k-1} + \cdots + Ra_{kn_k}\delta^{n_k}(Rc) = 0$. Therefore $Rf_kRc = R(a_{k0} + a_{k1}x + \cdots + a_{kn_{k-1}}x^{n_{k-1}})Rc = 0$. By a similar argument we obtain $Ra_{kn_j}R\delta^l(c) = 0, l \geq 0$ and $Ra_{kn_j}x^{n_j}Rc = 0$, for each $0 \leq j \leq n_k$. So $Rx^sa_{kn_j}x^{n_j}Rx^lc = 0$ for each $l, s \geq 0$ and $0 \leq j \leq n_k$. Therefore $R[x;\delta]f_kR[x;\delta]c = 0$. Thus $0 = Rg(x)Rc = R(f_1 + f_2x^{n_1+1} + \cdots + f_{n_{k-1}}x^{n_1+\cdots+n_{k-2}+1})Rc$. Similarly we deduce that $R[x;\delta]f_{k-1}R[x;\delta]c = 0$. We finally obtain that $R[x;\delta]f_iR[x;\delta]c = 0$, for each $0 \leq i \leq k$. Therefore $R[x;\delta]$ has right Property (A). The converse is clear. □

Theorem 3.13. *Let R be a δ-compatible ring. If R is strongly right AB and δ-skew McCoy, then $R[x;\delta]$ has right Property (A).*

Proof. Let $X = f(x)R[x;\delta] \subseteq Z_l([x;\delta])$, where $f(x) = a_0 + a_1x + \cdots + a_nx^n$. By hypothesis, there exists $g(x) \in R[x;\delta]$ such that $f(x)g(x) = 0$. Since R is δ-skew McCoy and δ-compatible, there exists $0 \neq c \in R$ such that $a_ic = 0$, for each i. Since R is strongly right AB, there exists an ideal J such that $a_iJ = 0$. For each $0 \neq d \in J$, we have $a_iRd = 0$, for each i. Since R is δ-compatible, $fR[x;\delta]d = 0$. This implies that $R[x;\delta]$ has right Property (A), by Proposition 3.12. □

Theorem 3.14. *Let R be a δ-compatible ring. If $R[x;\delta]$ is strongly right AB, then $R[x;\delta]$ has right Property (A).*

Proof. Let $X = f(x)R[x;\delta] \subseteq Z_l([x;\delta])$, where $f(x) = a_0 + a_1x + \cdots + a_nx^n$. By hypothesis, there exists $g(x) \in R[x;\delta]$ such that $f(x)g(x) = 0$. Since R is δ-skew McCoy and δ-compatible, there exists $0 \neq c \in R$ such that $a_ic = 0$, for each i. Since R is strongly right AB, by 2.18, there exists an ideal J such that $a_iJ = 0$. For each $0 \neq d \in J$, we have $a_iRd = 0$, for each i. Since R is δ-compatible, $fR[x;\delta]d = 0$. This implies that $R[x;\delta]$ has right Property (A), by Proposition 3.12. □

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